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Magnetized Anisotropic Bianchi Type-VI Cosmological Model Containing Dark Energy

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Abstract : *The paper summarizes the Bianchi Type VI cosmological model with electromagnetic field. Taking exponential scale factor and variable Lambda parameter the exact solutions of the field equations and different cosmological parameters have been found. Magnetized anisotropic model with dark energy has been found.*
Keywords: – *Bianchi Type VI, Electromagnetic field, Dark energy, Variable Lambda, Accelerated expansion*

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I. Introduction

The observational data obtained from various experiments like SNeIa, the CMB radiation anisotropies, LSS and X-ray experiments [1-8] indicates the discovery of accelerated expansion of the present day universe. Study of Bianchi type models shows that the models contain isotropic special cases and they permit arbitrarily small anisotropic levels at some instant of cosmic times. Bianchi type cosmological models are important due to their homogenous and anisotropic nature, from the theoretical point of view also, anisotropic universe has a general significance than isotropic models. The simplicity of the field equation made Bianchi space time useful in construction models of spatially homogenous and anisotropic cosmologies. Ellis and Mac Callum [9] obtained solutions of Einstein's field equations for a Bianchi type VI space-time in the case of a stiff-fluid. Collins [10] and Ruban [11] have also presented some exact solutions of Bianchi type VI for perfect fluid distributions satisfying specific equations of state. Some Bianchi VI cosmological models with gravitational field of the magnetic type has been studied by many authors [12-14]. Patel and Koppar [15] obtained some Bianchi type VI viscous fluid cosmological models. Bali, Pradhan and Hassan [16-17] are some researchers who studied Bianchi type VI magnetized string cosmological models in General Relativity. Pradhan and Bali [18-19] presented Bianchi type VI universe with decaying vacuum energy density. Bali, Banerjee and Banerjee [20] studied some LRS Bianchi type VI cosmological models with special free Gravitational fields. Asgar and Ansari [21] investigated spatially homogeneous and totally anisotropic Bianchi type-VI bulk viscous cosmological models in Lyra geometry. Abdel-Megied and Hegazy [22] studied Bianchi type VI cosmological model in the presence of electromagnetic field with variable magnetic permeability in the framework of Lyra geometry. In this paper, taking exponential scale factor and variable Lambda parameter Bianchi Type VI cosmological model with electromagnetic field has been studied.

II. Metric And Field Equations

We consider Bianchi type- VI metric in the form

$$ds^2 = -dt^2 + A^2dx^2 + B^2e^{-2ax}dy^2 + C^2e^{2ax}dz^2 \tag{2.1}$$

where A, B and C are function of cosmic time t and a is a constant parameter.

The Einstein's field equations (with $\frac{8\pi G}{c^4} = 1$) is given by

$$R^i{}_j - \frac{1}{2}g^i{}_jR + g^i{}_j\Lambda = -T^i{}_j \tag{2.2}$$

The energy momentum tensor for perfect fluid with electromagnetic field has the form

$$T_j{}^i = (\rho + p)u^iu_j + pg^i{}_j + E^i{}_j \tag{2.3}$$

Here ρ and p denote density and pressure respectively.

Also u^i is the four velocity vector satisfying $u^iu_i = -1$.

In Eq. (2.3), $E_j{}^i$ is the electromagnetic field given by Lichnerowicz [23]

$$E_j{}^i = \bar{\mu}\left[h_lh^l\left(u^iu_j + \frac{1}{2}g^i{}_j\right) - h^ih_j\right], \tag{2.4}$$

where $\bar{\mu}$ is the magnetic permeability and h_i the magnetic flux vector defined by

$$h_i = \frac{1}{\bar{\mu}} * F_{ji}u^j, \tag{2.5}$$

where the dual electromagnetic field tensor $*F_{ji}$ is defined by Synge [24]

$$* F_{ji} = \frac{\sqrt{-g}}{2} \varepsilon_{ij\ kl} F^{kl} . \tag{2.6}$$

Here F_{ij} is the electromagnetic field tensor and ε_{ijkl} is the Levi-Civita tensor density.

In the present model, the comoving coordinates are taken as

$$u^i = (0, \quad 0, \quad 0, \quad 1) . \tag{2.7}$$

The incident magnetic field is taken along z-axis so that $h_1 = 0 = h_2 = h_4, \ h_3 \neq 0$ (2.8)

The first set of Maxwell's equations

$$F_{[ij;k]} = 0, \tag{2.9}$$

$$\left[\frac{1}{\bar{\mu}} F^{ij}\right]_{;j} = J^i \tag{2.10}$$

Require that F_{12} is the function of x −alone and the magnetic permeability is the function of x and t both. Here semicolon represents a covariant differentiation.

Equation (2.9) leads to $F_{12} = constant.$ (2.11)

Here $F_{31} = F_{23} = F_{14} = 0.$, $F_{12} \neq 0.$ due to assumption of infinite electromagnetic conductivity. The only non-vanishing component of F_{ij} is F_{12}.

The non-vanishing components of ${E^i}_j$ are

$$ {E^1}_1 = \frac{1}{2\bar{\mu}} . \frac{{F_{12}}^2}{A^2 B^2 e^{-2ax}} = {E^2}_2 = -{E^3}_3 = -{E^4}_4 \tag{2.12}$$

Now using equation (2.3)-(2.12), the Einstein field equations (2.2) for the metric (2.1) reduces to following set of equations

$$\frac{\ddot{B}}{B} + \frac{\dot{B}\dot{C}}{BC} + \frac{\ddot{C}}{C} + \frac{a^2}{A^2} + \Lambda = -p - \frac{1}{2\bar{\mu}} . \frac{{F_{12}}^2}{A^2 B^2 e^{-2ax}} \tag{2.13}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{a^2}{A^2} + \Lambda = -p - \frac{1}{2\bar{\mu}} . \frac{{F_{12}}^2}{A^2 B^2 e^{-2ax}} \tag{2.14}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{a^2}{A^2} + \Lambda = -p + \frac{1}{2\bar{\mu}} . \frac{{F_{12}}^2}{A^2 B^2 e^{-2ax}} \tag{2.15}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}\dot{C}}{BC} - \frac{a^2}{A^2} + \Lambda = \rho + \frac{1}{2\bar{\mu}} . \frac{{F_{12}}^2}{A^2 B^2 e^{-2ax}} \tag{2.16}$$

$$\frac{\dot{B}}{B} - \frac{\dot{C}}{C} = 0 \tag{2.17}$$

The proper volume V and average scale factor S for Bianchi type-VI Space time is

$$V = S^3 = ABC \tag{2.18}$$

The generalized mean Hubble parameter H is given by

$$H = \frac{1}{3}(H_1 + H_2 + H_3) \tag{2.19}$$

where $H_1 = \frac{\dot{A}}{A}, \ H_2 = \frac{\dot{B}}{B}, \ H_3 = \frac{\dot{C}}{C}$ are directional Hubble parameters in x , y , z directions.

The scalar expansion Θ , shear scalar σ^2, anisotropy parameter Δ and the declaration parameter q have the following expressions

$$\Theta = 3H \tag{2.20}$$

$$\sigma^2 = \frac{1}{2}\left(\sum_{i=1}^{3} H_i^2 - 3H^2\right) \tag{2.21}$$

$$\Delta = \frac{1}{3}\sum_{i=1}^{3} \left(\frac{H_i - H}{H}\right)^2 \tag{2.22}$$

$$q = -\frac{S\ddot{S}}{\dot{S}^2} \tag{2.23}$$

III. Solution Of The Field Equations

From equation (2.17) we get $B = C$ (3.1)

Now let us assume, proper volume as

$$V = S^3 = ABC = c_2(2\sqrt{t} - c_1)^2 e^{\sqrt{t}} \tag{3.2}$$

where $c_1, \ c_2 > 0.$

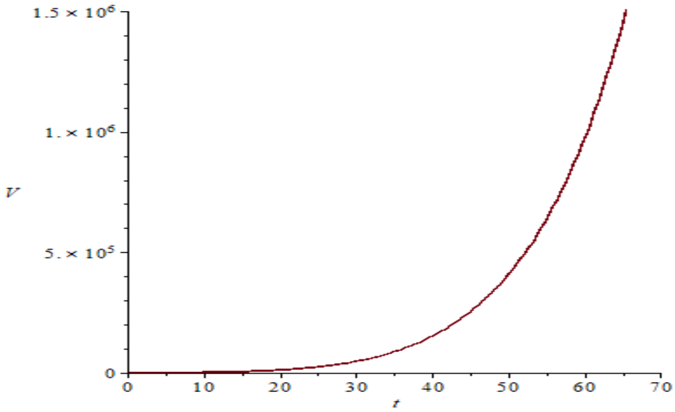

Figure 1: Graph of proper volume with respect to time for $c_1 = c_2 = 1$

To find a determinate solution, we first assume that $\frac{\Theta}{\sigma} = constant$. This leads to $A = C^n$ (3.3)

Then using (3.1) and (3.3) in (3.2) we get

$$B = \left[c_2(2\sqrt{t} - c_1)^2 e^{\sqrt{t}}\right]^{\frac{1}{n+2}} \tag{3.4}$$

$$C = \left[c_2(2\sqrt{t} - c_1)^2 e^{\sqrt{t}}\right]^{\frac{1}{n+2}} \tag{3.5}$$

$$A = \left[c_2(2\sqrt{t} - c_1)^2 e^{\sqrt{t}}\right]^{\frac{n}{n+2}} \tag{3.6}$$

Bianchi type- VI metric for this model is found to be in the form

$$ds^2 = -dt^2 + \left[c_2(2\sqrt{t} - c_1)^2 e^{\sqrt{t}}\right]^{\frac{2n}{n+2}} dx^2 + \left[c_2(2\sqrt{t} - c_1)^2 e^{\sqrt{t}}\right]^{\frac{2}{n+2}} \left[e^{-2ax} dy^2 + e^{2ax} dz^2\right] \tag{3.7}$$

On solving the field equations with the help of (3.4)-(3.6), we obtain following parameter values

$$p = -\left[\frac{n^2+n+1}{(n+2)^2}\left\{\frac{2\sqrt{t}-c_1+4}{2\sqrt{t}(2\sqrt{t}-c_1)}\right\}^2 - \frac{n+1}{n+2}\left\{\frac{(2\sqrt{t}-c_1)^2+4\sqrt{t}(2\sqrt{t}-c_1)+2\sqrt{t}}{4t^{\frac{3}{2}}(2\sqrt{t}-c_1)^2}\right\}\right] \tag{3.8}$$

$$\rho = \left[\frac{2n+1}{(n+2)^2}\left\{\frac{2\sqrt{t}-c_1+4}{2\sqrt{t}(2\sqrt{t}-c_1)}\right\}^2\right] - \frac{1}{2\bar{\mu}} \cdot \frac{F_{12}{}^2}{\left[c_2(2\sqrt{t}-c_1)^2 e^{\sqrt{t}}\right]^{\frac{2(n+1)}{n+2}} e^{-2ax}} \tag{3.9}$$

$$\Lambda = \frac{a^2}{\left[c_2(2\sqrt{t}-c_1)^2 e^{\sqrt{t}}\right]^{\frac{2n}{n+2}}} - \frac{1}{2\bar{\mu}} \cdot \frac{F_{12}{}^2}{\left[c_2(2\sqrt{t}-c_1)^2 e^{\sqrt{t}}\right]^{\frac{2(n+1)}{n+2}} e^{-2ax}} \tag{3.10}$$

Hubble parameter and Scalar expansion are given by

$$H = \frac{2\sqrt{t}-c_1+4}{6\sqrt{t}(2\sqrt{t}-c_1)} \tag{3.11}$$

$$\Theta = \frac{2\sqrt{t}-c_1+4}{2\sqrt{t}(2\sqrt{t}-c_1)} \tag{3.12}$$

Shear scalar and Anisotropy parameter are obtained as

$$\sigma^2 = \frac{(n-1)^2}{3(n+2)^2}\left[\frac{2\sqrt{t}-c_1+4}{2\sqrt{t}(2\sqrt{t}-c_1)}\right]^2 \tag{3.13}$$

$$\Delta = \frac{2(n-1)^2}{(n+2)^2} \tag{3.14}$$

Deceleration parameter is obtained as

$$q = -1 + \frac{\frac{1}{12}t^{-\frac{3}{2}} + \frac{1}{3}(2\sqrt{t}-c_1)^{-1}t^{-\frac{3}{2}} + \frac{2}{3}(2\sqrt{t}-c_1)^{-2}t^{-1}}{\left[\frac{1}{6}t^{-\frac{1}{2}} + \frac{2}{3}(2\sqrt{t}-c_1)^{-1}t^{-\frac{1}{2}}\right]^2} \tag{3.15}$$

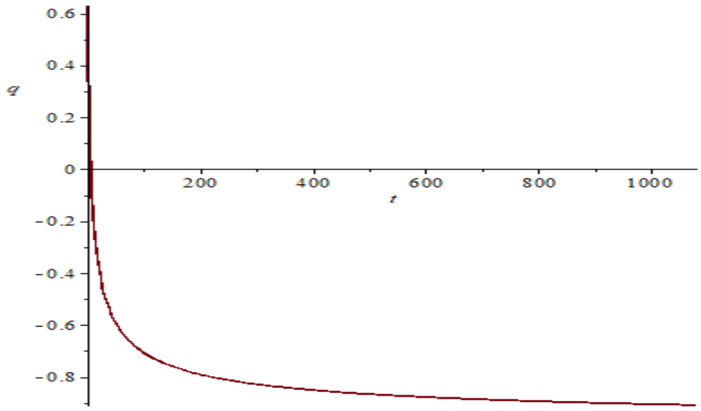

Figure 2: Graph of deceleration parameter with respect to time for $c_1 = 1$

IV. Conclusion

In this paper, the proper volume is considered as $V = c_2(2\sqrt{t} - c_1)^2 e^{\sqrt{t}}$ which is an exponential function of time t. From Figure 1, it is observed that the proper volume increases at very high rate as time increases. The proper volume of the model increases exponentially as $t \to \infty$; that is, the model is expanding with the increase of time. The model (3.7) starts expansion with a big-bang singularity from $t = -\infty$ and it goes on expanding as $t \to \infty$. Hubble's parameter and scalar expansion tend to zero as time tends to infinity. The value of deceleration parameter tends to -1 as time tends to infinity. From Eq. (3.9), it is found that ρ is a decreasing function of time and $\rho > 0$ for all times. The pressure is found to be negative in this model universe, which gives us a dark energy model with accelerated expansion of the universe. For $n \neq 1$, $\frac{\Theta^2}{\sigma^2} \neq 0$ which indicates that the model does not approach isotropy. From Eq. (3.10) , it is found that the value of Λ for the model is large at initial stage and tends to small positive value as time increases, which is supported by the results from various observational data from Cosmological Projects (Perlmutter et al. [6], Riess et al. [7,27], Garnavich et al. [25,26], Schmidt et al. [28]). With $F_{12} \neq 0$, a magnetized Bianchi Type VI dark energy model with decaying energy density has been found.

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