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A New Derivation Of Maxwell's Equation For Diffusion Current And Quantum Equation From Maxwell's Equation For Massive Photon

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Abstract:
Maxwell's equations accounting for diffusion current was derived.
Maxwell's equations are used to derive Klein-Gordon equation by replacing the electric field intensity by the wave function. Anew quantum equation which accounts for relativistic rest mass energy beside potential energy as well as medium friction is also derived.
Key words: *Maxwell's equations, massive photon*

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I. Introduction:

Quantum theory starts from the discovery of Max Plank, that light can be treated discrete quanta, known recently as photons. This means that waves can behave sometimes like particles. This encourages De Broglie to propose that particles can also behave like waves. This dual nature of microscopic particles, leads to proposing a new physical framework known as quantum mechanics (QM) [1, 2, 3].

The laws of quantum mechanics are now widely used to describe the behavior of atomic and subatomic particles beside nano particles [4, 5].

The spectrum of any atom beside some electrical and magnetic properties can be easily described by the laws of quantum mechanic [6, 7].

Despite these remarkable successes of quantum mechanic, it suffers from noticeable set backs. For instance, there is no full quantum theory that can describe the behavior of superconductors (SC). The behavior of nano systems are now far from being described fully by quantum mechanic.

The situation for elementary particles, fields is even worse. There is no theoretical model that can put gravity under the umbrella of quantum mechanic [8].

The dream of unification of forces is too difficult to be achieved within the present physical theories including quantum mechanics [9].

These failures may be related to mathematical and physical laws are based on the dual nature of wave beckets beside the energy expression in classical mechanics and relativity [10]. Unfortunately the energy expression take care of the effect of the field potentials only, without accounting other effects that can change the behavior of the particle under study. These effects include friction, collision and scattering effects that are closely related to the density of particles and relaxation time.

Thus there is a need for a quantum model that can accounts for the effect of the surrounding medium. One of the approaches is based on deriving quantum equations from Maxwell's equations as done by K. Algeilani and others [11].

This approach is reasonable, since Maxwell's equation have terms like conductivity and electric dipole moment (polarization) that account for medium density, relaxation time and internal electric charge [12]. Unfortunately Algeilani model does not accounts for the field effect through the potential term [13].

Maxwell's equation for diffusion current and polarized current is derived in section 2. A new approach based on Maxwell's equations is used to derive Klein-Gordon equation in section 3. Section 4 is devoted for deriving new generalized quantum equation based also on Maxwell's equations. Sections 5 and 6 are concerned with discussion and conclusion.

II. Maxwell's Electric Wave Equation:

From Maxwell's equation

$$\nabla\times H = J + G \qquad (1)$$

The equation of continuity takes the form

$$\nabla . J \quad + \quad \frac{\partial \rho}{\partial t} - \frac{\partial \rho_b}{\partial t} + c_d \nabla^2 \rho \quad = \quad 0 \qquad (2)$$

The current density J is assumed to result from external ohmic field J_0, beside bounded charge j_b and diffusion process j_d

$$J \;=\; J_0 \;+\; J_b \;+\; J_d \qquad (3)$$

Where

$$J_0 \;=\; \frac{-\partial D}{\partial t}$$

$$\Rightarrow \quad \nabla . J_0 \;=\; \frac{-\partial}{\partial t}(\nabla . D) \quad = -\frac{\partial \rho}{\partial t} \qquad (4)$$

$$J_b \;=\; \frac{-\partial P}{\partial t}$$

$$\Rightarrow \;\; \nabla . J_b \;=-\frac{\partial}{\partial t}(\nabla . P) \;=\; \frac{-\partial \rho_b}{\partial t} \qquad (5)$$

$$J_d \;=\; -c_d \nabla \rho$$

$$\Rightarrow \quad \nabla . J_d \;=\; -c_d \nabla^2 \rho \qquad (6)$$

Thus the divergence of both sides of equation (3) gives

$$\nabla . J \;=\; \nabla . J_0 \;+\; \nabla . \, J_b \;+\; \nabla . \, J_d \qquad (7)$$

In view of equations (4) , (5) and (6)

$$\nabla . J \;=\; -\frac{\partial \rho}{\partial t} \;+\; \frac{\partial \rho_b}{\partial t} - c_d \, \nabla^2 \, \rho \qquad (8)$$

By rearranging the above equation

$$\nabla . J \;+\; \frac{\partial \rho}{\partial t} \;-\; \frac{\partial \rho_b}{\partial t} \;-\; c_d \, \nabla^2 \, \rho \;=\; 0 \qquad (9)$$

To find the unknown G , one uses

$$\rho \;=\; \nabla . D \;=\; \varepsilon . \nabla . E \qquad (10)$$

$$\rho_b \;=\; -\nabla . P \qquad (11)$$

Taking the divergence of equation (1), one have

$$\nabla . \nabla \times H \;=0$$

$$\nabla . \nabla \times H \;=\; \nabla . J \;+\; \nabla . G \;=\; 0 \qquad (12)$$

Insert equation (12) in (8) yields

$$-\frac{\partial \rho}{\partial t} \;+\; \frac{\partial \rho_b}{\partial t} \;-\; c_d \, \nabla^2 \, \rho \;=\; -\nabla . G \qquad (13)$$

Using equation (10) and (11) yields

$$-\frac{\partial}{\partial t}(\nabla . D) \;+\; \frac{\partial}{\partial t}(-\nabla . P) \;-\; c_d \, \nabla . (\nabla \rho) \;=\; -\nabla . G \qquad (14)$$

But $\nabla . D = \rho$

Thus

$$\nabla \rho \;=\; \nabla . (\nabla . D) \qquad (15)$$

Using relations (10) and (15) yields

$$-\frac{\partial}{\partial t}(\nabla . \varepsilon E) \;+\; \frac{\partial}{\partial t}(-\nabla . P) \;-\; c_d \, \nabla . (\nabla (\nabla . D)) \;=\; -\nabla . G$$

$$-\frac{\partial}{\partial t}(\nabla . \varepsilon E) \;+\; \frac{\partial}{\partial t}(-\nabla . P) \;-\; c_d \, \nabla . (\nabla (\nabla . \varepsilon E)) \;=\; -\nabla . G$$

$$-\varepsilon\nabla.\frac{\partial E}{\partial t} - \nabla.\frac{\partial P}{\partial t} - \varepsilon c_d \nabla.(\nabla(\nabla.E)) = -\nabla.G$$

Comparing both sides of above equations yields

$$\varepsilon \frac{\partial E}{\partial t} + \frac{\partial P}{\partial t} + \varepsilon c_d \nabla(\nabla.E) = G$$

$$G = \varepsilon \frac{\partial E}{\partial t} + \frac{\partial P}{\partial t} + \varepsilon c_d \nabla(\nabla.E) \qquad (16)$$

Thus from equation (1) and the fact that $J = \sigma_0 E$

$$\nabla\times H = J + G$$

$$\nabla\times H = \sigma_0 E + \varepsilon \frac{\partial E}{\partial t} + \frac{\partial P}{\partial t} + \varepsilon c_d \nabla(\nabla.E) \qquad (17)$$

Also from Maxwell's equations we have

$$\nabla\times E = -\mu\frac{\partial H}{\partial t}$$

$$\nabla\times\nabla\times E = -\mu\frac{\partial(\nabla\times H)}{\partial t} \qquad (18)$$

From equation (16) and (1) one found that

$$\nabla\times H = J + \varepsilon \frac{\partial E}{\partial t} + \frac{\partial P}{\partial t} + \varepsilon c_d \nabla(\nabla.E) \qquad (19)$$

Multiplying both sides of equation (19) by μ and differentiate over time t yields

$$\mu\frac{\partial}{\partial t}(\nabla\times H) = \mu\frac{\partial J}{\partial t} + \mu\varepsilon\frac{\partial^2 E}{\partial t^2} + \mu\frac{\partial^2 P}{\partial t^2} + \varepsilon\mu c_d \nabla(\nabla.\frac{\partial E}{\partial t}) \qquad (20)$$

But

$$J = \sigma E \qquad (21)$$

$$\mu\frac{\partial}{\partial t}(\nabla\times H) = \mu\sigma\frac{\partial E}{\partial t} + \mu\varepsilon\frac{\partial^2 E}{\partial t^2} + \mu\frac{\partial^2 P}{\partial t^2} + \varepsilon\mu c_d \nabla(\nabla.\frac{\partial E}{\partial t}) \qquad (22)$$

Also we have

$$\nabla\times\nabla\times E = -\nabla^2 E + \nabla(\nabla.E) \qquad (23)$$

From equations (23), (22) and (18) yields

$$-\nabla^2 E + \nabla(\nabla.E) = \mu\varepsilon\frac{\partial^2 E}{\partial t^2} + \mu\sigma\frac{\partial E}{\partial t} + \mu\frac{\partial^2 P}{\partial t^2} + \varepsilon\mu c_d \nabla(\nabla.\frac{\partial E}{\partial t}) \qquad (24)$$

III. Derivation of Klein-Gordon Equation from Maxwell's Equation for a Massive Photon:

From Maxwell's equation

$$-\nabla^2 E + \mu\sigma\frac{\partial E}{\partial t} + \mu\varepsilon\frac{\partial^2 E}{\partial t^2} + \mu\frac{\partial^2 P}{\partial t^2} + \frac{m^2 c^2}{\hbar^2}E = 0 \qquad (25)$$

Neglecting polarization effect and considering the propagation in free space where

$$\left.\begin{aligned} \sigma &= 0 \\ \mu &= \mu_0 \\ \varepsilon &= \varepsilon_0 \end{aligned}\right] \qquad (26)$$

$$\mu_0\,\varepsilon_0 = \frac{1}{c^2} \qquad (27)$$

Where c is speed of light

Equation (25) reduce to

$$-\nabla^2 E + zero + \mu_0\varepsilon_0 \frac{\partial^2 E}{\partial t^2} + zero + \frac{m^2c^2}{\hbar^2} E = \quad 0 \qquad (28)$$

$$-\nabla^2 E + \mu_0\varepsilon_0 \frac{\partial^2 E}{\partial t^2} + \frac{m^2c^2}{\hbar^2} E = 0$$

$$\hbar^2(-\nabla^2 E + \mu_0\varepsilon_0 \frac{\partial^2 E}{\partial t^2}) + m^2c^2 = \ 0 \qquad (29)$$

inserting equation (27) in (29) ,one gets

$$-\hbar^2\nabla^2 E + \hbar^2 \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} + m^2c^2 = 0$$

Multiplying both sides of above equation by c^2

$$-\hbar^2 c^2\nabla^2 E + \hbar^2 \frac{\partial^2 E}{\partial t^2} + m^2c^4 E = 0 \qquad (30)$$

If the rest mass equals the relativistic mass, when no potential exist then,

$$m = m_0(1 - \frac{\upsilon^2}{c^2} + \frac{2\varphi}{c^2})$$

$$= m_0(1 - \frac{\upsilon^2}{c^2})$$

When $\upsilon <<<$

Thus equation (30) reduces to

$$m \quad = \quad m_0 \qquad (31)$$

$$-\hbar^2 \frac{\partial^2 E}{\partial t^2} \quad = \quad -c^2\hbar^2\nabla^2 E + m_0{}^2c^4 E \qquad (32)$$

Replacing E by ψ in equation (32), one gets

$$-\hbar^2 \frac{\partial^2 \psi}{\partial t^2} \quad = \quad -c^2\hbar^2\nabla^2 \psi + m_0{}^2c^4 \psi \qquad (33)$$

This is the ordinary Klein-Gordon Equation

IV. New generalized quantum equation:

Schrodinger equation deals only with non relativistic particles, thus it does not take into account the rest mass energy. On contrary Klein-Gordon equation can account for rest mass energy but does not have potential energy term for fields other than electromagnetic fields.

Thus there is a need to find a new quantum equation that accounts for rest mass energy, beside potential energy. This can be done with the aid of equation (25), where one uses the mass expression of the generalized special relativity which is given by:

$$m \quad = \quad m_0(1 + \frac{2\varphi}{c^2} - \frac{v^2}{c^2})^{\frac{1}{2}} \qquad (34)$$

$$m^2 \quad = \quad m_0{}^2(1 + \frac{2\varphi}{c^2} - \frac{v^2}{c^2})$$

$$m^2 \quad = \quad m_0{}^2 + 2m_0(\frac{m_0\varphi}{c^2}) - \frac{m_0{}^2 v^2}{c^2} \qquad (35)$$

But we have

$$m_0\varphi \quad = \quad V \qquad (36)$$

$$m_0 v \quad = \quad p \qquad (37)$$

Substituting equation (37) and (36) in (35), one gets

$$m^2 = m_0^{\ 2} + 2\, m_0 \frac{V}{c^2} - \frac{p^2}{c^2} \qquad (38)$$

Multiplying both sides of equation (38) by $E\, c^4$

$$m^2\, c^4\, E = m_0^{\ 2}\, c^4\, E + 2\, m_0\, c^2\, V\, E - p^2\, c^2\, E \qquad (39)$$

But for oscillating electric field

$$E = E_0\, e^{i(kx - \omega t)}$$

$$\frac{\partial E}{\partial x} = i\, k\ E_0\, e^{i(kx - \omega t)}$$

$$\nabla^2 E = \frac{\partial^2 E}{\partial x^2} = i^2\, k^2\, E_0\, e^{i(kx - \omega t)}$$

$$\nabla^2 E = -k^2\, E$$

$$\hbar^2\, \nabla^2 E = -\hbar^2\, k^2\, E$$

$$\hbar^2\, \nabla^2 E = -\ p^2\, E \qquad (40)$$

Thus equation (39) becomes

$$m^2\, c^4\, E = m_0^{\ 2}\, c^4\, E + 2\, m_0\, c^2\, V\, E - c^2\, \hbar^2\, \nabla^2 E \qquad (41)$$

By using the identity $\mu\, \varepsilon = \frac{1}{c^2}$ and inserting equation (41) in equation (25)

$$-c^2\, \hbar^2\, \nabla^2 E + c^2\, \hbar^2\, \mu\, \sigma \frac{\partial E}{\partial t} + \hbar^2 \frac{\partial^2 E}{\partial t^2} + m_0^{\ 2}\, c^4\, E + 2\ m_0\, c^2\, V\, E - c^2\, \hbar^2\, \nabla^2 E = 0$$

Replacing E by ψ and collecting similar terms leads to the new quantum equation of the form

$$-2\, c^2\, \hbar^2\, \nabla^2 \psi + c^2\, \hbar^2\, \mu\, \sigma \frac{\partial \psi}{\partial t} + \hbar^2 \frac{\partial^2 \psi}{\partial t^2} + m_0^{\ 2}\, c^4\, \psi + 2\ m_0\, c^2\, V\, \psi = 0 \qquad (42)$$

V. Discussion:

The fact that Maxwell's equation is used to derive Klein-Gordon equation is related to the fact that quantum mechanical laws are based on Plank quantum light equation. The replacement of the electric field intensity vector E by the wave function ψ is reasonable as far as the electromagnetic energy density which is related to the number of photons is proportional to E^2 , i.e.

$$n \propto E^2$$

While it is also related to $|\psi|^2$

I.e. $n \propto |\psi|^2$

Thus

$$E \rightarrow \psi$$

The new quantum mechanical law shown in equation (42) is more general than Schrödinger and Klein-Gordon equations. It consists of conductivity of the medium, which is related to the friction of the system. The conductivity term can also feels the existence of the bulk matter through the particle density term n , where

$$\sigma = \frac{n\, e^2\, \tau}{m}$$

Unlike Schrödinger equation the new quantum equation consists of a term representing rest mass energy. This equation is also more general than Klein-Gordon equation by having terms accounting for the effect of friction, collision through conductivity, besides having a potential term accounting for all fields other than electromagnetic field.

VI. Conclusion

Quantum equations derived from Maxwell's equations are very promising, since they reduce to Klein-Gordon equation. It also accounts for collision, friction and scattering processes.

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