

Modelling Of Pressure-Altitude Equation Covering Up To 2km Above The Sea Level In Lagos, Southwestern Nigeria

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Abstract

It is generally known that the atmospheric pressure varies inversely as altitude globally. In the absence of real time data, prediction of atmospheric pressure in the troposphere is based by the famous barometric equation which depicts exponential inverse relationship between these two variables. This work employed two years meteorological data of Lagos to study the relationship and model a localized equation. The research results show that the atmosphere is purely linear with a gradient of -10.1 m/hPa from the sea level to about 2 km. The modeled equation is useful for better prediction of atmospheric pressure for industrial, scientific and other purposes.

Keyword: Altitude, Pressure, Troposphere, Barometric equation

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I. Introduction

The Troposphere is the lowest level of the atmosphere where vast percentage of human and animals dwell in our planet. Its environmental conditions are determined by weather parameters such as temperature, pressure, humidity, windspeed amongst others. These parameters vary temporally and spatially to the extent that even for a particular location they also vary with altitudes. One of the major effects of these atmospheric parameters is the bending of radio signals, a phenomena known as refractivity. According to Smith and Weintraub (1995) [1], the refractivity at any altitude in the atmosphere is a function of its Pressure, Temperature as well as water vapor. Since these independent variables vary with altitude, the vertical profile of radio refractivity is thus, defined by refractivity gradient. For the purpose of inter-terrestrial wireless links within the troposphere, it is important to study the variation of pressure and temperature with respect to altitude. Several attempts have been made to model equations which predict atmospheric pressure and air temperature at various altitude and vice versa. Among all the atmospheric variables such as temperature, pressure, humidity, wind speed, vapour density etc pressure plays the most important role in the prediction of altitude. This is explicitly explained by the famous barometric equation

The barometric equation has been adopted over years for theoretical prediction of atmospheric pressure at various altitudes. However, recent research works, through experimental measurements of pressure-altitude at low intervals, have shown that this equation is not absolutely accurate especially in the lowest few kilometers above the earth's surface (Muath *et al.*, 2020) [2]. The major aim of this work is to study the dependence of pressure and temperature on altitude using Lagos as a case study.

It is generally known that atmospheric parameters such as pressure, temperature, humidity, wind speed vary spatially and temporally. Among all the layers of the atmosphere, the behavior of the troposphere is of great importance because this is where human and other animals dwell. The vertical profile of radio refractivity is uniquely determined by changes that occur in pressure, temperature and humidity in a given column of the atmosphere. In a well-mixed atmosphere, pressure, temperature and humidity decrease exponentially as a function of height (Falodun and Ajewole, 2006; Paul and Ferl 2007) [3-4]. Although, an altimeter can be used to measure altitude and pressure directly in a particular region, scarcity of the instrument has led to the adoption various models for computation altitude-pressure theoretically (Brombach, 1944) [5].

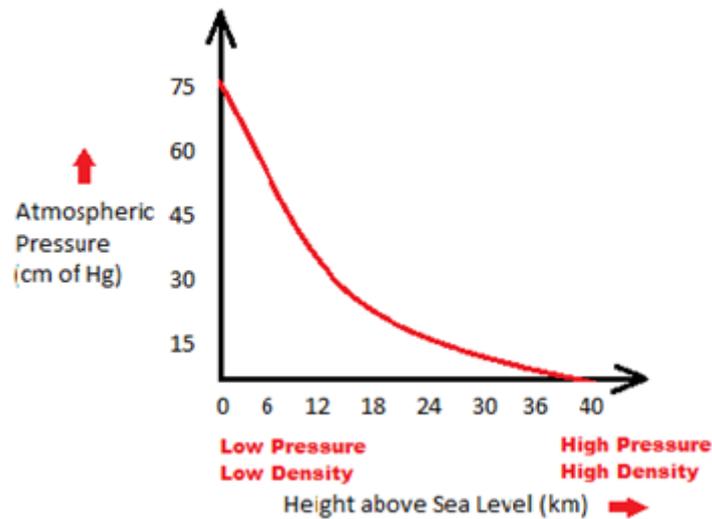


Figure 1: Exponential Variation of Atmospheric Pressure with Altitude

The famous barometric equation is commonly used for predicting pressure P at various altitudes z assume the vertical profile of the pressure completely exponential as shown in equation (1) (Mario *et al.*, 1997) [6]

$$P = P_o e^{-\frac{z}{H}} \quad (1)$$

Where P_o is the pressure at sea level, P is the pressure at altitude z . The scale height H is altitude at which atmospheric pressure reduces to $\frac{1}{e}$ of P_o . The scale height is a local weather parameter calculated from

$$H = \frac{KT}{mg} \quad (2)$$

Where K is the Boltzman constant ($1.38 \times 10^{-23} JK^{-1}$), T is the temperature at H in Kelvin, m is the mass of one air molecule in kg and g is the gravitational acceleration in ms^{-2} ($9.8 ms^{-2}$)

In the absence of upper air data, most of the previous works on atmospheric research applied equation (1) and (2) to predict atmospheric pressure at any altitude. This work studies the variational pattern of pressure P with respect to height h ; otherwise known as pressure gradient.

II. Methodology

Two years (2013-2014) data made of atmospheric pressures obtained through radiosonde measurement were collected from the archive of the Nigeria Meteorological Agency (NiMet). The study location is Oshodi, a metropolitan city in Lagos Nigeria which has a coordinate of $6.20^\circ N$, $3.45^\circ E$ (Agbonaye *et al.*, 2022) [7]. The pressure values and corresponding altitudes at interval of 50 mPa from the level up to 2 km altitude were measured by the radiosonde. Statistical analysis was carried out to study the correlation between the two sets of data. An empirical equation which is valid for the first 2 km from the ground surface were modeled. The pressure gradients obtained from the linear equations were categorized into dry and wet seasons.

III. Results and Discussion

Vertical Profile of Atmospheric Pressure

Figures 2a-r shows plots of the atmospheric pressure against height for typical months between 2013 and 2014 from sea level to 2km. The negative slopes obtained in all the graphs indicate that these two quantities have inverse relation. The linear regression equation of the plots shows that during the dry season, the gradient varies between -10.304 m/hPa in January 2013 to a maximum of -10.109 m/hPa in December 2014 with correlation coefficient of 0.998. During the wet season i.e November, December, January, February and March, a minimum slope of -10.1 m/hPa in June 2014 and has a peak of -9.619 m/hPa in August.

Seasonal variation

The two years average seasonal variation of the atmospheric pressure with altitude is presented in figure 2. The plot is double peak graph both occurring in the wet season. The values are generally low during the dry season typically but increases gradually at the onset of the wet season. Slightly low values of about -9.97 m/hPa is common within the months of July and August which could be attributed to the famous break. This is a period of temporal rain seizure which usually last for few days between July and August followed by frequent nominal rainfall which signifies the offset of wet season. The peak values of -9.89 and -9.75 m/hPa are observed in June and September respectively.

Over the two years considered, an average dry season gradient of -10.23 m/hPa was observed while the wet season value is -9.975 m/hPa. The overall gradient and intercept obtained are -10.1 m/hPa and 10.3 Km respectively. This is in close agreement with result of Mccord (2019) [8] which reported a gradient of about 10.1 kPa/km (-9.9 m/hPa) in the first 4-5 km. It should be noted that the intercept does not give accurate prediction of the altitude at which atmospheric pressure is reduced to zero because the regression equation is not valid at altitudes above 2 Km.

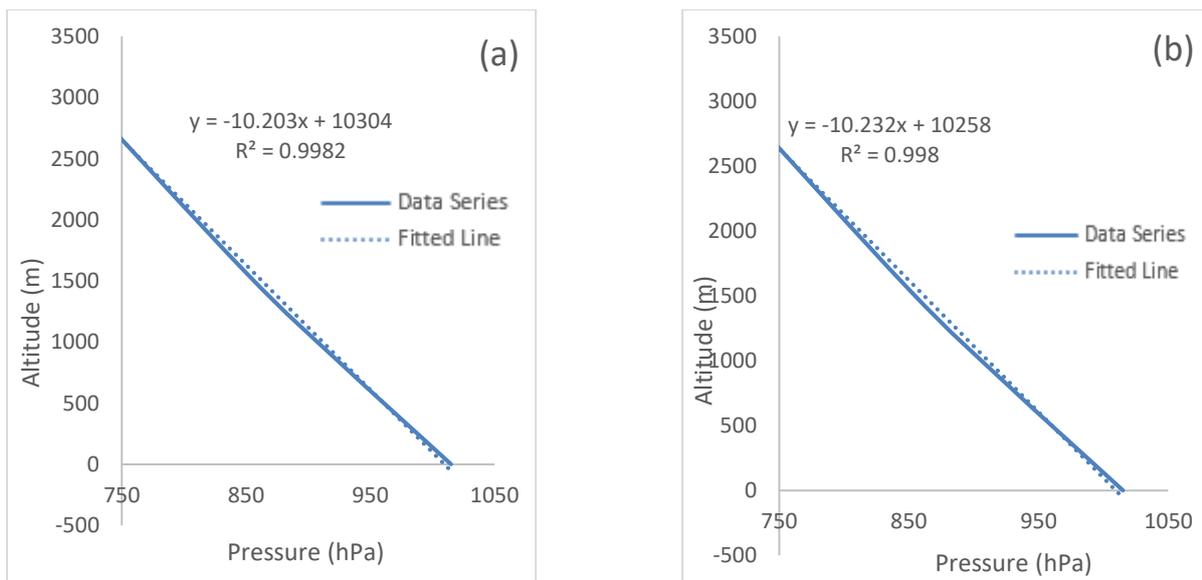


Figure 2a-b: Vertical Profile of Atmospheric Pressure for typical days in January: (a) 24 January 2013 (b) 9 January 2014

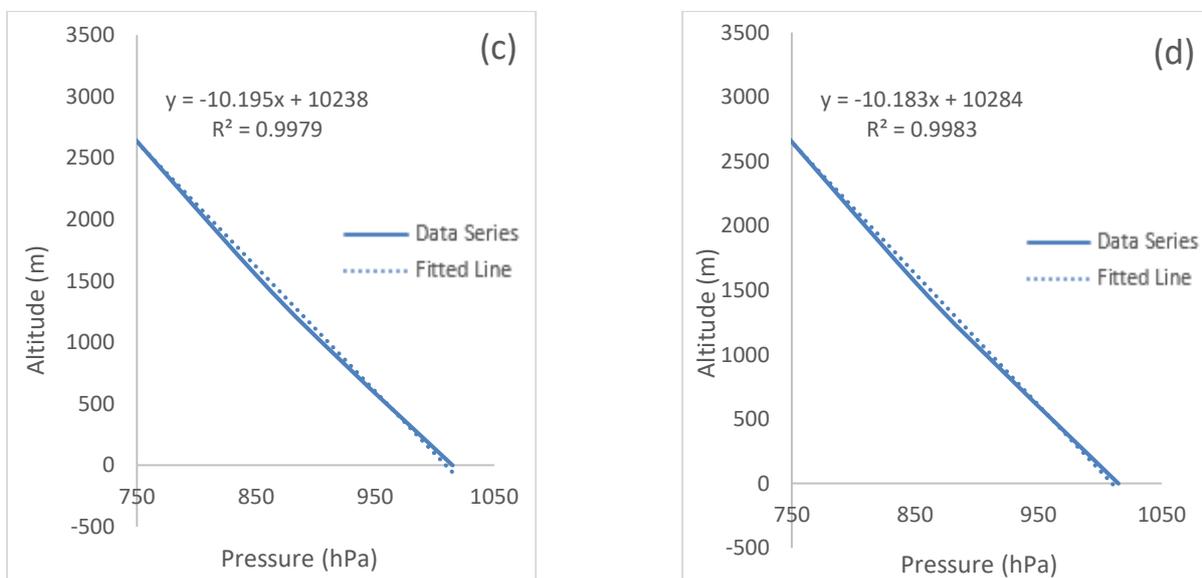


Figure 2c-d: Vertical Profile of Atmospheric Pressure for typical days in February: (c) 13 February 2013 (d) 7 February 2014

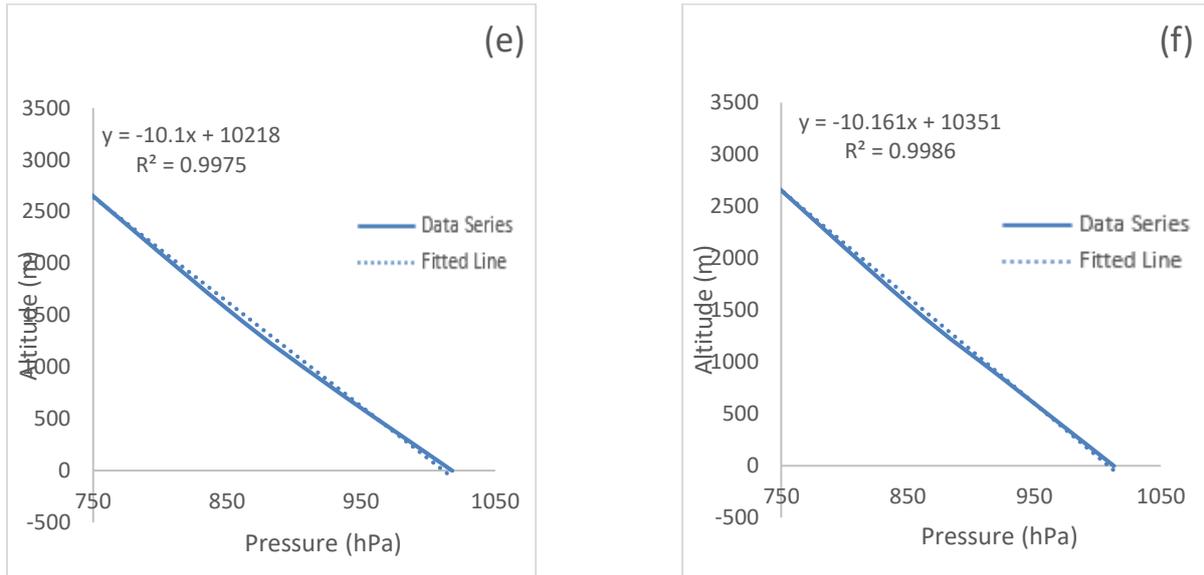


Figure 2e-f: Vertical Profile of Atmospheric Pressure for typical days in May (c) 9 May 2013 (d) 20 May 2014

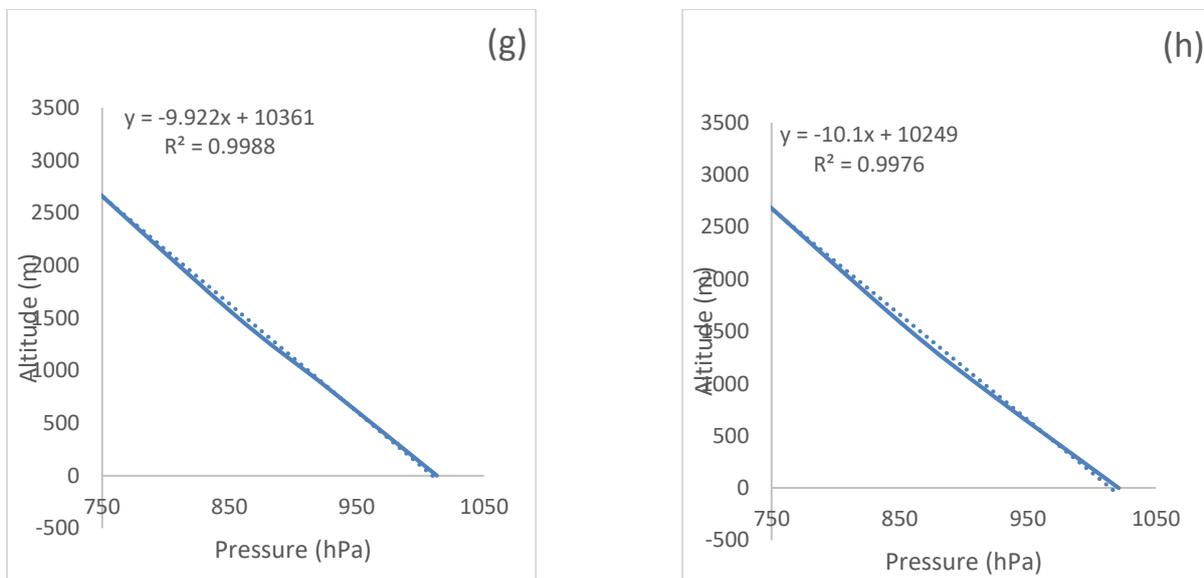


Figure 2g-h: Vertical Profile of Atmospheric Pressure for typical days in June (c) 30 June 2013 (d) 16 June 2014

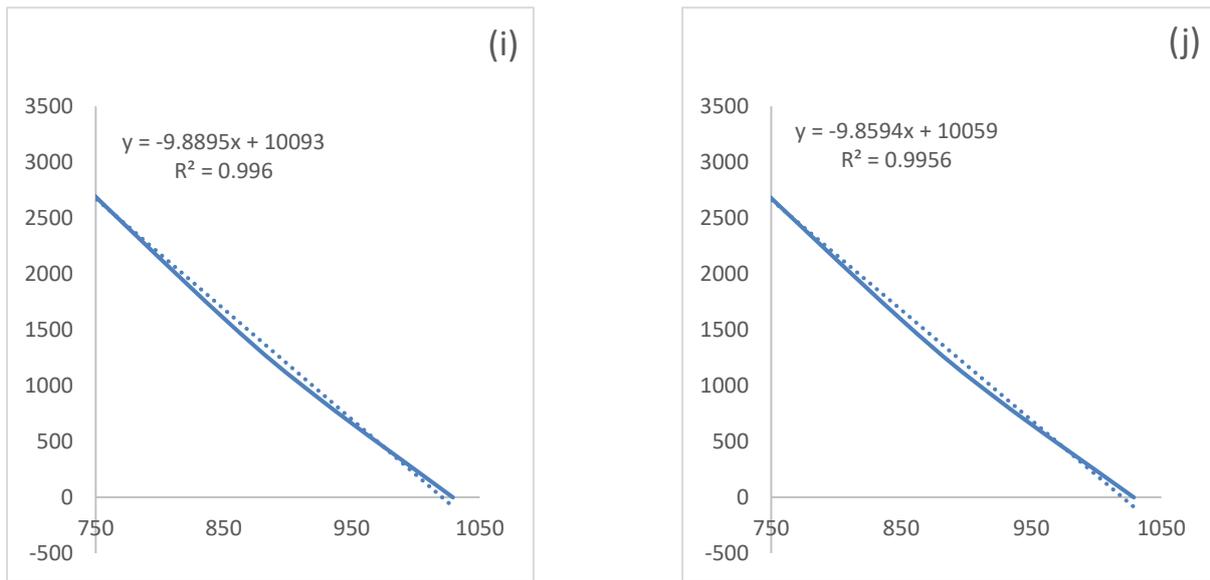


Figure 2i-j: Vertical Profile of Atmospheric Pressure for typical days in July (c) 11 June 2013 (d) 22 July 2014

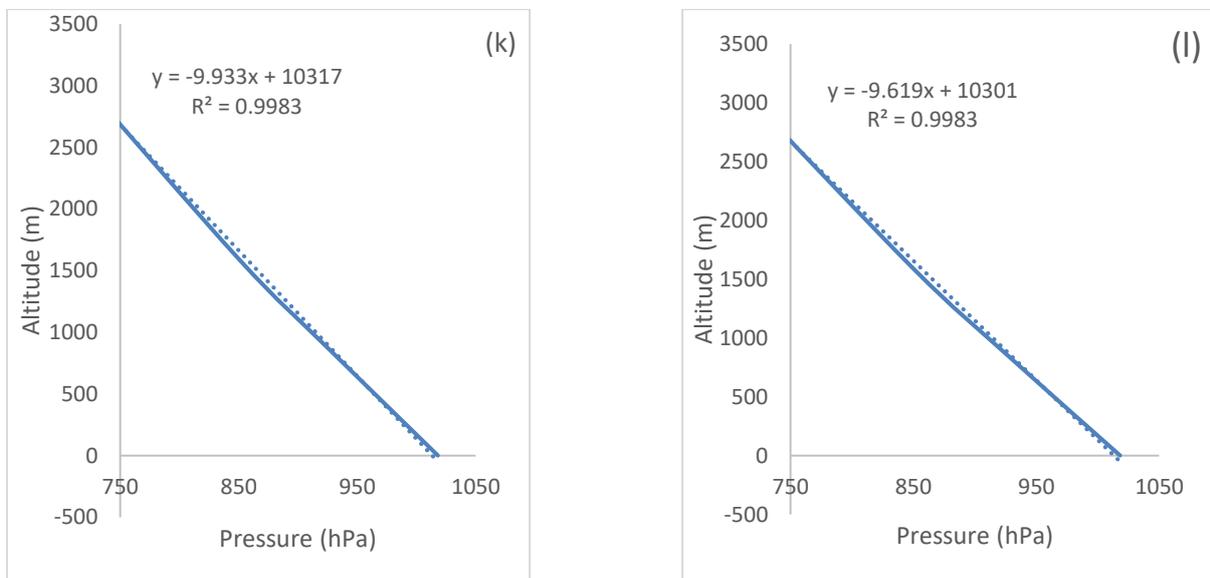


Figure 2k-l: Vertical Profile of Atmospheric Pressure for typical days in August (c) 30 August 2013 (d) 16 August 2014

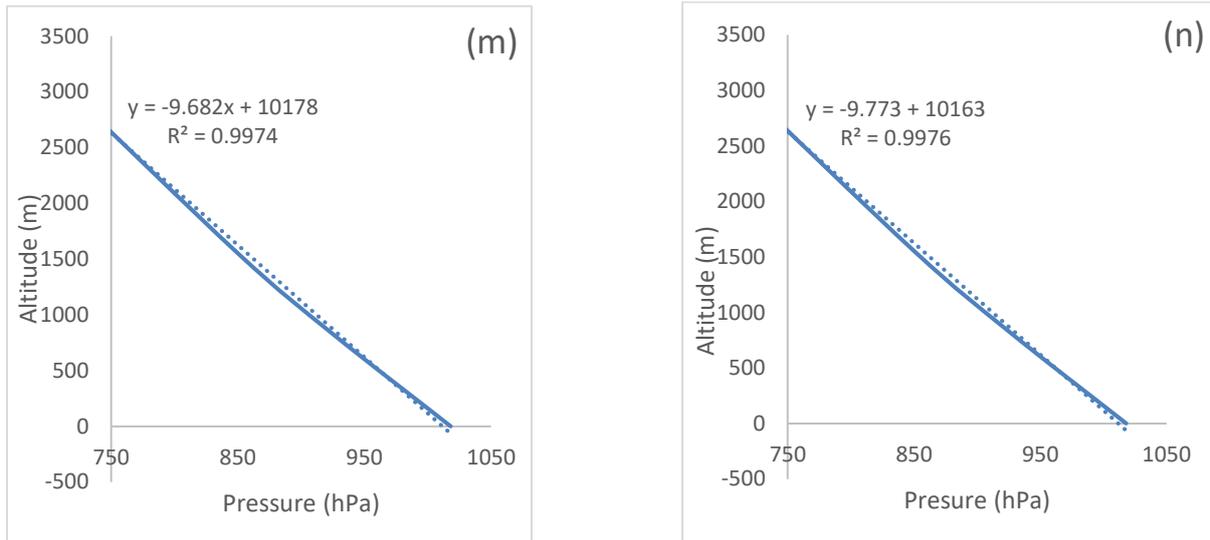


Figure 2m-n: Vertical Profile of Atmospheric Pressure for typical days in September: (m) 3 September 2013 (n) 17 September 2014

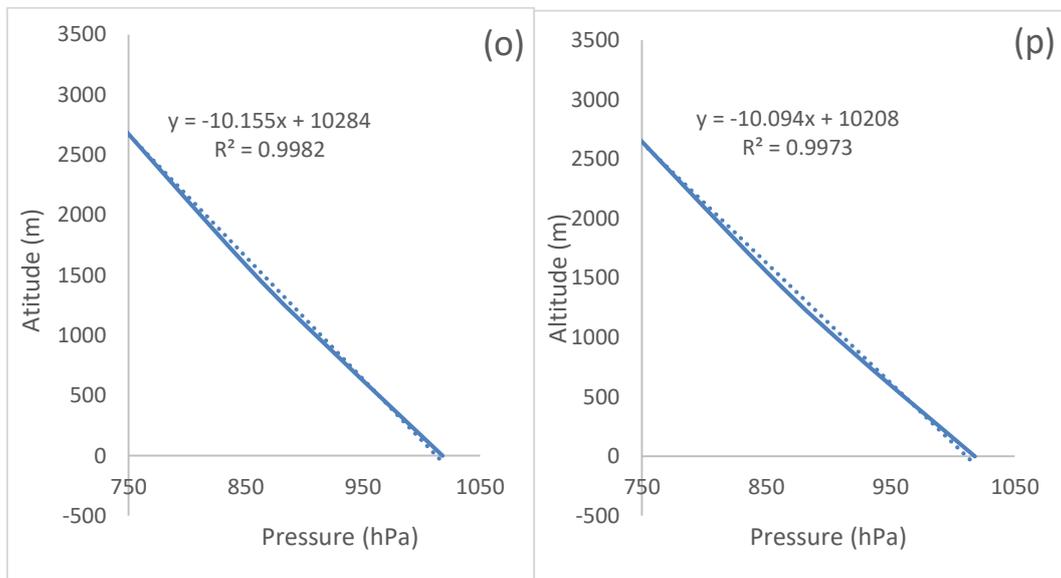


Figure 2o-p: Vertical Profile of Atmospheric Pressure for typical days in November: (o) 3 November 2013 (p) 17 November 2014

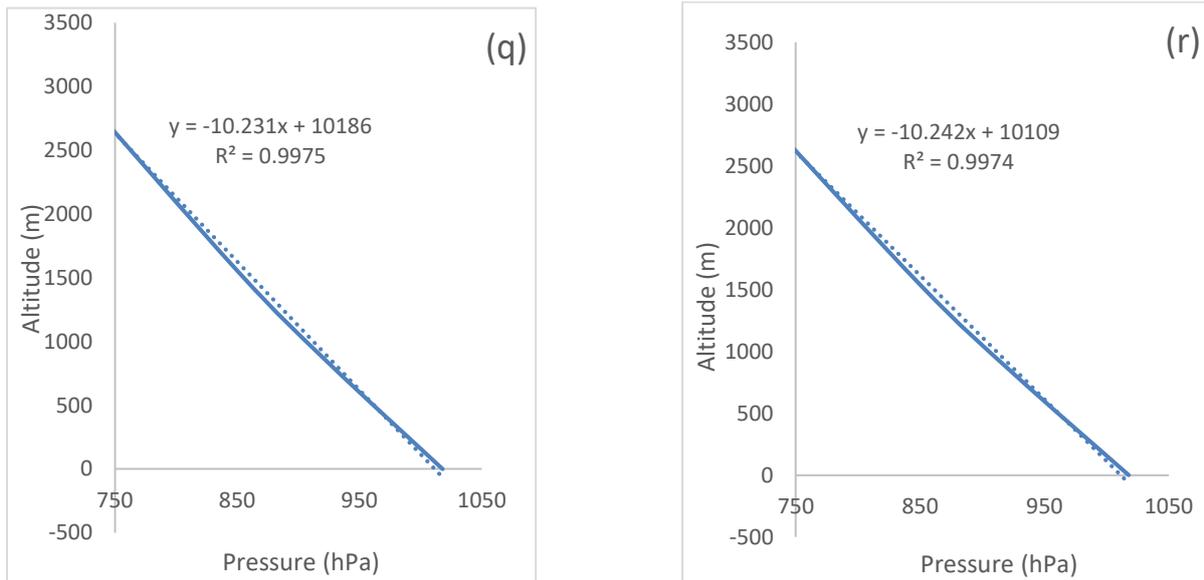


Figure 2q-r: Vertical Profile of Atmospheric Pressure for typical days in December: (q) 5 November 2013 (n) 21 November 2014



Figure 3: Seasonal Variation of Pressure Gradient

IV. Conclusion

In this research, using Lagos as a case study, it was established that the relationship between pressure and altitude is purely linear up to about 2km above the ground level. An overall all-season linear equation was derived with an average gradient of $-10.1m/hPa$ or $-9.9 kPa/km$ and correlation co-efficient of 0.998. The accuracy of results shows linear dependence of atmospheric pressure on altitude. The equation is therefore recommended for prediction of atmospheric pressure from 0-2 km above the sea level in Lagos State and Its environs. This study could be extended to other areas in Nigeria where similar data are available to ascertain the prevailing relationship between atmospheric pressure and altitude within the second kilometer in a locality. Atmospheric pressure gradient and intercept obtained for Lagos in this work could be compared with value obtained for other areas within the Lagos state or other states within the same geoclimatic zone of Nigeria.

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