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Harmonic Oscillator Solution for Free and Time Independent Potential String within the Framework of Dirac Special Relativistic Equation

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Abstract:*It is well known that Dirac relativistic equation describes the relativistic particle which is either free or move in any potential field. Even if one utilized the electromagnetic Hamiltonian which recognizes the effect of electromagnetic potential, the time dependent part of Dirac equation gives the same form which is free from the potential term if the potential is time independent. In this case the time dependent part gives a sine or cosine solution describing quantized string energy like that of harmonic oscillator but with no zero point energy. However if one follows De Broglie hypothesis and assumes that the string oscillator is in the form of a highly localized wave packet the time dependent part gives cosine solution with energy expression typical to that of Schrodinger equation which recognizes the existence of zero point energy. This means that when the wave packet islocalized in the form of a particle it has a rest mass energy corresponding to the zero point energy*

Key Words:*Klein Gordon equation, string theory, wave packet, harmonic oscillator, zero point energy, rest mass energy*

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I. Introduction:

String theory is one of the most well-known spectacular physical theories. It's main idea is based on the fact that elementary particles are in the form of vibrating oscillating strings [1]. A lot of different theoretical models were proposed to describe physical systems unfortunately these models, which utilizes complex mathematics, are unable to describe physical phenomena completely [2,3].

The problems of these models stems from the use of complex random action integrals and commutation relations which cannot be used to derive simple equation suitable for describing physical systems [4,5]

This motivates some authors and researchers to propose simple elegant models that can solve some long standing problems. One of the most promising problems are those which are proposed by M. Dirar, Lutfi, Inas and Nagwa, In most of these approaches the energy expression was obtained by treating particles as vibrating harmonic oscillating strings [6,7]. There models succeeded in solving many problems for many physical systems [8]. For example it was utilized to solve some super conductivity problems beside derivation of the radioactive decay law

The String Model Based on the Klein Gordon Equation

The function f in spherical coordinate takes the form

$$\nabla f = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\hat{\phi} (1)$$

If one considers f to depend on r only , one gets

$$\nabla f = \frac{\partial f}{\partial r}\hat{r} \qquad (2)$$

on the other hand Klein Gordon Equation takes the form

$$-\hbar\frac{\partial^2 \psi}{\partial t^2} = -c^2\hbar^2\nabla^2\psi + m_0^2c^4\psi \qquad (3)$$

To simplify this equation let:

$$\psi = u(r)f(t) = uf \ (4)$$

Thus separating time and spatial part gives

$$-\frac{\hbar^2}{f}\frac{\partial^2 f}{\partial t^2} = -\frac{1}{u}c^2\hbar^2\nabla^2 u + m_0^2c^4 = E^2 \qquad (5)$$

Where E is a constant parameter having the dimension of energy separating the time part, one gets

$$-\hbar^2\frac{\partial^2 f}{\partial t^2} = E^2 f \qquad (6)$$

This equation can be solved by suggesting the solution

$$f = A\sin\alpha t\ \dot{f} = \frac{\partial f}{\partial t} = \alpha\cos\alpha t\ \ddot{f} = -\alpha^2 f (7)$$

Thus inserting (7) in (6) gives

$$t\alpha^2\hbar^2 f = E^2 f$$

Thus

$$\alpha^2\hbar^2 = E^2 (8)$$

Let f be in the form of a pure wave. This it satisfy periodicity condition.

$$f(t+T) = f(t) \qquad (9)$$

Hence (7) gives

$$A\sin\alpha(t+T) = A\sin\alpha t$$

$$\sin\alpha t\cos\alpha T + \cos\alpha t\sin\alpha T = \sin\alpha t \ (10)$$

This requires

$$\cos\alpha T = 1$$

$$\sin\alpha T = 0 \qquad (11)$$

This requires

$$\alpha T = 2n\pi$$

$$\alpha = \frac{2\pi n}{T} nw (12)$$

In view of equation (8) one gets

$$E = \pm\hbar\alpha = \pm n\hbar w \qquad (13)$$

Thus if the proton satisfies periodicity condition, its energy is quantized consider also a solution

$$f = A\cos\alpha t \ (14)$$

Direct substitution in (4) gives

$$\hbar^2\alpha^2 f = E^2 f$$

Thus again

$$E = \pm\hbar\alpha (15)$$

Using the periodicity condition

$$f(t+T) = f(t)$$

$$\cos(\alpha t + \alpha T) = \cos\alpha t \ (16)$$

$$\cos\alpha T\cos\alpha t - \sin\alpha T\sin\alpha t = \cos\alpha t$$

Which requires

$$\cos\alpha T = 1 \qquad \sin\alpha T = 0 \qquad (17)$$

Again

$$\alpha T = 2n\pi$$

$$\alpha = \frac{2n\pi}{T} = nw \qquad (18)$$

According to equations (15) and (18) the photon energy is given by

$$E_p = E = \pm h\alpha = \pm n\hbar w (19)$$

The minus and plus signs may be related to the atomic transition

$$E_f = E_i + E_p \qquad (20)$$

Where the plus here means that n photons are absorbed by the medium, while the minus sign indicated emission of n photons by the medium. The energy E_f is the final energy of the medium while E_i is the initial energy of the medium.

These solutions are dependent on the periodic time change of the photons number, where the photons or oscillators behave as continuous electromagnetic waves.

Another approach or solution can be suggested if we use the De Broglie hypothesis by considering the wave function as a wave backet. If one assumed that the photon is a localized particle formed from a very large number of waves. The photon will look like a half wave.

Fig (1) a photon formed a very large number of particles thus is highly localized in the form of a half wave.

Consider now a solution of Eq. (6) in the form

$$f = A\cos\alpha t \ (21)$$

Then assume that just outside the localized wave backet the photon does not exist. This means that [see fig (1)]

$$a\cos\frac{1}{2}\alpha T = 0 \qquad (22)$$

$$\frac{1}{2}\alpha T = \left(n + \frac{1}{2}\right)\pi$$

$$\alpha = \frac{2\pi\left(n + \frac{1}{2}\right)}{T} = \left(n + \frac{1}{2}\right)w (23)$$

According to equation. This resembles the ordinary harmonic oscillator solution, where equations (15), (19) and (33), show that the string energy is given by

$$E = \pm\left(n + \frac{1}{2}\right) (24)$$

II. Discussion:

The ordinary Klein-Gordon in Eq. (3) is spitted into time dependent and time independent parts by suggesting the wave function to split into time dependent multiplied by time independent functions as shown by equation (4), to get pure time dependent equation [see eq (6)]. Assuming sine solution in (7) and assuming the oscillator behaving as a continuous or a train of discrete waves the periodicity condition in (9) and (10) shows that the energy is quantized in the form of discrete quanta as shown by eq (13), with plus and minus signs. Suggestion of a cosine solution in (14) and applying periodicity condition again leads to the harmonic oscillator energy in eq (19) but with no zero point energy.

However, when one considers the oscillating vibrating string in the form of a localized wave, the periodicity conditions becomes that of a wave packet rather than that of a wave train. With cosine function solution shown in eq (21) and the assumption that just outside the localized wave packet the particle d does not exist [see eq (23)], a new expression of energy was found. This energy expression in eq (24) shows that the string energy is typical to that of the ordinary harmonic oscillator.

The zero point energy corresponds to the rest mass energy of the string, i.e.

$$m_0c^2 = \frac{1}{2}\hbar w$$

Whereas the total relativistic energy takes the form

$$E = mc^2 = \left(n + \frac{1}{2}\right)\hbar w$$

III. Conclusion

The solution of the Klein Gordon equation shows that the energy is quantized by applying periodicity conditions and assuming the particles as continuous wave trains and as a localized wave group. Strikingly the solutions show that all particles behave as strings no matter what the potential is. When one consider strings to be highly

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