IOSR Journal of Applied Physics (IOSR-JAP)
e-ISSN: 2278-4861.Volume 17, Issue 3 Ser. 2 (May. – June. 2025), PP 34-36
www.iosrjournals.org

Quantum Teleportation: Alice and Bob experiment revisited

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Abstract:
In this article, the authors have discussed the Alice and Bob experimental set-up for the quantum teleportation, giving a detailed step-by step mathematical explanation of each unit. Quantum teleportation of a qubit requires the creation of an entangled quantum state between the sender's (Alice) and the receiver's (Bob) shared quantum state/s. At the end of this experiment, actual measurements of the Alice's share of quantum information are to be taken and shared with Bob via a classical communication channel. The teleportation is expected to open up a whole new regime of quantum communication, data encryption to be one of its applications.

***Key Word**: Quantum teleportation; Entanglement; Heisenberg uncertainty principle; EPR paradox; Alice and Bob;*

Date of Submission: 09-06-2025 Date of Acceptance: 20-06-2025

I. Introduction

In 1935, the "EPR paradox" (Bell 1964; Feynman 1971; Watson 2022) put a question mark on the Heisenberg's uncertainty principle yielding a hypothetical misconception when the information regarding the entangled quantum particles can be transferred over two different locations through a quantum channel at a speed greater than the velocity of light. The Bell-test-experiment also revealed that any quantum information cannot be teleported without incorporating a classical communication channel between the sender and the receiver. Hence resolving the EPR paradox that the teleportation can happen at a velocity greater than the velocity of light leading to acquiring more quantum information than is permissible as per the Heisenberg's principle. In this paper, the authors have made an attempt to explore the quantum teleportation starting from developing an understanding of the idea of teleportation using a simple model.

II. Quantum Circuit for Teleportation

Let's begin with an understanding of the problem in hand. A qubit $|A_T\rangle$ is to be teleported from some source to some destination (see Figure 1). Let's name the source as Alice and the destination as Bob. Before teleporting the qubit $|A_T\rangle$, Alice and Bob generate an "EPR pair" (Bell 1964) (also called "Bell state" (Watson 2022), explained below) by utilizing their one qubit each ($|A_e\rangle$ and $|B_e\rangle$). The qubit $|A_e\rangle$ is maintained by Alice and the qubit $|B_e\rangle$ is maintained by Bob wherever the two are residing in the universe. Before proceeding further, the authors would like to emphasize that a knowledge of linear algebra is a prerequisite to understand the mathematics involved in quantum computing (Aitken 2017; Lipschutz and Lipson 2009; Nielsen and Chuang 2010).

An EPR pair is a maximally entangled qubit pair which cannot be written as a linear combination of its subsystem states. The density matrix of an entangled system has finite off-diagonal terms signifying coherence. Figure 2 shows the quantum circuit to generate the EPR pairs or the Bell states. For different values of the input pair of qubits $|A_e, B_e\rangle = |00\rangle, |01\rangle, |10\rangle$, and $|11\rangle$, the corresponding Bell states $|\beta_{Ae,Be}\rangle$ obtained at the output in Figure 2 are $|\beta_{00}\rangle = \frac{(|00\rangle+|11\rangle)}{\sqrt{2}}$, $|\beta_{01}\rangle = \frac{(|01\rangle+|10\rangle)}{\sqrt{2}}$, $|\beta_{10}\rangle = \frac{(|00\rangle-|11\rangle)}{\sqrt{2}}$, and $|\beta_{11}\rangle = \frac{(|01\rangle-|10\rangle)}{\sqrt{2}}$ respectively.

Continuing with understanding the Figure 1, let the Bell state generated by Alice and Bob in the past be $|\beta_{00}\rangle$. Alice's task is to now teleport another qubit $|A_T\rangle$ to Bob at some point of time in future. Alice is not aware of the value of this third qubit $|A_T\rangle$ nor can she make any measurement to determine the same because she does not have any copy of $|A_T\rangle$ due to no-cloning theorem of the quantum states. The qubit $|A_T\rangle$ can be a pure state or a superposition state which has infinite possible values. The information qubit $|A_T\rangle$ is made to interact with Alice share of entangled qubit $|A_e\rangle$ as shown in Figure 1. The quantum states at various stages in Figure 1 are elaborated below:

$$|A_T\rangle = |\Psi\rangle = \alpha|0\rangle + \gamma|1\rangle \qquad (1)$$

Where α and γ are complex numbers such that $|\alpha|^2$ and $|\gamma|^2$ are the probabilities of $|A_T\rangle$ being in state $|0\rangle$ and $|1\rangle$ respectively.

$$|\beta_{00}\rangle = \frac{(|00\rangle+|11\rangle)}{\sqrt{2}} \qquad (2)$$

$$|\Psi_0\rangle = |A_T\rangle \otimes |\beta_{00}\rangle = (\alpha|0\rangle + \gamma|1\rangle\ ) \otimes \frac{(|00\rangle+|11\rangle)}{\sqrt{2}} \qquad (3)$$

$$|\Psi_0\rangle = \frac{1}{\sqrt{2}}(\alpha|0\rangle \otimes (|00\rangle + |11\rangle) + \gamma|1\rangle \otimes (|00\rangle + |11\rangle)) \qquad (4)$$

After the CNOT Gate, the target qubit (which is Alice qubit $|A_e\rangle$ in the entangled qubit in the EPR pair or the Bell state $|\beta_{Ae,Be}\rangle$) gets inverted whenever the control bit $|A_T\rangle$ is $|1\rangle$, *i.e.*,

$$|\Psi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} \alpha|0\rangle \otimes (|00\rangle + |11\rangle) + \gamma|1\rangle \otimes (|10\rangle + |01\rangle) \\ \uparrow \qquad\qquad\qquad\qquad \uparrow \qquad\quad \uparrow \qquad\quad \uparrow \\ control = 0 \qquad control = 1 \quad |\overline{A_e}\rangle \quad |\overline{A_e}\rangle \end{pmatrix} \qquad (5)$$

After the Hadamard Gate, the information qubit $|A_T\rangle = \alpha|0\rangle + \gamma|1\rangle$ which is now a substate of the quantum state $|\Psi_1\rangle$, gets modified to $\alpha|0\rangle \rightarrow \frac{\alpha}{\sqrt{2}}(|0\rangle + |1\rangle)$ and $\gamma|1\rangle = \frac{\gamma}{\sqrt{2}}(|0\rangle - |1\rangle)$. Hence, $|\Psi_2\rangle$ becomes:

$$|\Psi_2\rangle = \frac{1}{\sqrt{2}}\left(\frac{\alpha}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes (|00\rangle + |11\rangle) + \frac{\gamma}{\sqrt{2}}(|0\rangle - |1\rangle) \otimes (|10\rangle + |01\rangle)\right) \qquad (6)$$

Simplifying Eqn. (6):

$$|\Psi_2\rangle = \frac{1}{2}[\alpha(|000\rangle + |011\rangle + |100\rangle + |111\rangle) + \gamma(|010\rangle - |110\rangle + |001\rangle - |101\rangle)] \qquad (7)$$

In all the three-qubit states in Eqn. (7), the first qubit corresponds to $|A_T\rangle$, the second qubit corresponds to $|A_e\rangle$, and the third qubit corresponds to $|B_e\rangle$. The linear algebra allows the complex constants α and γ to be shifted to any of the three qubits. Hence Eqn. (7) can be rewritten as:

$$|\Psi_2\rangle = \frac{1}{2}[|00\rangle\alpha|0\rangle + |01\rangle\alpha|1\rangle + |10\rangle\alpha|0\rangle + |11\rangle\alpha|1\rangle + |01\rangle\gamma|0\rangle - |11\rangle\gamma|0\rangle + |00\rangle\gamma|1\rangle - |10\rangle\gamma|1\rangle] \qquad (8)$$

Eqn. (8) can be re-adjusted by grouping the terms as per first two qubits which belong to Alice:

$$|\Psi_2\rangle = \frac{1}{2}[|00\rangle\ (\alpha|0\rangle + \gamma|1\rangle) + |01\rangle\ (\alpha|1\rangle + \gamma|0\rangle) + |10\rangle\ (\alpha|0\rangle - \gamma|1\rangle) + |11\rangle\ (\alpha|1\rangle - \gamma|0\rangle)] \qquad (9)$$

Eqn. (9) has four terms: each term has first two qubits belonging to Alice, followed by Bob's qubit which is now in a superposition state. Table 1 details the bifurcation of each of the four terms. At this point in Figure 1, there are two measurement devices M_1 and M_2 which detect the two qubits belonging to Alice, *i.e.*, $|A_T\rangle$ and $|A_e\rangle$ respectively. The double lines indicate classical wires which take this information to Bob. When $|M_1M_2\rangle = |00\rangle$, which is the first term in Eqn. (9), Bob receives $(\alpha|0\rangle + \gamma|1\rangle)$ which is the original qubit that Alice had to teleport to Bob. The classical information $|M_1M_2\rangle = |00\rangle$ communicated to Bob conveys that he has received the teleported qubit in its original form. When Bob receives the classical information that measurements on Alice qubits has yielded $|M_1M_2\rangle = |01\rangle$, then teleported qubit received by Bob is $(\alpha|1\rangle + \gamma|0\rangle)$ which is not the original qubit that Alice had to teleport to Bob. The classical information $|M_1M_2\rangle = |01\rangle$ tells Bob to apply an X-Gate to the teleported qubit received by him, in order to retrieve the original teleported qubit. The third term in Eqn. (9) submits the classical information $|M_1M_2\rangle = |10\rangle$ to Bob. Bob now needs to apply a Z-Gate to extract the teleported information sent to him. And lastly, if $|M_1M_2\rangle = |11\rangle$ then Bob has to apply an X-Gate followed by a Z-Gate to recover the teleported info. After taking the measurements, the quantum state is $|\Psi_3\rangle$ which is the same as $|\Psi_2\rangle$. Afterapplying correction matrices/Gates, the quantum state $|\Psi_4\rangle = |\Psi\rangle = |A_T\rangle$. Hence the teleportation mission is accomplished!

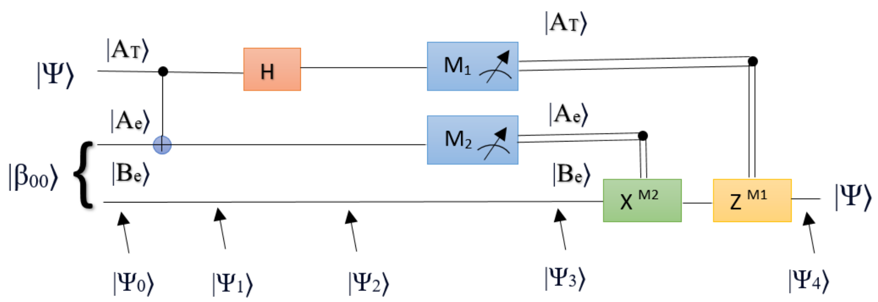

Figure 1: Quantum Circuit showing three qubit system (three single lines), with upper two qubits ($|A_T\rangle$ and $|A_e\rangle$) correspond to Alice and the third ($|B_e\rangle$) corresponds to Bob. The double lines show classical wires, the rectangular boxes represent Hadamard Gate (orange box), measurement instruments (M_1 and M_2 in blue boxes), Pauli's X-Gate (green box), and the Pauli's Z-Gate (yellow box). The purple circle represents the CNOT gate.

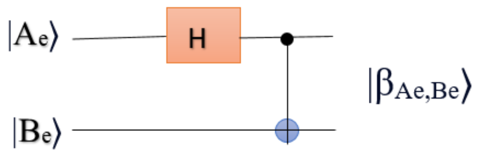

Figure 2: Quantum Circuit for generating Bell states consists of a Hadamard Gate (the orange box H) and a CNOT Gate (the purple circle with a plus sign inside it, signifying the XOR operation). The input qubits are $|A_e\rangle$ and $|B_e\rangle$ and the output state is a maximally entangled state $|\beta_{Ae,Be}\rangle$.

Table 1: Details of the four terms of Eqn. (9). Note that Matrix and Gate terms are used interchangeably.

S. No.	Term	Alice qubits $\vert M_1M_2\rangle$	Bob qubit	How to extract the teleported qubit: Matrix/Gate to be applied on Bob qubit
1.	$\vert 00\rangle\ (\alpha\vert 0\rangle + \gamma\vert 1\rangle)$	$\vert 00\rangle$	$(\alpha\vert 0\rangle + \gamma\vert 1\rangle)$	Apply [I] matrix; (I=identity matrix)
2.	$\vert 01\rangle\ (\alpha\vert 1\rangle + \gamma\vert 0\rangle)$	$\vert 01\rangle$	$(\alpha\vert 1\rangle + \gamma\vert 0\rangle)$	Apply X-Gate/matrix
3.	$\vert 10\rangle\ (\alpha\vert 0\rangle - \gamma\vert 1\rangle)$	$\vert 10\rangle$	$(\alpha\vert 0\rangle - \gamma\vert 1\rangle)$	Apply Z-Gate/matrix
4.	$\vert 11\rangle\ (\alpha\vert 1\rangle - \gamma\vert 0\rangle)$	$\vert 11\rangle$	$(\alpha\vert 1\rangle - \gamma\vert 0\rangle)$	Apply X-Gate followed by X-Gate

III. Discussion

The authors have discussed the mathematics of quantum teleportation in detail with suitable diagrams giving emphasis on the tensor of the qubits. The step by step evolution of the three qubits has been portrayed very elegantly. The authors are working very diligently towards the hardware implementation of Figure 1 and shall publish the same in a separate paper.

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