

Relation between Power Loss of a Motor Rotating a Cylinder in a Viscous Fluid and its Speed of Rotation

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I. Introduction

Driving a car was new to me this summer. It was an exciting experience, one which I had sought for a long time. When driving, I always felt that I was in control. This was until I drove through sand. One day as I was cruising in the car, the wheels got stuck into the deep sand. Pressing on the gas pedal seemed to not make much of a difference, as the tires did not force the vehicle forward. "God! Imagine how exhausting this much be for the car engine," I told myself and continued to wonder. I finally got the car out by forcing it through by going at higher and higher speeds. Now ever since I took physics class last year, I became interestingly attached to mechanical work and electricity. I was always interested in rotational motion, power, voltages, current and all these concepts. So I thought to myself, how much power does a given motor need to exert, when the medium it is working in is more viscous. I related the difference between sand and cement to differing liquid viscosities. This is when I applied my problem to liquids. Time and time again, I started dropping objects in honey and water and observing the differences as they traveled to the bottom of the container. These mediocre experiments tied with the sand problem I had over the summer, which incorporated the rotational motion of the tires, led me to think of my Internal Assessment idea. What if a cylindrical object was rotated in viscous fluid such as oil? How would the power used by the motor be affected by increasing the voltage?

II. Research Question:

When a rotating cylinder is immersed into a liquid, what is the relation between the power loss due to viscosity of the liquid and the speed of rotation of the cylinder which is proportional to the voltage supplied?

III. Background

When a sphere falls through a liquid such as water, it moves much faster compared to the situation where it falls through another liquid such as honey. It is the viscosity of the liquid that affects the sphere while falling as mentioned. The viscosity of a liquid is defined as the force between two layers of the liquid of unit area when the separation between them is unity and the difference between their speeds is unity. In other words, if we assume a layer of a certain area A of the liquid sliding with a speed v with respect to another layer of the same area separated by a distance d from the first layer, the viscous force between these two layers would be given by the following formula:

$$F = \frac{\eta v A}{d}$$

Where η is the coefficient of viscosity of the liquid, A is the area of the layers mentioned, v is the relative speed between the layers, and d is the separation between the layers. The coefficient of viscosity η is related to the force between the particles of the liquid. It is also very much dependent on the temperature. As the temperature increases, the separation between the particles of the liquid increases, and therefore the forces between them decreases. This is why cold honey is much more viscous compared to warm honey. In the experiment, it is expected that increasing the speed of rotation of the motor will increase the torque due to the viscous force acting on the surface of the rotating cylinder.

A DC motor is a motor powered by a direct current. The power delivered to a DC motor is the product of the voltage between its terminals and the current drawn by the motor. Ideally, when there is zero friction, the motor would rotate without drawing any current. This is due to the back emf (electromotive force). To understand this phenomena, we think about the rotating coil of the motor, which acts as a generator that creates a back emf opposing that of the battery and cancels it so that the net current is zero. This situation, however, is practically impossible. Any friction in the rotating parts of the motor will cause the coil to rotate slower, thus producing a back emf less than that of the battery so that it is not completely canceled and the net emf is not

zero leading to an amount of current being drawn by the motor. In the extreme case, when the motor shaft is fixed and not allowed to rotate at all, there is zero back emf and the net emf is only that of the battery so that the drawn current is maximum, which might damage the coil. In summary, as long as the motor does not do any useful work, the current drawn by the motor, and therefore the power delivered to the motor, is due to the frictional losses of the rotating parts of the motor.

In addition, the speed of rotation of a DC motor is related to the voltage applied between its terminals. Therefore, the voltage supplied to the motor can be taken as a measure for its speed of rotation. Now, does increasing the voltage and thereby increasing the speed of rotation, affect the power loss?

IV. Procedure

A. Variable Selection

1. **Independent Variable:** The speed of rotation of the motor measured using a photogate counting the number of revolutions per second. The speed is changed by changing the voltage supplied to the motor.
2. **Dependent Variable:** The power dissipated by the motor calculated by multiplying the current through the motor and the voltage supplied to the motor.
3. **Controlled Variables:**
 - The temperature of the liquid should be kept constant throughout the experiment since increasing the temperature would reduce the viscosity significantly.
 - The diameter of the rotating cylinder as well as the length submerged into the liquid determine the area that is in contact with the liquid.
 - These parameters must be constant throughout the experiment.

Hypothesis

- As the speed of rotation increases, the power loss is expected to increase.

Materials:

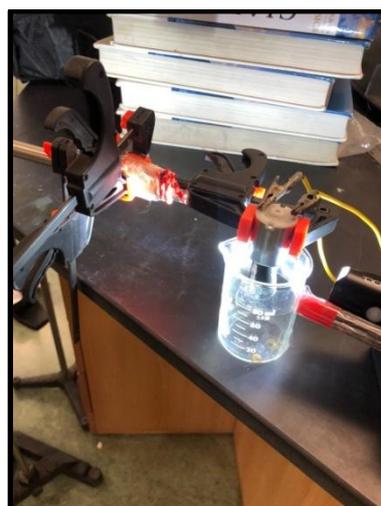
1. A DC motor with a metal cylinder of known dimensions connected to the rotating shaft of the motor.
2. A cylindrical container to hold the viscous liquid. The inner diameter of the cylindrical container should be around 1 cm larger than the outer diameter of the rotating cylinder.
3. A variable voltage source to supply the motor with different voltages to achieve different speeds of rotation.
4. A voltmeter and an ammeter to measure the voltage and current of the motor.
5. Clamps to hold apparatus in place
6. Superglue to secure cylinder to motor
7. Metal stand to hang motor from





Steps:

1. Fill the cylindrical container with the viscous liquid.
2. Fix the motor with its axis of rotation being vertical and the rotating cylinder being immersed into the liquid.
3. Connect the terminals of the motor to the voltage source.
4. Connect the voltmeter in parallel with the motor and the ammeter in series with the motor.
5. Set the voltage source to its minimum output voltage then start increasing the voltage gradually until the motor starts rotating.
6. Interpret the rotation speed by implementing fact that velocity is proportional to voltage
7. Measure the values of the voltmeter and ammeter and calculate the power.
8. Record the voltage of the motor and the power then increase the applied voltage and measure the voltage and power for five other applied voltages.



V. Data Collection

Raw Data:

Table 1 below shows how the current through the motor varied with the voltage across its terminals. The first column, consisting of two sub column presents the voltage applied by the power supply, as well as its absolute error. The second main column shows the the current, I , in milliamperes, (mA), across all five trials. The third main column shows the voltage across the motor across all five trials. The voltage across the motor terminals differs from that supplied by the source due to the internal resistance of the measuring devices as well as the resistance of the connecting wires. All data was calculated as seen in the table for seven different applied voltages. The power used by the motor when the rubber cylinder was immersed in the oil, and rotated about a

stable axis, is calculated as the product of the voltage across the motor and the current through it. The motor, being connected to a power supply through two wires, would allow the transformer to display the voltage and current. Seven different voltages were used, and each tried five times in order to minimize random errors. For instance, the first trial modeled the current and voltage through the motor, when the voltage applied by the transformer was 0.5 V. As seen in the data, trial 1, 2, 3, 4, and 5 obtained the currents of 192.5, 229.8, 221.4, 257.7 and 238.2 mA, respectively. However, given the limitations of the apparatus used, there were certain uncertainties in the data calculated. The absolute error of the voltage was 0.1V as seen below, since the digital voltmeter reading was given to 1 decimal place.

Table 1: Voltage and Current

V _{Applied} (V)		I (mA)					V _{Motor} (V)				
Value	Abs. Err.	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5
0.5	0.1	192.5	229.8	221.4	257.7	238.2	0.4931	0.4846	0.4789	0.5234	0.5215
0.7	0.1	286.8	305	300.9	302.6	294.1	0.6755	0.6775	0.6796	0.6519	0.658
0.9	0.1	379.8	385.8	391.9	416.9	413.1	0.9075	0.8961	0.9018	0.8851	0.9224
1.1	0.1	414.6	460.1	431	507.5	470.1	1.103	1.105	1.079	1.084	1.075
1.3	0.1	487.3	514.1	489.1	585.7	540.3	1.314	1.287	1.272	1.272	1.3
1.5	0.1	555.5	586.8	553.7	663.7	605.3	1.449	1.474	1.505	1.496	1.504
1.7	0.1	685.2	656.4	616.9	743.4	673.3	1.695	1.666	1.651	1.744	1.659

Processed Data

In Table 2, the variation of voltages and currents is once again depicted, however, this time with a calculated average for the five trials, as well as an absolute error for each calculation. As observed in the graphs, as the voltage applied increases, the voltage of the motor, quite obviously increased as well. This also leads to an increase in current. In order to calculate the average current for each voltage applied value, the data from trials 1, 2, 3, 4 and 5, were added together and divided by five. Upon calculating an average current, the absolute error in the calculation was recorded. The calculation of this is as follows: the minimum trial value for the current was subtracted from the maximum value, and then divided by two. These calculations were repeated for all seven voltages applied. In regards to the voltage of the motor, the same process in essence took place. To calculate the average value of the voltage across the motor, the data from all five trials was added, and then divided by five. To calculate the absolute error in the data, the minimum motor voltage was subtracted from the maximum value, and then divided by two.

Table 2: Voltage and Current, with averages and uncertainties

V _{Applied} (V)		I (mA)							V _{Motor} (V)						
Value	Abs. Err.	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Avg.	Abs. Err.	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Avg.	Abs. Err.
0.5	0.1	192.5	229.8	221.4	257.7	238.2	227.92	32.6	0.4931	0.4846	0.4789	0.5234	0.5215	0.5003	0.02225
0.7	0.1	286.8	305	300.9	302.6	294.1	297.88	9.1	0.6755	0.6775	0.6796	0.6519	0.658	0.6685	0.01385
0.9	0.1	379.8	385.8	391.9	416.9	413.1	397.5	18.55	0.9075	0.8961	0.9018	0.8851	0.9224	0.90258	0.01865
1.1	0.1	414.6	460.1	431	507.5	470.1	456.66	46.45	1.103	1.105	1.079	1.084	1.075	1.0892	0.015
1.3	0.1	487.3	514.1	489.1	585.7	540.3	523.3	49.2	1.314	1.287	1.272	1.272	1.3	1.289	0.021
1.5	0.1	555.5	586.8	553.7	663.7	605.3	593	55	1.449	1.474	1.505	1.496	1.504	1.4856	0.028
1.7	0.1	685.2	656.4	616.9	743.4	673.3	675.04	63.25	1.695	1.666	1.651	1.744	1.659	1.683	0.0465

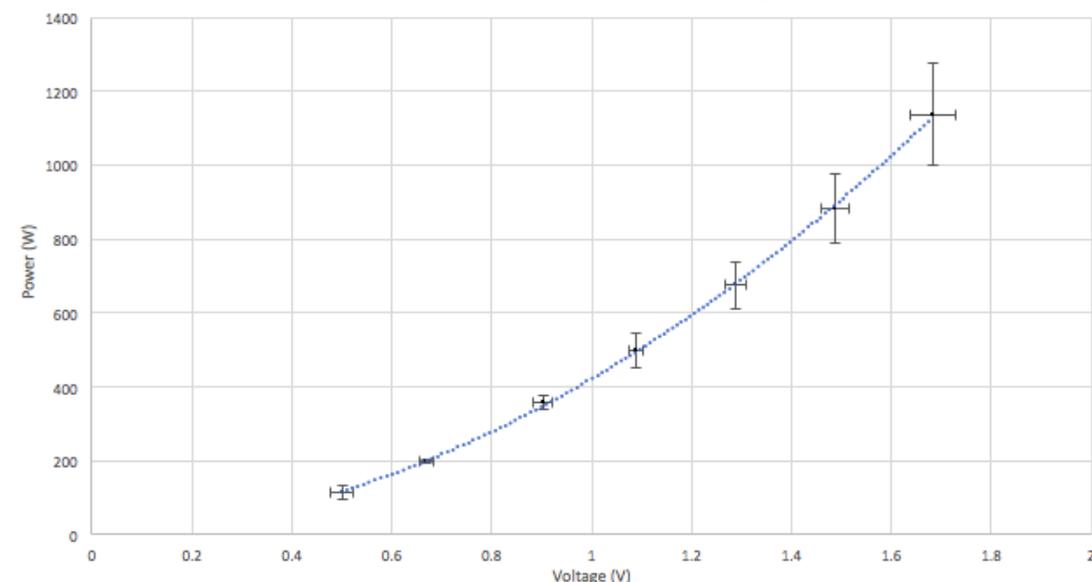
Below in Table 3, a column for the power used by the motor is added. Power is given in milliWatts and is calculated as the product of the current and voltage of the motor. The power used by the motor was calculated for five trials, across the seven various voltages. As hypothesized, the power should increase as the applied voltage increases. As seen in the table below, this was indeed the case. For each trial, to calculate the power, the voltage and current from that same trial in Table 2 were multiplied by each other. Afterwards, an average value for power was calculated per each value of applied voltage. This was done by adding the power for each trial then dividing by five. Once again, the uncertainties were also measured for each calculated average power value. In order to calculate the absolute error, the minimum power value was subtracted from the maximum, and then the obtained value was divided by two. In order to calculate the relative error, the absolute error for the same applied voltage, was divided by the average power for the same applied voltage.

Table 3: Voltage and Power, with averages and uncertainties

V _{Motor} (V)							Power (mW)							
Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Avg.	Abs Err.	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Avg.	Abs Err.	Rel. Err.
0.4931	0.4846	0.4789	0.5234	0.5215	0.5003	0.02225	94.92175	111.36108	106.02846	134.88018	124.2213	114.282554	19.979215	0.174822966
0.6755	0.6775	0.6796	0.6519	0.658	0.6685	0.01385	193.7334	206.6375	204.49164	197.26494	193.5178	199.129056	6.55985	0.032942706
0.9075	0.8961	0.9018	0.8851	0.9224	0.90258	0.01865	344.6685	345.71538	353.41542	368.99819	381.04344	358.768186	18.18747	0.050694211
1.103	1.105	1.079	1.084	1.075	1.0892	0.015	457.3038	508.4105	465.049	550.13	505.3575	497.25016	46.4131	0.093339538
1.314	1.287	1.272	1.272	1.3	1.289	0.021	640.3122	661.6467	622.1352	745.0104	702.39	674.2989	61.4376	0.091113303
1.449	1.474	1.505	1.496	1.504	1.4856	0.028	804.9195	864.9432	833.3185	992.8952	910.3712	881.28952	93.98785	0.106648097
1.695	1.666	1.651	1.744	1.659	1.683	0.0465	1161.414	1093.5624	1018.5019	1296.4896	1117.0047	1137.39452	138.99385	0.122203728

The data from Table 3, was graphed as seen below in Figure 1. Power in mW, was graphed as a function of Voltage in V. Obviously, the relation does not seem to be linear. Instead, the graph is closer to a quadratic relationship.

Figure 1: Power as a Function of Voltage



However, in order to more easily interpret the quadratic relationship, the data was linearized. This was done by plotting the square root of the power versus the voltage. The values for the square root of the power are listed in the table below.

Table 4: Voltage and Square Root of Power, with absolute error

VoltageMotor (V)		√ Power (W)		
Avg.	Abs Err.	Avg.	Abs Err.	Rel. Err.
0.50	0.02	10.7	0.9	0.09
0.67	0.01	14.1	0.2	0.02
0.90	0.02	18.9	0.5	0.03
1.09	0.02	22.3	1.0	0.05
1.29	0.02	26.0	1.2	0.05
1.49	0.03	29.7	1.6	0.05
1.68	0.05	33.7	2.1	0.06

The data in the table above is plotted in Figure 2 below

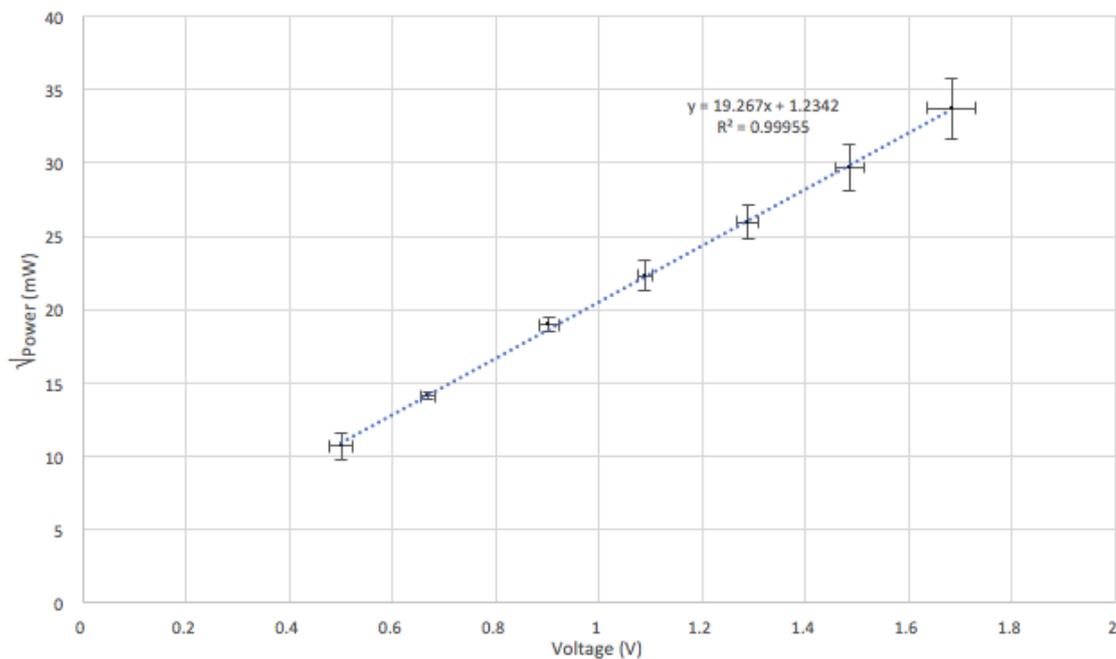
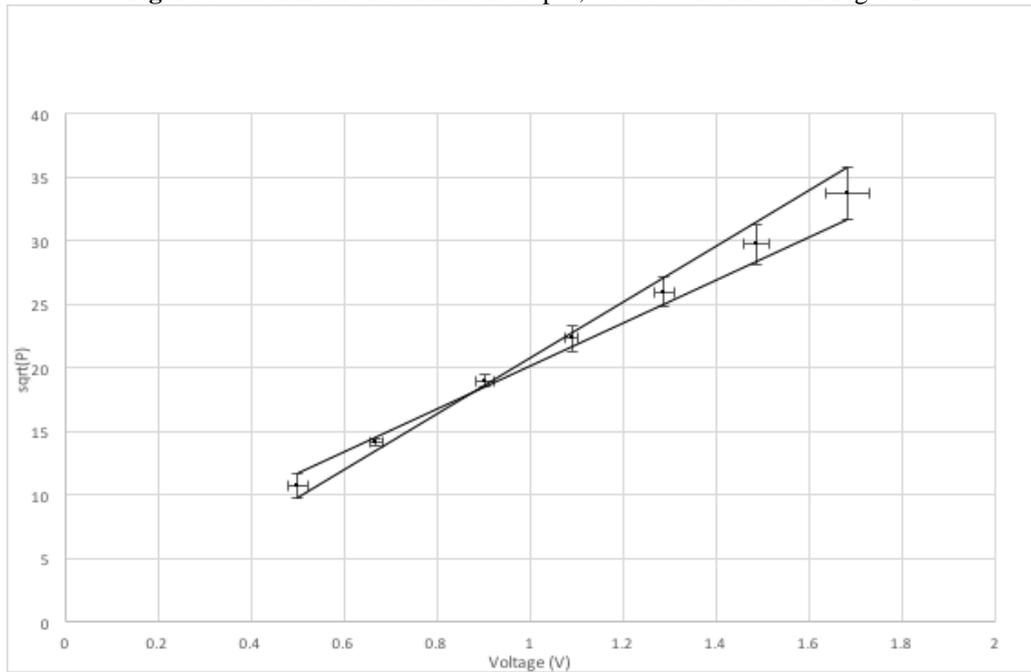


Figure 2: Linearization of P(V) Function

The slope of the line is 19.3 and the y-intercept is 1.2. The line of best fit passes through all points within the error bars indicating a linear relationship between the square root of the poer and the applied voltage. The R^2 value of 0.99955 further verifies the linear relationship. The uncertainty of the slope will be calculated using maximum and minimum slopes, shown in Figure 3.

Figure 3: Maximum and minimum slopes, with best fit line from Figure 2



$$\begin{array}{rcl}
 \text{maxslope} & - \text{bestfitslope} & = 22.0 - 19.3 = 2.8 \\
 \text{bestfitslope} & - \text{minimumslope} & = 19.3 - 16.9 = 2.4 \\
 \text{slope} & \text{h uncertainty} & = 19.3 \pm 2.8
 \end{array}$$

The maximum and minimum slope calculated were: 22.0 and 16.9, respectively. However, in Figure 2, the slope of the line obtained from linearization was $19.3 \sqrt{W}/V$. On the other hand, in Figure 3 above, the maximum and minimum slopes were obtained and illustrated the uncertainties. By subtracting the slope of the best fit line from the maximum slope, the value obtained is 2.8. When the minimum slope is subtracted from the slope of the best fit line, the value obtained is 2.4. This suggests that the slope of the best fit line is more accurately presented as: $19.3 \pm 2.8 \sqrt{W}/V$.

Safety

During the experiment multiple safety precautions were set up. This was to ensure that nothing or nobody is harmed during the process. In order to minimize splashing effects, the motor was held tightly and centered in the middle of the beaker. The cylinder was rotating about a stable axis, as to not hit the sides of the beaker. Safety gloves were utilized with the purpose of preventing any oil from staining the hands. Safety glasses were also used as a means of preventing any splashing from contaminating the eyes. The environment around the apparatus was kept dry as to limit the exposure of the power supply and wires to liquids such as water.

VI. Evaluation

The graph in Figure 1 shows the relationship between the power consumed by the motor as function of the applied voltage. It has been seen that increasing the voltage caused the power used by the motor to increase. This agrees with the hypothesis. In fact, as the applied voltage increases, the speed of rotation increases. The power loss due to friction between the outer surface of the rotating cylinder, and the fluid, is the the product of the linear speed of a point on the outer surface and the frictional force. According to the equation mentioned previously:

$$F = \eta v A / d$$

the frictional force itself is proportional to the linear speed of the outer surface of the rotating cylinder. Now, the power is proportional to the speed and also proportional to the force which is proportional to the speed. Therefore, the power is proportional to the square of the speed. And for a DC motor, it can be assumed that the speed is proportional to the applied voltage. This explains the parabolic shape of the curve obtained in

Figure 1. Furthermore, when plotting the square root of the power versus voltage, an almost linear relationship has been obtained. The line of best fit was within the error bars and the R-squared value was close to 1, which verifies the previous analysis.

During the experiment, several limitations have been observed. First, when the motor starts rotating, water started to splash away from the container causing the depth of the rotating cylinder that is immersed into the liquid, to change. This affected the frictional force between the liquid and the rotating cylinder. This problem can be solved by using a container with higher walls so that the water drops that splash from the liquid are not lost and go back to the liquid. In addition, the rotating cylinder was always vibrating in such a way that the upper surface of the liquid can not be considered flat. Again, the depth of the rotating cylinder under water surface can not be determined accurately. In order to solve this problem, the motor and the rotating cylinder must be fixed in such a way to avoid extreme vibrations.

VII. Conclusion

Upon reviewing the data and curves obtained, it was established that the power is proportional the voltage squared. To restate the research question: When a rotating cylinder is immersed into a liquid, what is the relation between the power loss due to viscosity of the liquid and the speed of rotation of the cylinder which is proportional to the voltage supplied? Taking into consideration that the speed of rotation of a DC motor can be considered proportional to the applied voltage, it was hypothesized that the power loss will increase with increasing speed, thus with increasing voltage. However, a quadratic relationship was established which was interpreted and theoretically explained.

Extension

One way of improving the results of this experiment, is to measure the speed of rotation directly rather than measuring the voltage and taking it as a measure for the speed. This can be done by using a photogate and attaching light obstacle that cuts the beam once every revolution. Then by measuring the time between the cuts, the period can be measured from which the speed of rotation can be calculated.

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