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The Phase Transitions and the Moduli Space of Particles from D-Brane

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Abstract:
The study of the birth of the Universe is closely related to the study of phase transitions in high-energy physics implemented in the framework of the theory of derived categories. Using the theory of D-branes and superstrings, the properties of the C^3/Z_3 *orbifold as a space of extra dimensions were studied. In the framework of the criterion of stability of the D-brane as bound state of fractional branes* O_{P2} *and* $O_{P2}(-3)$ *the impossibility of phase transition of one sheaf into another one was shown. The category of distinguished triangles with objects - McKay quivers and morphism between them -* $Ext^q(A,B)$*-group was used for the calculation of the number of vibrational modes of the string presented by Poincare supergravity with* $N = 4$*.*

Key Word*: Phase transitions; Moduli space; D-brane; Orbifold; McKay quiver.*

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I. Introduction

The study of the origin and evolution of the Universe is one of the key issues in experimental and theoretical high-energy physics. The experiment is associated with searches for microscopic black holes, extra dimensions, Kaluza-Klein partners of gravitons, supersymmetry at the LHC, [1], and in cosmology with Planck experiment of the polarized cosmic microwave background, [2]. These exotic particles are associated with the vibrational modes of closed or open superstrings with ends fixed on D-branes. The moduli space of such strings is connected with gauge symmetry from D-branes and contains information about the matter content of the early Universe.

The theory of superstrings and D-branes, [3], AdS/CFT correspondence, [4]and the theory of black holes, [5]are linked to the theoretical constructions of the evolution of the Universe. The LHC experiments can have sensitivity to extra dimensions through the production of new particles which move in the space of the size of about 10^{-17}cm. Due to the extra dimensions, microscopic black holes may be detected at the LHC. Ultimately the physics of extra dimensions affects on the astroparticle physics assuming that there are dark energy, dark matter and cosmic inflation. Collider data and theoretical investigations will penetrate deep into explanation of this connection.

So, our investigations are connected with the application of the theory of superstrings and D-branes to the further study of the relationship between micro and macrophysics for obtaining the corresponding conclusions in high energy physics connected with number of particles of moduli space.

The article is devoted to the study of the coherent sheaf on C^3/Z_3 in terms of McKay quiver associated to the orbifold singularity, its phase transition into another space, as well as to the study of the number of fields associated with moduli space of coherent sheaf on C^3/Z_3.

II. Ideology of phase transitions in superstring and D-brane theory

In superstring theory, [6] the construction of realistic superstring models requires dimensional reduction from 10-dimensional space to the observed 4 dimensions of space- time. Constraints on the theories are connected with the restrictions on the compactified space which must be 6-dimensional Calabi-Yau manifold or degenerate examples of Calabi-Yau manifolds - orbifold. An important role in fundamental description of non-perturbative string theory is played by non-abelian gauge fields and conformal field theory, [7]. Studies of theoretical models in the space-time with extra dimensions proposed byT.Kaluza and O.Klein, [8] in the 20th gave the origin to a theoretical approach and to the conclusion that the extra dimensions proposed by physicists must be too small for us to observe them directly and they have an important effect on the observed physical phenomena. In string theory, this connection between microscopic properties of space and observable physical phenomena is evident as extra dimensional geometry defines fundamental physical properties such as particle masses and charges. To receive vibrational string modes suitable for Standard Model particles it is necessary to have string theory with the coresponding extra dimensional space of Calabi-Yau type. The advantage of such a theory is in inclusion of gravitational interaction into a quantum mechanical scheme. The description of gravity at lengths of the order of the Planckian, requires a revision of the theory of general relativity. The concept of Riemannian geometry, which lies at the heart of general relativity must be modified at small distances in string theory. Such approach is valid only when the structure of the Universe is considered on a sufficiently small scale in the early stages of the Universe. At the lengths on the order of the Planck length, a new geometry called quantum geometry must be consistent with the new physics of string theory. The fluctuations in space at short distances suggest that punctures and breaks could be common phenomena in the microscopic structure of space. Such concept is connected with space-time wormholes within the original Calabi-Yau space, [9]. Webbing in space is connected with the contraction of the sphere to one point, (Fig.1a), which expands and rebuilds into another spherical figure (Fig. 1b), of normal size.

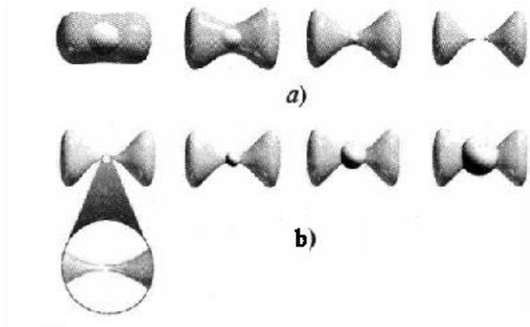


Fig.1. *The evolution of the sphere inside Calabi-Yau space to one point a) and its subsequent rebuilding intoanother spherical figure b), from [10].*

Mathematically, this procedure consists of the blowing off rational curves on a Calabi-Yau manifold. Everything happens as if a bouncy ball is "turned inside out" inside another Calabi-Yau space. Tian, Yau and other mathematicians have shown that under certain conditions the new Calabi-Yau manifold will be topologically different from the original, [11]. In 1991, some string physicists had a clear sense that the fabric of space could break, which is connected with the question of whether flop rearrangements may be in the Universe described by string theory. But the collapse of the three-dimensional sphere inside the Calabi-Yau space can have catastrophic consequences for our Universe. In 1995 Andrew Strominger, [12] showed that three-dimensional string theory objects, i.e.3-branes, can envelop and completely cover three-dimensional spheres, precisely compensating for the potentially catastrophic consequences of a possible collapse of the three-dimensional sphere. Such phase transition from the initially nonzero mass of three-dimensional sphere or black hole to zero during the evolution of the Calabi-Yau manifold in the language of string theory is described by the new massless vibrational mode of a string. Thus, in string theory a direct, precise connection between black holes and elementary particles is established. The description of particle interactions in a multidimensional space-time by the notion of localization of fields on branes was proposed in [13]. The model provides the dynamical localization of fields on the hyperplane or three-dimensional space which is referred to as 3-brane embedded into the four-dimensional space. The most important development in string theory is the discovery of D-brane as a subspace of the target space-time on which open strings may end [14]. So, schematically the picture of black hole evolution can be presented byFig.2

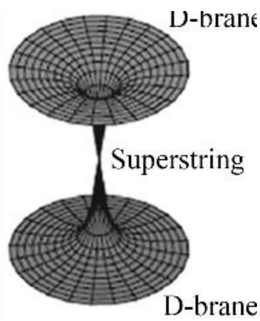


Fig.2. *Schema of phase transition of one D-brane into another one through superstring.*

We are interested in a string propagating on the orbifold C_d/G which have played an important role in the understanding of stringy geometry. So, D-branes on orbifolds provides further insight into the properties of D-branes. According to Aspinwall, [14], D-branes on the orbifold C_d/G, are fractional branes, and open strings between them are described by the derived category of McKay quiver representations [15].

It was assumed that the holomorphic vector bundles representing D-branes could be described in terms of coherent sheaves - elements of derived category, [16]. An object of the derived category is a complex of sheaves

$$\ldots \to C^2(\mathcal{F}) \to C^1(\mathcal{F}) \to C^0(\mathcal{F}) \to \ldots$$

with sheaves $C(\mathcal{F})$ and morphisms between them presented by arrows. Non-locally-free sheaves correspond to some phases of the moduli space. So, phase transitions from one to another D-brane could be presented by the arrows in terms of the moduli space.

We will define two sets of D-branes at the origin of C^3, and the Z_3quotient of $C^3$and talk about morphisms or open strings connecting D-branes in terms of Ext groups between D-brane configurations, [17]. According to [18] the partial resolutions of orbifold can be described as moduli spaces of representations of the McKay quiver, Fig. 3.

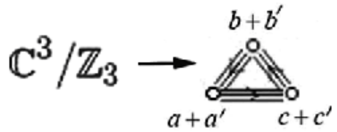


Fig.3. *Orbifold resolution through McKay quiver.*

So, it would be interesting to find the elementary particles, which are treated as vibrational modes of strings,connected with a consistent quantum theory, Fig.4.

Fig.4. *Quiver representations of D-branes and morphism between them by Ext-group.*

Phase transition from the initially nonzero mass of three-dimensional sphere or black hole to zero during the evolution of the Calabi-Yau manifold is connected in our case of orbifold compactifications with procedure of blowing up into fully smooth Calabi-Yau manifolds. Using toric geometry, [19], we can resolve the singularity to $O_{P_n}(-3)$ sheaf in order to access the entire moduli space of the compactified manifold. The toric fan for C^3/Z_3 is given in Fig. 5, where the fan is spanned by the vectors v1,v2, v3 (left) and one interior lattice point, (0; 0; 1) which causes the singularity (center). The singularity is resolved (right) by introducing **w** vector and the associated exceptional divisor.

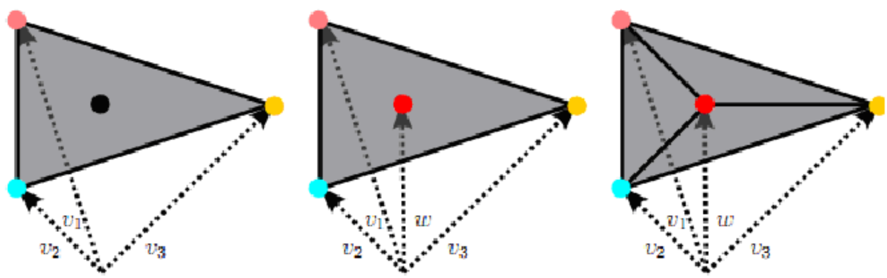


Fig.5. *The procedure of blowing up (from toric diagram through refined toric diagram to triangulated toric diagram) of the orbifold C^3/Z_3 singularity, from [19].*

III. The study of the phase transition near the singularity of the orbifold C^3/Z_3

The phase transition of orbifold C^3/Z_3 into another space can be studied using the homological algebra, through the category of complexes of coherent sheaves. As was said above, the resolution of orbifold singularity for getting information about moduli space is realized by blowing up procedure to O_{P_n} (-3) sheaf.

Such sheaves are called fractional D-branes and mathematically described by Serre's twisted sheaves, [20], defined asfollows:

$$\mathcal{O}_{P_n}(k) = \begin{cases} \mathcal{O}_{P_n}(1)^{\otimes k} & for\ k > 0 \\ \mathcal{O}_{P_n}(-1)^{\otimes k} & for\ k < 0 \end{cases},$$

where $\mathcal{O}_{P_n}(-1)$ satisfies the exact sequence

$$0 \to \mathcal{O}_{P_n}(-1) \to \mathcal{O}_{P_n} \to \mathcal{O}_{P_{n-1}} \to 0$$

with $\mathcal{O}_{P_n}$ structure sheaf of the complex projective space P_n. Dual to sheaf $\mathcal{O}_{P_n}(-1)$ is

$$\mathcal{O}_{P_n}(1) = Hom(\mathcal{O}_{P_n}(-1), \mathcal{O}_{P_n}) = \mathcal{O}_{P_n}(-1)^{\star}.$$

All these sheaves are stable and correspond to fractional D-branes with RR-charge. Since these objects are defined on compact space, the Dirac operator theorem, known as the Euler characteristic theorem is applied to them. Let's calculate the Euler matrix, which is associated with RR-charges for the sheaves:

$$\chi(\mathcal{O}_{P_2}(l), \mathcal{O}_{P_2}(m)) = \chi(\mathcal{O}_{P_2}(m-l))$$

In particular, the Euler matrix for sheaves, $\mathcal{O}_{P2}$, $\mathcal{O}_{P2}(1)$, $\mathcal{O}_{P2}(2)$ looks like this

$$\chi(\mathcal{O}_{P_2}(l), \mathcal{O}_{P_2}(m)) = \begin{pmatrix} 1 & 3 & 6 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix}$$

During transposition, this matrix becomes a matrix

$$\begin{pmatrix} 1 & 3 & 6 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix} \xrightarrow{T} \begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 6 & 3 & 1 \end{pmatrix}$$

The rows of matrices are RR-charges that characterize the sheaves:

$$\mathcal{O}_{P_2}(-3) = (6\ 3\ 1), \mathcal{O}_{P_2}(-2) = (3\ 1\ 0), \mathcal{O}_{P_2}(-1) = (1\ 0\ 0),$$
$$\mathcal{O}_{P_2} = (0\ 0\ 1), \mathcal{O}_{P_2}(1) = (0\ 1\ 3), \mathcal{O}_{P_2}(2) = (1\ 3\ 6)$$

These sheaves can be recorded through large volume charges (Q_4 Q_2 Q_0)

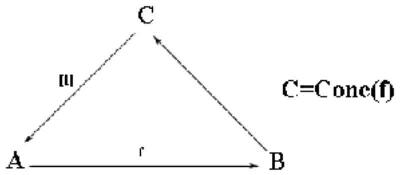
$$Q_4 = n_1 - 2n_2 + n_3,\ Q_2 = -n_1 + n_2,\ Q_0 = \frac{n_1 + n_2}{2}.$$

Therefore the next sheaves describe fractional D-branes:

$$\mathcal{O}_{P_2}(-3) = (1\ -3\ 9/2), \mathcal{O}_{P_2}(-2) = (1\ -2\ 4/2), \mathcal{O}_{P_2}(-1) = (1\ -1\ 1/2),$$
$$\mathcal{O}_{P_2} = (1\ 0\ 0), \mathcal{O}_{P_2}(1) = (1\ 1\ 1/2), \mathcal{O}_{P_2}(2) = (1\ 2\ 4/2)$$

If Q_4= 0 then we get a D2-brane, and if Q_4= 0, Q_2= 0 then we get a D0-brane.

We turn to the problem of phase transition or stability of D-branes, which are potential bound states of fractional D-branes. Let's use the construction of distinguished triangle (1) [21]



(1)

To determine the stability of the sheaf C, we use the grading concept

$$\xi(A) = \frac{1}{\pi} arg\ Z(A)$$
$$\xi(B) = \frac{1}{\pi} arg\ Z(B)$$

To study the stability of the sheaf C it is necessary to calculate the square of the mass of the string fconnecting sheaves A and B, [22]

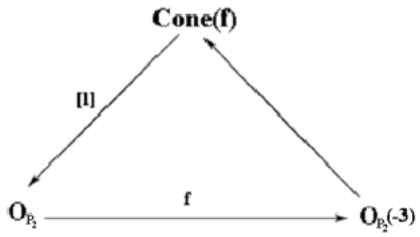
$$m^2 = \frac{1}{2}(\xi(B) - \xi(A) + q - 1) \tag{2}$$

The criterion of stability of the D-brane C says:

(*)D-brane is stable with respect to decay into D-branes A and B **if** $q < 1$;

(**)D-brane is unstable with respect to decay into D-branes A and B **if** $q > 1$.

Suppose we have the distinguished triangle



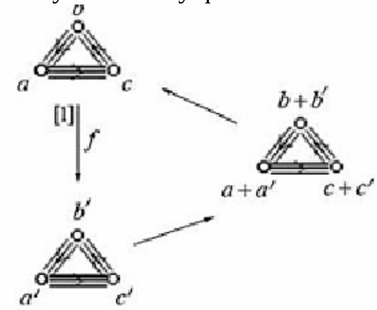
where *Cone*(*f*) = $\mathcal{O}_{P2}[1] \oplus \mathcal{O}_{P2}(-3)$ is the bound state of two sheaves $\mathcal{O}_{P2}$ and $\mathcal{O}_{P2}(-3)$. In this case $Ext^q(A,B)$-group is morphism or open string connecting sheaves $\mathcal{O}_{P2}$ and $\mathcal{O}_{P2}(-3)$:

$$Ext^2(\mathcal{O}_{P_2}, \mathcal{O}_{P_2}(-3)) = H^2(P^2, \mathcal{O}^\star_{P_2} \otimes \mathcal{O}_{P_2}(-3))$$

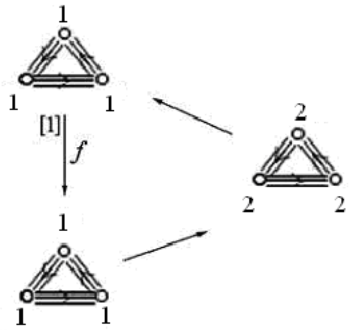
and according to Serre's duality theorem it is equal to $H^0(P^2, \mathcal{O}_{P2})$, which is the space of sections of structure sheaf $\mathcal{O}_{P2}$. If q = 2 the mass $m^2>0$ and D-brane *Cone*(*f*) is unstable and decay into $\mathcal{O}_{P2}$ and $\mathcal{O}_{P2}(-3)$. So, the investigation of the phase transition of D-brane *Cone*(*f*) demonstrates its decay into the fractional branes and shows the impossibility of phase transition of $\mathcal{O}_{P2}$ into $\mathcal{O}_{P2}(-3)$ sheaf.

IV. The study of the spectrum near the singularity of the orbifold C^3/Z_3

For studying the spectrum near the singularity of the orbifold C^3/Z_3, let's consider the distinguished triangle (1) in the case of *A*,*B*,*C*, sheaves are replaced by the McKay quivers for orbifold spaces:



Since the phase transition to the same state is one of the most interesting from the point of view of both high-energy physics and cosmology [5], let's consider the phase transition represented by a string f of the following form



This state called D0-brane [23] is characterized by following RR-numbers:

$$Q_4 = n_1 - 2n_2 + n_3,\ Q_2 = -n_1 + n_2,\ Q_0 = \frac{n_1 + n_2}{2}\ ,$$
$$n_1 = a,\ n_2 = b,\ n_3 = c$$

with $Q_4 = Q_2 = 0$. According to [24] we have the following correspondence between objects of M-theory presented by supergraviton with $p_1 1 = 1/R$ and D0-brane, Table 1:

Table 1. *Correspondence between objects in M-theory and IIA string theory.*

M-theory	IIA
KK photon	RR gauge field A_μ
supergraviton with $p_1 1 = 1/R$	D0-brane
wrapped membrane	IIA string
unwrapped membrane	IIA D2-brane
wrapped 5-brane	IIA D4-brane
unwrapped 5-brane	IIA NS5-brane

An open string *f* connects two D-branes and has states described by Ext groups [17]. The type of states or massless particles determined in vacuum depends on the geometric configuration of the D-brane bundles connected by the string *f*. For C^3/Z_3 we have the following set of states presented byExt-gropus,

$$\begin{aligned}
Ext^0_{[C^3/Z_3]}(i_\star\mathcal{E}, i_\star\mathcal{F}) &= C^{aa'+bb'+cc'},\\
Ext^1_{[C^3/Z_3]}(i_\star\mathcal{E}, i_\star\mathcal{F}) &= C^{3ba'+3cb'+3ac'},\\
Ext^2_{[C^3/Z_3]}(i_\star\mathcal{E}, i_\star\mathcal{F}) &= C^{3ca'+3ab'+3bc'},\\
Ext^3_{[C^3/Z_3]}(i_\star\mathcal{E}, i_\star\mathcal{F}) &= C^{aa'+bb'+cc'},
\end{aligned}$$

Ext^0- and Ext^2-groups represent the number of bosons, but Ext^1 - and Ext^3 – groups represent the number of fermions. So, as we have

$$a = a^{'} = b = b^{'} = c = c^{'} = 1,$$

therefore,

- the number of bosons is equal to 12
- the number of fermions is equal to 12 .

According to formula (2) this state is unstable and the transition between these D-branes is accompanied by the emission of particles, classified by *SU*(1,1|3) algebra with R-symmetry SU(3)× U(1), [24]. Group *SU*(1,1|3) can be represented as

$$SU(1,1|3) = SU(1,1) \times SU(3).$$

It is known that SU(1, 1)-invariance arises in Poincare supergravity with *N* = 4, [25]. The generalized special unitary group SU (1, 1) is isomorphic to the symplectic group Sp(2, R) and to SL(2, R) - the groupof all linear transformations of the space R^2 that preserve the oriented area. The massless states, whose dynamics is described by 2 × 2 supersymmetric SU(1, 1)-group are classified according to SU(3) group of spirality.

V. Conclusions

The question of the birth of the Universe is closely related to the question of phase transitions in elementary particle physics at high energies. String and D-brane theory brings quantum consistency to physics with an elegant mathematical construction, called derived category based on the sheaves on the spaces of extra dimensions. The questions about the form of extra dimensions, their shapes and sizes are open. The validity of D-brane theory requires the special type of extra dimensional space - orbifold, which should be related to fundamental energy scales of particle physics: the cosmological scale. Using the homological algebra, which studies the category of complexes of coherent sheaves, we have considered the exact sequence of Serre's twisted sheaves received after blowing up of orbifold space C^3/Z_3 in toric geometry. In the framework of the criterion of stability of the D-brane as bound state of fractional branes $\mathcal{O}_{P2}$ and $\mathcal{O}_{P2}(-3)$ we showed the impossibility of phase transitionof $\mathcal{O}_{P2}$ sheaf into $\mathcal{O}_{P2}(-3)$ sheaf. To study the question of the type of the new particles associated with extra dimensions we used the category of distinguished triangles and the predictions about the Ext-group association of morphism between sheaves or McKay quivers in the case oforbifold. So, the number of vibrational modes of string can be calculated through $Ext^q(A,B)$-group. We have calculate the number of bosons and fermions for particular type of orbifold numbers

$$a = a^{'} = b = b^{'} = c = c^{'} = 1,$$

and found the group of their representations, *SU*(1,1|3), which connected with Poincare supergravity with *N* = 4.

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