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Unextendible Maximally Entangled Bases In $C^9 \otimes C^9$

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Abstract: *We investigate the unextendible maximally entangled bases in $C^d \otimes C^d$ and present a 72-number UMEB in $C^9 \otimes C^9$. Moreover, we show the specific construction of the UMEB construction in $C^9 \otimes C^9$ from that in $C^3 \otimes C^3$.*

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I. Introduction

In 2009, S. Bravyi and J. A. Smolin [1] generalized the notion of the unextendible product bases(UPB) to the unextendible maximally entangled basis (UMEB), they showed that there did not exist UMEB in $C^2 \otimes C^2$, and constructed a 6-member UMEB in $C^3 \otimes C^3$ and 12-numbers UMEB in $C^4 \otimes C^4$.

Then , Y. L. Wang and S.M. Fei [2] constructed a 30-member UMEB in $C^6 \otimes C^6$, they also showed that for an given N- number UMEB in $C^d \otimes C^d$, there is a $\tilde{N} = (qd)^2\text{-}q(d^2\text{-}N)$- member UMEB in $C^{qd} \otimes C^{qd}$ for any $q \in \mathrm{N}$.

In this paper, in order to using higher dimension UMEB more directly in the future, we present theconstruction of a 72-member UMEB in $C^9 \otimes C^9$ by using the known 6-member UMEB in $C^3 \otimes C^3$.

II. UMEBs In $C^9 \otimes C^9$

A set of states $\{ |\varphi_a\rangle \in C^d \otimes C^d : a = 1,2\ldots,n, n < d^2 \}$ is called an n- member UMEB [1] if and only if

(i) $|\phi_a\rangle, a = 1,2,\ldots,n,$ are all maximally entangled;

(ii) $\langle \phi_a | \phi_b \rangle = \delta_{ab}, a, b = 1,2,\ldots,n$;

(iii) If $\langle \phi_a | \psi \rangle = 0$ for all $a = 1,2,\ldots,n$, then $|\psi\rangle$ cannot be maximally entangled.

Here under computational basis a maximally entangled state $|\phi_a\rangle$ in $C^d \otimes C^d$ can be expressed as

$$|\phi_a\rangle = (I \otimes U_a)\frac{1}{\sqrt{d}}\sum_{i=0}^{d\text{-}1}|i\rangle \otimes |i\rangle \qquad (1)$$

where I is the $d \times d$ identity matrix, U_a is any unitary matrix. According to (1), a set of unitary matrices $\{U_a \in M_d(C) | a = 1,\ldots,n\}$ gives an n- number UMEB in $C^d \otimes \mathrm{C}^d$ if and only if

(i) $n < d^2$;

(ii) $Tr(U_a^{+}U_b) = d\delta_{ab}, \quad a, b = 1,2,\ldots,n$;

(iii)For any $U_a \in M_d(C)$, if $Tr(U_a^{+}U) = 0, \quad a = 1,2,\ldots,n$, then U cannot be unitary.

In the following we construct a 72-member UMEB in $C^9 \otimes \mathrm{C}^9$. Set $\{|0\rangle, |1\rangle, |2\rangle\}$ be the computational basis of C^3, and

$$U_{nm} \triangleq \sum_{k=0}^{2} \omega^{kn} |k \oplus m\rangle\langle k|, n,m = 1,2,3$$

$$U_{nm}^{j} = \delta_j \otimes U_{nm}, n,m = 1,2,3, j = 1,\ldots,6, \quad (2)$$

and

$$U_i^p = \eta_p \otimes U_i, i = 1,\ldots,6, p = 1,2,3, \quad (3)$$

where $\omega = e^{\frac{2\pi i}{3}}$, $k \oplus m$ denotes $k+m$ mod 3, and

$$\delta_1 = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \delta_2 = \begin{pmatrix} 0 & 0 & 1 \\ \omega & 0 & 0 \\ 0 & \omega^2 & 0 \end{pmatrix}, \delta_3 = \begin{pmatrix} 0 & 0 & 1 \\ \omega^2 & 0 & 0 \\ 0 & \omega & 0 \end{pmatrix},$$

$$\delta_4 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \delta_5 = \begin{pmatrix} 0 & \omega & 0 \\ 0 & 0 & \omega^2 \\ 1 & 0 & 0 \end{pmatrix}, \delta_6 = \begin{pmatrix} 0 & \omega^2 & 0 \\ 0 & 0 & \omega \\ 1 & 0 & 0 \end{pmatrix},$$

$$\eta_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \eta_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}, \eta_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix},$$

$\{U_i\}_{i=1}^{6}$ are the following unitary matrices constructed in $C^3 \otimes C^3$ in Ref.[1]:

$$U_i = I - (1 - e^{i\theta})|\psi_i\rangle\langle\psi_i|, \quad i = 1,\ldots,6 \quad (4)$$

where

$$|\psi_{1,2}\rangle = N(|0\rangle \pm \alpha|1\rangle), |\psi_{3,4}\rangle = N(|1\rangle \pm \alpha|2\rangle), |\psi_{5,6}\rangle = N(|2\rangle \pm \alpha|0\rangle),$$

with $\alpha = \frac{(1+\sqrt{5})}{2}$, $N = \frac{1}{\sqrt{1+\alpha^2}}$.

Then, it is easy to check that $\{U_{nm}^{j}, n,m = 1,2,3; j = 1,2,..,6\}$ and $\{U_i^p, i = 1,2,...,6; p = 1,2,3\}$ constitute a 72-member UMEB in $C^9 \otimes C^9$. Next, we present the specific expressions of the above 72 states as follows:

Firstly, we show the 54 states in $\{U_{nm}^{j}\}$ in (2)

$$|\phi_{1,2,3}\rangle = \frac{1}{3}(|06'\rangle + \alpha|17'\rangle + \alpha^2|28'\rangle + |30'\rangle + \alpha|41'\rangle + \alpha^2|52'\rangle + |63'\rangle + \alpha|74'\rangle + \alpha^2|85'\rangle)$$

$$|\phi_{4,5,6}\rangle = \frac{1}{3}(|07'\rangle + \alpha|18'\rangle + \alpha^2|26'\rangle + |31'\rangle + \alpha|42'\rangle + \alpha^2|50'\rangle + |64'\rangle + \alpha|75'\rangle + \alpha^2|83'\rangle)$$

$$|\phi_{7,8,9}\rangle = \frac{1}{3}(|08'\rangle + \alpha|16'\rangle + \alpha^2|27'\rangle + |32'\rangle + \alpha|40'\rangle + \alpha^2|51'\rangle + |65'\rangle + \alpha|73'\rangle + \alpha^2|84'\rangle)$$

$$|\phi_{10,11,12}\rangle = \frac{1}{3}(|06'\rangle + \alpha|17'\rangle + \alpha^2|28'\rangle + \omega|30'\rangle + \omega\alpha|41'\rangle + \omega\alpha^2|52'\rangle + \omega^2|63'\rangle + \omega^2\alpha|74'\rangle + \omega^2\alpha^2|85'\rangle)$$

$$|\phi_{13,14,15}\rangle = \frac{1}{3}(|07'\rangle + \alpha|18'\rangle + \alpha^2|26'\rangle + \omega|31'\rangle + \omega\alpha|42'\rangle + \omega\alpha^2|50'\rangle + \omega^2|64'\rangle + \omega^2\alpha|75'\rangle + \omega^2\alpha^2|83'\rangle)$$

$$|\phi_{16,17,18}\rangle = \frac{1}{3}(|08'\rangle + \alpha|16'\rangle + \alpha^2|27'\rangle + \omega|32'\rangle + \omega\alpha|40'\rangle + \omega\alpha^2|51'\rangle + \omega^2|65'\rangle + \omega^2\alpha|73'\rangle + \omega^2\alpha^2|84'\rangle)$$

$$|\phi_{19,20,21}\rangle = \frac{1}{3}(|06'\rangle + \alpha|17'\rangle + \alpha^2|28'\rangle + \omega^2|30'\rangle + \omega^2\alpha|41'\rangle + \omega^2\alpha^2|52'\rangle + \omega|63'\rangle + \omega\alpha|74'\rangle + \omega\alpha^2|85'\rangle)$$

$$\left|\phi_{22,23,24}\right\rangle=\frac{1}{3}(\left|07'\right\rangle+\alpha\left|18'\right\rangle+\alpha^2\left|26'\right\rangle+\omega^2\left|31'\right\rangle+\omega^2\alpha\left|42'\right\rangle+\omega^2\alpha^2\left|50'\right\rangle+\omega\left|64'\right\rangle+\omega\alpha\left|75'\right\rangle+\omega\alpha^2\left|83'\right\rangle)$$

$$\left|\phi_{25,26,27}\right\rangle=\frac{1}{3}(\left|08'\right\rangle+\alpha\left|16'\right\rangle+\alpha^2\left|27'\right\rangle+\omega^2\left|32'\right\rangle+\omega^2\alpha\left|40'\right\rangle+\omega^2\alpha^2\left|51'\right\rangle+\omega\left|65'\right\rangle+\omega\alpha\left|73'\right\rangle+\omega\alpha^2\left|84'\right\rangle)$$

$$\left|\phi_{28,29,30}\right\rangle=\frac{1}{3}(\left|03'\right\rangle+\alpha\left|14'\right\rangle+\alpha^2\left|25'\right\rangle+\left|36'\right\rangle+\alpha\left|47'\right\rangle+\alpha^2\left|58'\right\rangle+\left|60'\right\rangle+\alpha\left|71'\right\rangle+\alpha^2\left|82'\right\rangle)$$

$$\left|\phi_{31,32,33}\right\rangle=\frac{1}{3}(\left|04'\right\rangle+\alpha\left|15'\right\rangle+\alpha^2\left|23'\right\rangle+\left|37'\right\rangle+\alpha\left|48'\right\rangle+\alpha^2\left|56'\right\rangle+\left|61'\right\rangle+\alpha\left|72'\right\rangle+\alpha^2\left|80'\right\rangle)$$

$$\left|\phi_{34,35,36}\right\rangle=\frac{1}{3}(\left|05'\right\rangle+\alpha\left|13'\right\rangle+\alpha^2\left|24'\right\rangle+\left|38'\right\rangle+\alpha\left|46'\right\rangle+\alpha^2\left|57'\right\rangle+\left|62'\right\rangle+\alpha\left|70'\right\rangle+\alpha^2\left|81'\right\rangle)$$

$$\left|\phi_{37,38,39}\right\rangle=\frac{1}{3}(\omega\left|03'\right\rangle+\omega\alpha\left|14'\right\rangle+\omega\alpha^2\left|25'\right\rangle+\omega^2\left|36'\right\rangle+\omega^2\alpha\left|47'\right\rangle+\omega^2\alpha^2\left|58'\right\rangle+\left|60'\right\rangle+\alpha\left|71'\right\rangle+\alpha^2\left|82'\right\rangle)$$

$$\left|\phi_{40,41,42}\right\rangle=\frac{1}{3}(\omega\left|04'\right\rangle+\omega\alpha\left|15'\right\rangle+\omega\alpha^2\left|23'\right\rangle+\omega^2\left|37'\right\rangle+\omega^2\alpha\left|48'\right\rangle+\omega^2\alpha^2\left|56'\right\rangle+\left|61'\right\rangle+\alpha\left|72'\right\rangle+\alpha^2\left|80'\right\rangle)$$

$$\left|\phi_{43,44,45}\right\rangle=\frac{1}{3}(\omega\left|05'\right\rangle+\omega\alpha\left|13'\right\rangle+\omega\alpha^2\left|24'\right\rangle+\omega^2\left|38'\right\rangle+\omega^2\alpha\left|46'\right\rangle+\omega^2\alpha^2\left|57'\right\rangle+\left|62'\right\rangle+\alpha\left|70'\right\rangle+\alpha^2\left|81'\right\rangle)$$

$$\left|\phi_{46,47,48}\right\rangle=\frac{1}{3}(\omega^2\left|03'\right\rangle+\omega^2\alpha\left|14'\right\rangle+\omega^2\alpha^2\left|25'\right\rangle,\omega\left|36'\right\rangle+\omega\alpha\left|47'\right\rangle+\omega\alpha^2\left|58'\right\rangle+\left|60'\right\rangle+\alpha\left|71'\right\rangle+\alpha^2\left|82'\right\rangle)$$

$$\left|\phi_{49,50,51}\right\rangle=\frac{1}{3}(\omega^2\left|04'\right\rangle+\omega^2\alpha\left|15'\right\rangle+\omega^2\alpha^2\left|23'\right\rangle+\omega\left|37'\right\rangle+\omega\alpha\left|48'\right\rangle+\omega\alpha^2\left|56'\right\rangle+\left|61'\right\rangle+\alpha\left|72'\right\rangle+\alpha^2\left|80'\right\rangle)$$

$$\left|\phi_{52,53,54}\right\rangle=\frac{1}{3}(\omega^2\left|05'\right\rangle+\omega^2\alpha\left|13'\right\rangle+\omega^2\alpha^2\left|24'\right\rangle+\omega\left|38'\right\rangle+\omega\alpha\left|46'\right\rangle+\omega\alpha^2\left|57'\right\rangle+\left|62'\right\rangle+\alpha\left|70'\right\rangle+\alpha^2\left|81'\right\rangle)$$

where $\alpha = 1, \omega, \omega^2$.

	0'	1'	2'	3'	4'	5'	6'	7'	8'
0				1	2	3	1	2	3
1		U_i		3	1	2	3	1	2
2				2	3	1	2	3	1
3	1	2	3				1	2	3
4	3	1	2		U_i		3	1	2
5	2	3	1				2	3	1
6	1	2	3	1	2	3			
7	3	1	2	3	1	2		U_i	
8	2	3	1	2	3	1			

Next, we will show the constructions of 18 states in $\{U_i^p\}$ in (3), before that we first present all the specific matrix forms of U_i in (4) from different $\left|\psi_i\right\rangle, i = 1,2,\ldots,6$. As follows:

$$U_1=\begin{pmatrix}1-(1-e^{i\theta})N^2 & -(1-e^{i\theta})N^2\alpha & 0\\ -(1-e^{i\theta})N^2\alpha & 1-(1-e^{i\theta})N^2\alpha^2 & 0\\ 0 & 0 & 1\end{pmatrix}, \quad U_2=\begin{pmatrix}1-(1-e^{i\theta})N^2 & (1-e^{i\theta})N^2\alpha & 0\\ (1-e^{i\theta})N^2\alpha & 1-(1-e^{i\theta})N^2\alpha^2 & 0\\ 0 & 0 & 1\end{pmatrix}$$

$$U_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1-(1-e^{i\theta})N^2 & -(1-e^{i\theta})N^2\alpha \\ 0 & -(1-e^{i\theta})N^2\alpha & 1-(1-e^{i\theta})N^2\alpha^2 \end{pmatrix}, U_4 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1-(1-e^{i\theta})N^2 & (1-e^{i\theta})N^2\alpha \\ 0 & (1-e^{i\theta})N^2\alpha & 1-(1-e^{i\theta})N^2\alpha^2 \end{pmatrix}$$

$$U_5 = \begin{pmatrix} 1-(1-e^{i\theta})N^2\alpha^2 & 0 & -(1-e^{i\theta})N^2\alpha \\ 0 & 1 & 0 \\ -(1-e^{i\theta})N^2\alpha & 0 & 1-(1-e^{i\theta})N^2 \end{pmatrix}, U_6 = \begin{pmatrix} 1-(1-e^{i\theta})N^2\alpha^2 & 0 & (1-e^{i\theta})N^2\alpha \\ 0 & 1 & 0 \\ (1-e^{i\theta})N^2\alpha & 0 & 1-(1-e^{i\theta})N^2 \end{pmatrix}$$

Then, according to (1) in Ref[1], we can get a matrix with a dimension of 81.Therefore, we can get 18 states from them. Take ϕ_1, ϕ_2, ϕ_3 for example. Then we can get the rest of the other states.

$$|\phi_1\rangle = (\varphi_{11}, \varphi_{12}, \varphi_{13})^T$$
$$\varphi_{11} = (A,B,1,A,B,1,A,B,1,A,B,1,A,B,1,A,B,1,A,B,1,A,B,1,A,B,1)^T$$
$$\varphi_{12} = (A,B,1,A,B,1,A,B,1,A,B,1,A,B,1,A,B,1,A,B,1,A,B,1,A,B,1)^T$$
$$\varphi_{13} = (A,B,1,A,B,1,A,B,1,A,B,1,A,B,1,A,B,1,A,B,1,A,B,1,A,B,1)^T$$
$$|\phi_2\rangle = (\varphi_{21}, \varphi_{22}, \varphi_{23})^T$$
$$\varphi_{21} = (A,B,1,A,B,1,A,B,1,A,B,1,A,B,1,A,B,1,A,B,1,A,B,1,A,B,1)^T$$
$$\varphi_{22} = \omega(A,B,1,A,B,1,A,B,1,A,B,1,A,B,1,A,B,1,A,B,1,A,B,1,A,B,1)^T$$
$$\varphi_{23} = \omega^2(A,B,1,A,B,1,A,B,1,A,B,1,A,B,1,A,B,1,A,B,1,A,B,1,A,B,1)^T$$
$$|\phi_2\rangle = (\varphi_{21}, \varphi_{22}, \varphi_{23})^T$$
$$\varphi_{31} = (A,B,1,A,B,1,A,B,1,A,B,1,A,B,1,A,B,1,A,B,1,A,B,1,A,B,1)^T$$
$$\varphi_{32} = \omega^2(A,B,1,A,B,1,A,B,1,A,B,1,A,B,1,A,B,1,A,B,1,A,B,1,A,B,1)^T$$
$$\varphi_{33} = \omega(A,B,1,A,B,1,A,B,1,A,B,1,A,B,1,A,B,1,A,B,1,A,B,1,A,B,1)^T$$

where $A = [1-(1-e^{i\theta})N^2] + [-(1-e^{i\theta})N^2\alpha]$ $B = [-(1-e^{i\theta})N^2\alpha] + [1-(1-e^{i\theta})N^2\alpha^2]$

Then we can get the rest of the other states.

III. Conclusion

We present a 72-member UMEB in $C^9 \otimes C^9$ from construction of an UMEB in $C^3 \otimes C^3$ in detail. Thus give a example for the construction of an UMEB in high dimension bipartite space and illustrate the main idea of paper[2].

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