

Self-Energy and Radius of Black Holes and Elementary Particles Using Potential Dependent Special Relativity

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Abstract

The singularity black hole problem and the infinite self-mass of elementary particles are one of the long standing problems in physics. Using the expression of energy in a potential dependent special relativity an advanced model has been constructed. According to this model the energy at which both radius and mass give minimum energy, determine the self-mass and minimum radius. The self-mass was found to be finite, dependent on vacuum energy and coupling constant. The radius was found to be dependent on the short range field coupling constant.

Key words: black hole, elementary particle potential, special relativity, self-mass radius, minimum energy.

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I. Introduction:

General relativity (GR) is one of the big achievements which enables scientists to understand the nature of our universe much [1]. Its theoretical big bang (BB) cosmological model which is designed to describe the universe evolution [2]. The BB model succeeded in describing a large number of astronomical observations including light red shift, relic microwave back ground radiation, deflection of light by the sun and gravitational waves [3, 4]. According to the big bang model the light red shift indicates the universe expansion, which decreases matter density and temperature [5]. The BB model also explains galaxy formation and evolution of stars. The evolution of stars results from the nuclear energy consumption and the effect of pressure and attractive gravitational force. This evolution of neutron stars, pulsars, while dwarfs, red giant stars and black holes. The behavior of these exotic astronomical objects needs promoting the cosmological models [6,7]. Many physical phenomena and long standing problems are associated with these exotic objects. One of these main problems are associated with the singularity problem, infinite self-mass and generation of gravitational waves [8,9, 10]. This work is concerned with using generalized potential dependent special relativity (PSR) to construct nonsingular black hole model with finite self-mass of black holes and elementary particles generate inside it. The mathematical model beside discussion are presented in section (2). Section (3) is devoted for conclusion.

II. Black hole and Elementary particle self-energy and radius

When the field is weak, the time metric is given by

$$g_{00} = \left(1 + \frac{2\phi}{c^2}\right)^{\frac{1}{2}} \quad (1)$$

The total energy can be given by

$$E = \frac{g_{00}m_0c^2}{\sqrt{g_{00} - \frac{v^2}{c^2}}} \quad (2)$$

With m_0 , c , v standing for the rest mass, speed of light in vacuum, and the particle velocity respectively.

Consider the particle to be at rest, as far as we need the minimum energy. Thus according to equations (1) and (2) and since ($v=0$). It follows that

$$E = \left(1 + \frac{2\phi}{c^2}\right)^{\frac{1}{2}} m_0 c^2 \tag{3}$$

When the elementary particle is affected by short range nuclear field ϕ_n and gravitational field ϕ_g , in this case

$$\phi = \frac{c_0}{r} e^{c_1 r} - \frac{GM}{r} \tag{4}$$

Where the nuclear and gravitational potentials per unit mass are given by

$$\phi_n = \frac{c_0}{r} e^{c_1 r} \tag{5}$$

$$\phi_g = -\frac{GM}{r} \tag{6}$$

Then by defining

$$m_0 c^2 = c_2 \tag{7}$$

Equation (3) and (4) beside (7) gives

$$E = \left(1 + \frac{2c_0}{c^2 r} e^{c_1 r} - 2\frac{GM}{c^2 r}\right)^{\frac{1}{2}} c_2 \tag{8}$$

The radius for which the energy is minimum can be found by minimizing E

$$\frac{dE}{dr} = \frac{c_2}{2} \left(1 + \frac{2c_0}{c^2 r} e^{c_1 r} - 2\frac{GM}{c^2 r}\right)^{-\frac{1}{2}} \left(\frac{-2c_0}{c^2 r} e^{c_1 r} + \frac{2c_0 c_1}{c^2 r} e^{c_1 r} + 2\frac{GM}{c^2 r^2}\right) \tag{9}$$

The radius which make the energy minimum is thus given by setting E is minimum when

$$\frac{dE}{dr} = 0 \tag{10}$$

This is satisfied when

$$\frac{-2c_0}{c^2 r} e^{c_1 r} + \frac{2c_0 c_1}{c^2 r} e^{c_1 r} + 2\frac{GM}{c^2 r^2} = 0 \tag{11}$$

Since the radii of elementary particles are so small, or if one consider small radius black hole, one has

$$r \rightarrow 0 \tag{12}$$

Thus one can use the Taylor expansion for the exponential function to get

$$e^{c_1 r} \cong 1 + c_1 r \tag{13}$$

A direct substitution of this in (11) gives

$$-c_0(1 - c_1 r)(1 + c_1 r) + GM = 0 \tag{14}$$

$$1 - c_1^2 r^2 = \frac{GM}{c_0} \tag{15}$$

$$r^2 = \frac{c_0 - GM}{c_0 c_1^2}$$

Thus the radius at which the energy is minimum is given by

$$r_0 = r = \frac{1}{c_1} \sqrt{1 - \frac{GM}{c_0}} \tag{16}$$

It is very interesting to note that for massive body additional the radius become smaller due to strong gravity attraction. Means that the radius of massive black hole is very small. To this find the additional self-mass one can consider the energy in equation (3) to having vacuum term, which is given by

$$\phi_v = \frac{V_v}{M} \tag{17}$$

Thus according to equation (4) and (17) the total potential is given by

$$\phi = \phi_n + \phi_g + \phi_v = \frac{c_0}{r} e^{c_1 r} - \frac{GM}{r} + \frac{V_v}{M} \tag{18}$$

Inserting (18) in (3) gives

$$E = m_0 c^2 \left(1 + \frac{2\phi}{c^2}\right)^{\frac{1}{2}} = m_0 c^2 \left(1 + \frac{2c_0 c_1}{c^2 r} e^{c_1 r} + 2\frac{GM}{c^2 r^2} + \frac{2V_v}{c^2 M}\right)^{\frac{1}{2}} \tag{19}$$

Thus the mass which makes the energy minimum, requires

$$\frac{dE}{dM} = 0 \tag{20}$$

$$\frac{dE}{dM} = \frac{1}{2} m_0 c^2 \left(1 + \frac{2}{c^2} \left(\frac{c_0}{r} e^{c_1 r} + \frac{GM}{r} + \frac{V_v}{M}\right)\right)^{\frac{1}{2}} \left(-\frac{G}{r} - \frac{V_v}{M^2}\right) \left(\frac{2}{c^2}\right) \tag{21}$$

$$\begin{aligned} \frac{G}{r} + \frac{V_v}{M^2} &= 0 \\ \frac{V_v}{M^2} &= -\frac{G}{r} \\ \frac{M^2}{M^2} &= -\frac{r}{V_v} \\ \frac{M^2}{V_v} &= -\frac{r}{G} \\ M^2 &= -\frac{rV_v}{G} \end{aligned}$$

$$M = \sqrt{-\frac{rV_v}{G}} \tag{22}$$

Since the mass is real and not imaginary. This requires vacuum energy to be attractive, i.e

$$V_v = -V_0 \tag{23}$$

$$M = \sqrt{\frac{rV_0}{G}} \tag{24}$$

It is clear that the mass increases when vacuum energy increases which means that mass is generated by vacuum. Thus more vacuum energy generates more massive body

Where r is the radius for minimum energy i.e (see equation 16)

$$r = r_0 = \frac{1}{c_0} \sqrt{1 - \frac{GM}{c_0}} \tag{25}$$

$$M = \sqrt{\frac{r_0 V_0}{G}} \tag{26}$$

$$M^2 = \frac{r_0 V_0}{G} = \frac{V_0}{GC_1} \sqrt{1 - \frac{GM}{c_0}} \tag{27}$$

Since short range nuclear force is much stronger than the gravity force, it follows that $\frac{GM}{c_0} \ll 1$ (28)

$$M = \sqrt{\frac{V_0}{GC_1}} \tag{29}$$

This equation again shows that mass is generated by vacuum. The more vacuum energy generates more massive body, but more generally the mass can be found by squaring equation (27) to get

$$M^4 = \frac{V_0^2}{(GC_1)^2} \left(1 - \frac{GM}{c_0}\right)$$

$$\frac{(GC_1)^2}{V_0^2} M^4 + \frac{G}{c_0} M - 1 = 0 \tag{30}$$

Consider now elementary particles.

Since the mass of elementary particles is so small. Thus

$$M^4 \ll M \tag{31}$$

Therefore

$$M = \frac{c_0}{G} \tag{32}$$

However for massive black holes

$$M^4 > M \tag{33}$$

Thus equation (30) gives

$$M^4 = \frac{V_0^2}{(GC_1)^2}$$

$$M = \pm \sqrt{\frac{V_0}{GC_1}} \tag{34}$$

Which again gives an expression similar to equation (29) but with additional negative mass solution. This gives possibility of generating anti particles which conforms with elementary particles theories which propose that particles are created by photons in pairs. However black holes have positive mass. But one can also propose existence of anti-particles black holes with negative masses.

One can also try to find the radius of elementary particles and black holes in the presence of vacuum and strong nuclear force. This is done when the generalized potential special relativity (gpsr) energy is given by

$$E = m_0 c^2 \left(\phi_0 - \frac{c_2}{r} e^{c_1 r} \right)^{\frac{1}{2}} \tag{35}$$

Where

$$c_2 = \frac{2c_0}{c^2}, \quad \phi_0 = 1 + \frac{2}{c_2} \phi_v \tag{36}$$

Here outside the body ($c_0 = 0$)

It is important to note that, the short range force in equation (33) is the attractive gravity force, to find the minimum radius under the effect of a short range attractive gravity force, one applies the condition of minimum energy, where

$$\frac{dE}{dr} = \frac{1}{2} m_0 c^2 \left(\phi_0 - \frac{c_2}{r} e^{c_1 r} \right)^{-\frac{1}{2}} \left(\frac{c_2}{r^2} e^{c_1 r} - \frac{c_2 c_1}{r} e^{c_1 r} \right) \tag{37}$$

For minimum energy

$$\begin{aligned} \frac{dE}{dr} &= 0 \\ \frac{1}{2} m_0 c^2 \left(\phi_0 - \frac{c_2}{r} e^{c_1 r} \right)^{-\frac{1}{2}} \left(\frac{c_2}{r^2} e^{c_1 r} + \frac{c_2 c_1}{r} e^{c_1 r} \right) &= 0 \end{aligned}$$

$$\frac{c_2}{r^2} [e^{c_1 r} - c_1 r e^{c_1 r}] = 0 \quad (38)$$

For the black hole r may by sometimes large, thus

$$\begin{aligned} e^{c_1 r}(1 - c_1 r) &= 0 \\ 1 - c_1 r &= 0 \end{aligned}$$

$$r = \frac{1}{c_1} \quad (39)$$

Thus the minimum radius is given by

$$r_0 = \frac{1}{c_1} \quad (40)$$

For (out-side the body $c_0 = c$) for small distances

$$e^{c_1 r} = (1 + c_1 r) \quad (41)$$

$$\begin{aligned} (1 + c_1 r)(1 - c_1 r) &= 0 \\ (1 - c_1^2 r^2) &= 0 \\ c_1^2 r^2 &= 1 \end{aligned}$$

$$r = \pm \frac{1}{c_1} \quad (42)$$

Again the minimum radius is given by

$$r_0 = \frac{1}{c_1} \quad (43)$$

In view of equation (35) it is clear that stronger attractive force requires c_1 to be large. This makes r_0 very small. This is quite reasonable, since stronger attractive force causes the body to be shrunked more thus causing its radius to be very small. Equation (42) gives additional possibility by all awing a possibility of having short attractive force which diminish itself away from the source. Thus requires according to equation (35)

$$c_1 = -c_3 \quad (44)$$

Thus according to equation (42) the minimum radius is given by

$$r_0 = \frac{1}{c_3} \quad (45)$$

III. Conclusion:

The theoretical model based on potential dependent special relativity can successfully construct an singular model to describe the behavior and find the self-energy and the radius of generated elementary particles and black holes. According to this model the self-mass is dependent on vacuum energy and gravitational and short range force coupling constants. The radius depends on the short range coupling constant, as well as the mass in some cases also.

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