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The Wave Function and the Energy of Crystalline Regular Nano Particles in A uniform Crystal Field for Spherically Symmetric System According to Sting Theory

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Abstract: *Schrodinger equation was used for nano particles are considered as spherical crystals that are uniformly distributed. The solution shows that the wave function predicts uniform distribution of particles which conform to the assumptions. Applying periodic conditions the energy is shown to be quantized when thermal force exceeds the. External vibrating force opposing it.Unlike ordinary models the energy decreases as the quantum number increases.*

Key words*: spherical symmetry, Schrödinger Equation, nano particles, crystals, energy, quantization*

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I. Introduction

The building block of matter is the atom. Quantum theory is one of the important physical theories that describe the behavior of atoms. Quantum theory starts from the Plank hypothesis to describe black body radiation spectral distribution. He proposes that light does not behave as a wave but as discrete quanta, carrying energy which is proportional to the electromagnetic frequency. In other words, he proposed that light behaves as particles [1, 2]. This Plank hypothesis succeeded in describing many physical phenomena, like photoelectric, Compton Effect and pair production. This encourages Bohr in 1913 to construct a model that predicts energy quantization. This model explains the spectra of some simple atoms. Later on De Broglie suggest in 1925 that particles behaves as waves, this particle-wave duality is the corner state of quantum mechanics [3, 4].It is used together with Hamilton and Lagrangian formalism to construct Schrödinger, Klein-Gordon and Dirac quantum laws [5].Quantum mechanics succeeded in describing a wide variety of atomic phenomena. These include atomic spectra, scattering processes beside matter-radiation interactions for many processes. However quantum laws face some difficulties in describing some phenomena associated with bulk matter, the so called many body problem. Now a day the so called high temperature super conductor’s phenomena cannot be completely explained by using ordinary quantum laws [6, 7]. The same hold for the behavior of nano materials [8]. This requires searching for new models to cure these setbacks on of the most promising notions is the string theory [9, 10]. This motivates constructing a quantum model for spherically symmetric systems affected by oscillatory potential.

Spherically Symmetric

Schrödinger Equations for Vibrating String The radial part of Schrödinger equation in spherical coordinate gives a

$$\ddot{u} - \frac{2m}{\hbar^2}(V - E)u = \frac{2m}{\hbar^2}\frac{u}{r^2} \qquad (1)$$

Where the wave function ψ and the radial part $R(r)$ satisfies

$\psi = R(r)O(\theta,\phi)$

$R(r) = \frac{u}{r}$ (2)

For atoms inside the nano particle it can be subjected to uniform potentialV_0. Assuming that electrons are in the form of string in the radial direction the total potential V takes the form

$V = V_0 + \frac{1}{2}kr^2 = V_0 + \frac{1}{2}mw^2r^2$ (3)

Thus equation (1) can be written in the form:

$\ddot{u} - \frac{c_1}{r_1} + (k^2 - k_0^2)u - c_2r^2u = 0$ (3)

Where

$c_1 = \frac{2m}{\hbar^2} = \frac{l(l+1)}{\hbar^2},\ k^2 = \frac{2m}{\hbar^2}E, \quad k_0^2 = \frac{2m}{\hbar^2}V_0, \quad c_2 = \frac{m^2w^2}{\hbar^2}$ (4)

One can solve equation (3) by suggesting

$u = ge^{ikr}\,\dot{u} = \dot{g}e^{ikr} + ikge^{ikr}\,\ddot{u} = \ddot{g}e^{ikr} + 2ik\dot{g}e^{ikr} - k^2ge^{ikr}$ (5)

A direct substitution in equation (3) gives

$\ddot{g} + 2ik\dot{g} - k_0^2g - c_2r^2g - \frac{c_1}{r^2}g = 0$ (6)

To solve equation (6) assume that

$g = Ae^f, \ \ \dot{g} = A\dot{f}e^f, \ \ \ddot{g} = \left(\ddot{f}e^f + \dot{f}^2e^f\right)A$ (7)

Inserting (7) in (6) gives

$\ddot{f} + \dot{f}^2 + 2ik\dot{f} - k_0^2 - c_2r^2 - \frac{c_1}{r^2} = 0$ (8)

To solve equation (8) assume that

$f = c_3\ln r + c_4r^2 + c_5r$ (9)

$$\dot{f} = \frac{c_3}{r} + 2c_4r + c_5$$

$$\ddot{f} = -\frac{c_3}{r^2} + 2c_4$$

$$\dot{f}^2 = \left(\frac{c_3}{r} + 2c_4r + c_5\right)\left(\frac{c_3}{r} + 2c_4r + c_5\right)$$

$$\frac{c_3^2}{r^2} + 2c_3c_4 + \frac{c_5c_3}{r} + 2c_3c_4 + 2c_4c_5r + \frac{c_3c_5}{r} + 2c_4c_5r + c_5^2$$

$= c_5^2 + \frac{c_3^2}{r^2} + 4c_4^2r^2 + 4c_3c_4 + \frac{2c_3c_5}{r} + 4c_4c_5r$ (9)

Inserting (9) in (8) to get

$-\frac{c_3}{r^2} + 2c_4 + c_5^2 + \frac{c_3^2}{r^2} + 4c_4^2r^2 + 4c_3c_4 + \frac{2c_3c_5}{r} + 4c_4c_5r + \frac{2ikc_3}{r} + 4ikc_4r + 2ikc_5 - k_0^2 - c_2r^2 - \frac{c_1}{k^2} = 0$ (10)

$\frac{1}{r^2} = -c_3 + c_3^2 - c_1 = 0$ (11)

$\frac{1}{r} = 2c_3c_5 + 2ikc_3 = 0$ (12)

$free = 2c_4 + c_5^2 + 4c_3c_4 + 2kc_5i - k_0^2 = 0$ (13)

$r = 2c_4 + c_5^2 + 4c_3c_4 + 2ikc_4 = 0$ (14)

$r^2 = 4c_4^2 - c^2 = 0$ (15)

From (15)

$c_4 = \pm\frac{1}{2}\sqrt{c_2}$ (16)

From (14)

$c_5 = -ik$ (17)

From (12) also

$c_5 = -ik$ (17)

From (11)

$$c_3^2 - c_3 - c_1 = 0$$

$c_3 = \frac{1\pm\sqrt{1+4c_1}}{2}$ (18)

From (13) and (17)

$$k_0^2 = 2c_4 + c_5^2 + 4c_3c_4 + 4c_5ik$$

$= 2c_4 - k^2 + 4c_3c_4 + 2k^2 = k^2 + 2c_4 + 4c_3c_4$ (19)

$k^2 = k_0^2 - 2c_4 - 4c_3c_4$ (20)

For zero angular momentum, equation

$l = 0 \qquad c_1 = 0$ (21)

Thus equation (18) gives

$$c_3 = \frac{1 \pm \sqrt{1}}{2}$$

Either

$c_3 = 0$ (22)

$c_3 = 1$ (23)

Take (23) and use it in (20) to get

$$6c_4 = k_0^2 - k^2$$

$c_4 = \frac{1}{6}(k_0^2 - k^2)$ (24)

If one assumes that $c_3$vanishes as in equation (22), equation (20) thus gives

$c_4 = \frac{1}{2}(k_0^2 - k^2)$ (25)

Using equations (9), (7), (5) and (2) the radial part is gives by

$r = \frac{u}{r} = \frac{ge^{ikr}}{r} = \frac{Ae^f}{r}e^{ikr} = \frac{Ae^{c_3 \ln r + c_4 r^2 + c_5 r + ikr}}{r}$ (26)

Using relation (17) in (26) yields

$R = \frac{A}{r}e^{\ln r^{c_3} + c_4 r^2} = Ar^{c_3 - 1}e^{c_4 r^2}$ (27)

It is well known that the wave function ψ should be finite. Thus R should be finite too. This requires R to be finite at any point including$(r = 0)$. Thus it $(c_3 = 0)$ as in equation (22), thus

$$R \to \infty$$

When

$$r \to 0$$

However when, one chooses $(c_3 = 1)$ as in equation (23), thus

$$R = Ae^0 = A$$

Which is a finite quantity, which gives finite probability. Thus equation (27) together with (23) gives

$R = Ae^{c_4 r^2}$ (28)

Let us apply this solution to regular crystalline nano spherical particle with distanced between two successive Lattice points. Thus the wave function is periodic according to Bloch hypothesis. i.e.

$R(r + d) = R(r)$ (29)

Thus equation (28) gives

$e^{c_4(r+d)^2} = c^{c_4 r^2}$ (30)

$e^{c_4 r^2 + 2c_4 rd + c_4 d^2} = e^{c_4 r^2}$ (31)

$e^{2c_4 rd + c_4 d^2} = 1$ (32)

Near the origin, i.e. at

$$r \to 0$$

$e^{c_4 d^2} = 1$ (33)

The potential in equation (3) can be modified by adding new term representing natural thermal oscillation of electrons with frequency w_0 and with the restoring force opposite to that of w, which can be assumed to be generated by sound waves or bilding energy or even one can assume that there are two forces (thermal and sound), which are 180^0 out of phase, in this case

$x = A \sin wt$

$x_0 = A \sin(wt + 180)$

$= A[\sin wt \cos 180 + \cos wt \sin 180]$

$-A \sin wt = -x$ (34)

Thus, the total restoring force is

$$F = -kx - k_0 x_0 = -kx + k_0 x$$

$= -(k - k_0)x = -m(w^2 - w_0^2)x = -c_0 x$ (35)

$$V(oscollation) = -\int F dx = c_0 \int x dx$$

$= \frac{1}{2}c_0 x^2 = \frac{1}{2}m(w^2 - w_0^2)x$ (36)

Thus c_2 in equation (4) becomes

$c_2 = \frac{m^2(w^2 - w_0^2)}{\hbar^2}$ (37)

Thus from (16) and (37)

$c_4 = I\frac{1}{2}\frac{m}{\hbar}\sqrt{w^2 - w_0^2}$ (38)

When the thermal oscillation becomes larger then binding or sound oscillation, i.e. when

$w_0 > w$ (39)

Equation (38) can be rewritten as

$$c_4 = \pm i \frac{m}{2\hbar}\sqrt{w_0^2 - w^2} = \pm i c_6 \qquad (40)$$

Thus equation (33) becomes

$$e^{c_6 d^2} = 1$$

$$\cos c_6 d^2 + i \sin c_6 d^2 = 1 \qquad (40)$$

This is satisfied when

$$\cos c_6 d^2 = 1$$
$$\sin c_6 d^2 = 0 \qquad (41)$$

Which requires

$$c_6 d^2 = 2n\pi$$

$$n = 0, 1, 2 \ldots \qquad (42)$$

Thus from (40)

$$w_0^2 - w^2 = \frac{4n^2\pi^2}{d^4}\left(\frac{4\hbar^2}{m^2}\right)$$

$$w^2 = w_0^2 - \frac{16n^2\pi^2\hbar^2}{m^2 d^4} \qquad (43)$$

But

$$k = \frac{2\pi}{\lambda} = \frac{2\pi f}{\lambda f} = \frac{w}{c}$$

$$k^2 = \frac{w^2}{c^2} \qquad (44)$$

Thus according to equation (4), (43) and (44)

$$E = \frac{\hbar^2 k^2}{2m}$$

And

$$E = \hbar^2 w_0^2 - \frac{8\pi^2\hbar^2}{m^3 d^4} \qquad (45)$$

II. Discussion

Treating atoms and nano particles as spherically symmetric systems and considering electrons as vibrating strings embedded in a uniform constant field, the radial Schrödinger equation part was constructed in equation (3). The solutions were suggested as spatially oscillating in equation (5). Solving Schrodinger equation the constant unknown coefficients were found in equations (17), (18) & (20).The coefficients become simple for zero angular momentum as shown by equations (22) & (23). Assuming nano particles as small nano crystals that uniformly distribute themselves certain restrictions are imposed on the solution in equations (30) & (31).treating electrons as affected by thermal collisional force apposing an external vibrating force the quantization condition in equations (40) up to (43) requires that the energy is quantized when thermal energydominates. This energy decreases as the quantum number increases

III. Conclusion

Considering nano particles as uniformly distributed small crystals Schrödinger equation for spherically symmetric particles was solved. The solution shows that the energy is quantized, and decreases upon increasing the quantum number when the thermal energy exceeds the external one.

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