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Quark Model Three Body Calculations for the Hypertriton Bound State

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Abstract: *Hyperspherical three body calculations are performed to study and review the various properties of the hypertriton bound state nucleus ${}^{3}_{\Lambda}H$ in the quark model using Λ-N potentials. In these calculations we study the different effects of the Λ-N potentials on the hypertriton bound states as well as the separation energy B_{Λ}. A combination of realistic two body N-N potentials with various Λ- N potentials are considered. Complete symmetric and mixed symmetric wave functions are introduced. using the renormalized Numerov method. The agreement between the calculated ${}^{3}_{\Lambda}H$ binding energies and the available experimental data basically depends on the type of the Λ-N interactions used in the calculations. It was found that the Λ-N potentials are the most effective part in the hypertriton binding energy as well as the separation energy B_{Λ} where the Λ-N potentials is very effective to bound or unbound the ${}^{3}_{\Lambda}H$ hyper nucleus*

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I. Introduction

One way to get information about the hyperon nucleon interactions is to study the bound state of the hyperons ,namely the hypernuclei, The most suitable nucleus for such study would be the lightest of these hypernuclei, the hypertriton ${}^{3}_{\Lambda}H$. It is well known that the hypertriton is a bound state which consists of proton, neutron and Λ hyperon. Interest in the hypertriton study comes mainly from the fact that it is the lightest and the most loosely bound hypernucleus. As the hypertriton consists of proton, neutron and Λ hyperon, it may be considered as deuteron and a Λ particle bound in a state with binding energy $B_{\Lambda} = 0.13 \pm 0.05$ MeV [1]. The deuteron binding energy $B_d = 2.2246$ MeV and the binding energy of the hypertriton is B = 2.35 MeV. The hypertriton being the lightest hypernucleus is the first system in which Λ- N potential can be tested in the nuclear medium. This is also supported by the fact that the strength of theΛ- N interaction is not sufficient to produce a bound two body system where the available Λ- N scattering data are still extremely poor. Therefore the hypertriton is expected to play important role in hypernuclear physics similar to that of the deuteron in the conventional nuclear physics.

The hypertriton or approximately the Λ-deuteron system is a very loosely bound system which has a root mean square radius of $[r^2] = 0.2$ fm with reduced mass $\mu = 0.780\ m_N$ is about 314 of the nucleon mass $mc^2 = 939$ MeV. Thus the Λ-deuteron system is 5 times larger than the root mean square radius $R_d = 1.971$ fm of the deuteron. Thus the hypertriton is a very simple system but it is close to the Λ- dripline and therefore it is very sensitive to the Λ- N interaction which is limited by the hypertriton properties. Because of the experimental difficulties however, the existing information on the hypertriton is mostly from old measurements [2]-[4]. The hypertriton study includes a combination of simplicity, sensitivity and fundamental interest in connection with the interactions between nucleons and strange particles. Therefore the theoretical investigations of hypertriton have attracted considerably attentions in nuclear physics study over the last four decades .The interest is reflected in a number of investigations where different techniques and models have been used. Faddeev calculations were performed in order to study and investigate the hypertriton using various Λ- N and N- N interactions [5]-[16]. In these calculations a combinations of the Λ- N and N- N interactions which came out with the results that the hypertriton is unbound [5],[6] ,while in using the Nijmegen potentials the calculations led to a bound state of the hypertriton [7]. Faddeev quark model calculations were performed [8]-[14] using quark model Λ- N interactions combined with various N- N interactions which also came out with the result that the they unbound the hypertriton except for one or two Λ- N interactions which could bound the hypertriton [12]-[15].Variational calculations have been made [16]-[19] in order to study the hypertriton bound state using various Λ- N and N- N interactions. Some of the calculations turned out to unbound the hypertriton and others led to a hypertriton bound state [20].The powerful and exact hyperspherical expansion method was also used to investigate the different properties of the hypertriton bound state using various Λ- N and N- N interactions [22]-[26].The common factor in all the above different methods of calculations is that they have led to considerable

variation in the hypertriton binding energy values from the (observed) bound three body system to the (wrong) unbound Λ-deuteron system [6],[7].

It is the aim of our work to investigate and review the different properties of the hypertriton bound state nucleus ${}_{\Lambda}^{3}H$ in the quark model using Λ- N and N-N potentials using hyperspherical three body calculations. In these calculations we study the different effects of the Λ- N potentials on the bound states of the hypertriton as well as the separation energy B_{Λ}. A combination of the two body N - N potentials with various Λ- N potentials are considered. In the hyperspherical expansion formalism, a complete symmetric and mixed symmetric wave functions are introduced. using the renormalized Numerov method. The agreement between the calculated ${}_{\Lambda}^{3}H$ binding energies and the available experimental data basically depends on the type of the Λ-N interactions used in the calculations. It was found that the Λ-N potentials are the most effective part in the hypertriton binding energy as well as the separation energy B_{Λ} where the Λ-N potentials are very effective to bound or unbound the ${}_{\Lambda}^{3}H$ hyper nucleus. The Fabre optimal subset [27]-[29] is adopted to obtain fast and good convergence for the calculated binding energy using the renormalized Numerov method. [30],[31]. The (HH) method is a powerful tool for the ab initio solution of the few-body Schrodinger equation for a given set of interaction potentials among the constituent particles. In the (HH) method, the wave function describing a system of N particles (in the center of mass system) is expanded in terms of a complete set of orthonormal functions of 3N-4 variables. The expansion coefficients are functions of a single variable that represents the length of 3N-3 dimensional vector. By substituting the wave function expansion into the Schroedinger equation describing the system, one obtains an infinite set of coupled differential equations for the expansion coefficients. The resulting set of coupled differential equations can be solved numerically by the renormalized Numerov method [30],[31] A multi pole potential is also obtained by expanding the two body interaction on a complete set of hyperspherical harmonics. This multi pole potential is very helpful and useful when used in the Schroedinger equation. As for the three-body system, the angular harmonics are functions of five angular variables. In order to determine the potential matrix of the three-body Schroedinger equation, the matrix elements of the multi pole potentials between a pair of such hyperspherical harmonics were calculated. The symmetry of the system under study rules out some harmonics from appearing in the set of coupled equations. Further, the centrifugal barrier terms occurring in the set of coupled equations grow considerably with higher harmonics. One can therefore, truncate this infinite set [27]-[29] and work with a finite set (Fabre optimal subset) of coupled differential equations or a corresponding one dimensional integral equation. The (HH) method is essentially an exact one and more reliable than other methods. It involves no approximation except for a possible truncation of the expansion basis. By gradually expanding the expansion basis and checking the rate of convergence, any desired precision in the binding energy can, in principle, be achieved. However, the number of coupled differential equations and, therefore, the complexity in the numerical solution increases rapidly as the expansion basis is increased by including larger hyper-angular-momentum quantum number. The numbers of equations that have to be retained in any calculation using the (HH) method will, of course, depend on the nature of the potential used.

In the present work, the Fabre optimal subset [27]-[29] was used to obtain a converged set of coupled differential equations in a single variable, namely, the hyper-radius. By numerically solving these equations, the eigenvalues and eigenfunctions of the hyper-nucleus ${}_{\Lambda}^{3}H$ wave function were determined. In section 2 the (HH) method is presented including the different equations used in our calculation. In section 3 the numerical work and results are presented while the discussion and conclusion are given in section 4.Section 5 is devoted for references.

II. Theoretical Work

Let the position vectors of the two nucleons denoted by r_1 and r_2, respectively, and their masses by m_N. The position vector of the Λ particle is denoted by r_3 and its mass by m_{Λ}.The total mass of the ${}_{\Lambda}^{3}H$ hyper-nucleus is $M = m_{\Lambda} + 2m_N$ and that for the lambda Λ particle is taken to be $m_{\Lambda} = (6/5)\, m$, where m is the nucleon (proton or neutron) mass. The Jacobi coordinates set used by Clare and Levinger [32] was chosen here:

$$\eta = \alpha(r_1 - r_2) \qquad (1)$$

$$\xi = \beta(r_1 + r_2 - 2r_3) \qquad (2)$$

where $\alpha = \sqrt{\dfrac{3m_N}{2(2m_N + m_{\Lambda})}}$ *and* $\beta = \alpha\sqrt{\dfrac{m_N}{2m_N + m_{\Lambda}}}$

The inter-particle separation are expressed as:

$$r_{12} = \frac{\eta}{\alpha}, \quad r_{13} = \frac{1}{2\beta}\xi + \frac{1}{2\alpha}\eta, \quad \text{and} \quad r_{23} = \frac{1}{2\beta}\xi - \frac{1}{2\alpha}\eta \qquad (3)$$

Now, the hyper-spherical coordinates ρ and θ are introduced, where ρ is the hyper-radius and θ is the hyper-spherical angle. As a result, the following relations are obtained

$$\eta = \rho\sin\theta \quad , \quad \xi = \rho\cos\theta \quad , \quad \rho^2 = \eta^2 + \xi^2 \qquad (4)$$

with $$\tan\theta = \frac{\eta}{\xi} \qquad (0 \le \theta \le \frac{\pi}{2}) \qquad (5)$$

The non relativistic Schroedinger equation for the hyper-nucleus ${}_{\Lambda}{}^{3}\mathrm{H}$, after separating out the motion of the center of mass, can be written as :

$$\left\{\frac{-\hbar^2}{2\mu}(\nabla_\eta^2 + \nabla_{\xi)}^2 + \sum_{i\succ j} V(r_{ij}) - E\right\}\Psi(\eta,\xi) = 0 \qquad (6)$$

where μ is chosen to be μ = M/3 and $V(r_{ij})$ is the two-body central potential. Therefore, the Schroedinger equation for the hyper-nucleus ${}_{\Lambda}{}^{3}\mathrm{H}$ system expressed in terms of the hyper-spherical coordinates becomes:

$$\left\{\frac{-\hbar^2}{2\mu}\nabla^2 + V(\rho) - E\right\}\Psi(\rho) = 0 \qquad (7)$$

Expanding the wave function Ψ(ρ) on the HH basis, $Y_{[L]}(\Omega)$, gives

$$\Psi(\rho) = \rho^{\frac{-5}{2}} \sum_{[L]} Y_{[L]}(\Omega) U_{[L]}(\rho) \qquad (8)$$

where Ω is a set of 5 angular coordinates describing the position of a point at the surface of the unit hypersphere. $U_{[L]}(\rho)$ are the renormalized hyper radial partial wave functions, [L] stands for the set of quantum numbers including spin and isospin defining the state of grand orbital L. Substituting the expansion (8) into Eq.(7), yields an infinite set of second order coupled differential equations written as:

$$\{\frac{-\hbar^2}{\mu}[\frac{d^2}{d\rho^2} - \frac{(L+2)^2 - \frac{1}{4}}{\rho^2}] - E\}U_{[L]}(\rho) + \sum_{[L]}\langle Y_{[L]}(\Omega)V(\rho)Y_{[L']}\rangle U_{[L']}(\rho) = 0 \qquad (9)$$

which is subsequently truncated in order to be treated numerically. This can be done by using the Fabre optimal subset [27]-[29]. As a result, the infinite set of coupled equations is transformed into a finite set of coupled ones to be solved numerically. For the case of central potential, the ground state of the hyper-nucleus ${}_{\Lambda}{}^{3}\mathrm{H}$ nucleus is described by even number of the grand orbital momentum L= 2K+l, due to the parity conservation. Then, the finite set of coupled equations, for orbital momentum l = 0, becomes:

$$[T_{2K} - E]U_{2K}(\rho) + \sum_{K'}(-)^K V_{2K''}^{2K'}(\rho)U_{2K'}(\rho) = 0 \qquad (10)$$

with the kinetic energy operator and the potential matrix elements being expressed as:

$$T_{2K} = \frac{-\hbar^2}{\mu}[\frac{d^2}{d\rho^2} - \frac{(2K+2)^2 - \frac{1}{4}}{\rho^2}] \qquad (11)$$

and

$$V_{2K''}^{2K'}(\rho) = \sum_{K'''} C_{2K''}^{2K'}(K'',0,C)V_{2K''}(\rho) \qquad (12)$$

respectively. The geometrical coefficients, $C_{2K''}^{2K'}(K'',0,C)$, appearing in Eq.(12) couple the set of equations with the main equation for which K = 0 for each component of the central components of the two-body potentials. Explicit expressions for these coefficients are given in Ref. [27]. The multi-pole potentials, $V_{2K''}(\rho)$, given in Eq.(12) introduce the multi-poles of the central parts of the two-body potential and are expressed as :

$$V_{2K''}(\rho) = \sum_{i\succ j} V_{2K''}^{ij}(\rho) \qquad (13)$$

with $$V_{2K}^{ij}(\rho) = \frac{16}{\pi}\int_0^\infty dx\sqrt{1-(\frac{x}{\rho})^2}\,x^2 V(x^2)\,{}_2F_1(-K, K+2, \frac{3}{2}, (\frac{x}{\rho})^2) \qquad (14)$$

where V(x) = V(r_{ij}) is the two-body potential. In the hyper-nucleus $_{\Lambda}^{3}$H the two body potentials refers to the Λ–Λ and the Λ-α interactions. Equation (14) may be written as:

$$V_{2K}^{ij}(\rho)=\frac{16}{\pi}\int_0^1 du(1-u^2)^{\frac{1}{2}}u^2V(\rho u)_2F_1(-K,K+2,\frac{3}{2},u^2) \qquad (15)$$

with V(ρu) = V(r_{ij}) , which is more useful in our numerical calculations.

III. Numerical Calculations and Results

In order to carry out the numerical calculations, the set of coupled differential equations represented by Eq. (10) were written in matrix form as:

$$\left\{[I]\frac{d^2}{d\rho^2}+[Q]\right\}U(\rho)=0 \qquad (16)$$

where [I] is the unit matrix and the column vector U(ρ) contains the partial waves $U_{2K}(\rho)$ as its components. Also the matrix element [Q] is given by:

$$[Q(\rho)]=\frac{\mu}{\hbar^2}\{E[I]-V_{eff}(\rho)\} \qquad (17)$$

The components of the effective potential matrix is given by:

$$V_{eff}(\rho)=V_{2K}^{2K'}(\rho)+\frac{\hbar^2}{\mu}[\frac{2K(K+1)+\frac{15}{4}}{\rho^2}] \qquad (18)$$

where $V_{2K}^{2K'}(\rho)$ is given by Eq.(12).

The renormalized Numerov method [30],[31] was then used to solve [27]- [29] the set of coupled equations (16). In order to obtain fast convergence of the hyper-nucleus $_{\Lambda}^{3}$H energy eigenvalue the Fabre optimal subset [27]-[29] have been used, using a different Gaussian forms of the realistic N-N and quark model Λ-N interactions. The convergence of $_{\Lambda}^{3}$H eigenvalue have been attained by including terms up to K= $K_{\{max\}}$. Therefore we have solved K=20 coupled equations in order to obtain the hyper-nucleus $_{\Lambda}^{3}$H binding energy values for the considered realistic N-N and quark model Λ-N interactions.

In calculating the binding energy of the $_{\Lambda}^{3}$H hyper nucleus we have used Gaussian form potential for the realistic two body N- N as well as the Λ-N quark model interactions. For the N- N interactions, we have considered two different sets for N- N potentials to be combined with the suggested Λ -N potentials. The first set is the Minnesota N-N potentials MI [33] for which parameters were determined so as to produce the low energy N- N scattering data. The second set is the modified Volkov two body potential V^x [34],[35] for which the singlet and triplet parts are unequal. These two body potential sets satisfy the requirements that they both produce reasonably well the binding energy and size for the trinucleon systems which a necessary condition for our present calculations. For the Λ- Λ interactions**:** It is well known that the Λ- N interactions are constructed in such a way that the calculations with the Λ- N interactions reproduce the experimentally known bound states of the hypernuclei and it is difficult to perform scattering experiments with the Λ- particles because of their short lifetimes (~ 10^{-10} Sec) and the extremely low intensity beams that can be obtained. It is also well known that even when scattering data are available in full, it is impossible to construct Λ- N interactions in a unique way namely one can always obtain different phases and equivalent potentials. According to the above requirements we consider two different sets of quark model Λ- N interactions which reproduce the Λ- P existing scattering data and reproduce the low energy behavior of the Λ- N phase shift obtained in the full coupled calculations. The first one is a three terms Gaussian shape Λ- N interaction QM1 [36] and the second on is the two term Gaussian shape QM2[37] . All the considered types of Λ-N and N-N interactions $_{\Lambda}^{3}$H are of Gaussian shape and given by the following relation:

$$V^{N(\Lambda)}(r)=\sum_{i=1}^{6}V_i\exp(-\frac{r^2}{d_i^2})$$

where the parameters V_i and d_i are given in table (1).

Table (1)

Potn.	i	1	2	3	4	5	6
MI [33]	V_i MeV	200.0	–91.85	–178.0			
	d_i Fm	1.487	0.465	0.639			
V^x [35]	V_i MeV	130.00	-110.0	130.00	-65.30		
	d_i Fm	0.8	1.5	0.8	1.5		

QM1 36]	V_i MeV	200.0	-109.8	-121.3			
	d_i Fm	1.638	0.7864	0.7513			
QM2[37]	V_i MeV	1015.80	-128.0	1072.8	-50.245		
	d_i Fm	5.3830	0.8908	13.740	0.75310		

Table (1). Parameters for the different sets of Gaussian shape N-N and Λ-N interactions

In studying the different properties of the ${}_{\Lambda}{}^{3}H$ hypernucleus bound state, we have considered a combination of the sets of the N-N and the sets of the Λ-N interactions. The calculated ${}_{\Lambda}{}^{3}H$ binding energies and the separation energy B_Λ obtained by the combinations are given in table2. The first four partial waves of the ${}_{\Lambda}{}^{3}H$ hypernucleus wave functions generated by combining MI and QM1 potential are presented in Fig (1.a) and those generated by combining V^x and QM1 potential are presented in Fig(1.b). Similarly the first three partial waves of the ${}_{\Lambda}{}^{3}H$ hypernucleus wave functions generated by combining MI and QM2 potential are presented in Fig (2.a) and those generated by combining V^x and QM2 potential and are presented in Fig(2.b).

N-N	MI [33]		V^x [35]		
Λ-N	QM1[36]	QM2[37]	QM1[36]	QM2[37]	Exp.[1]
E (MeV)	2.8566	2.2207	2.6491	2.1871	2.3556
B_Λ (MeV)	0.632	0.039	0.4245	0.0375	0.131±0.05

Table (2): Calculated binding energies E for the ${}_{\Lambda}{}^{3}H$ hypernucleus and separation energy B_Λ for the Λ particle in ${}_{\Lambda}{}^{3}H$ hypernucleus using different combinations of the realistic N-N and quark model Λ-N interactions. The experimental binding energy value for deuteron E_d is given as $E_d = 2.2246$ MeV

IV. Discussion and Conclusions

In calculating the different properties o the ${}_{\Lambda}{}^{3}H$ hypernucleus we have used the exact powerful and reliable hyperspherical expansion method which is successfully applied in solving and calculating properties of the three and many body problems. We have calculated the ${}_{\Lambda}{}^{3}H$ binding energy E and the separation energy of the Λ particle B_Λ using different combinations of realistic N-N and quark model Λ- N interactions. In order to get a good and fast convergence for the ${}_{\Lambda}{}^{3}H$ binding energy ,Fabre optimal subset [27],[28] was considered in solving the finite set of coupled differential equations, where up to up to $K_{max} = 20$ coupled equations have been solved. The calculated binding energy E and separation energy B_Λ for the ${}_{\Lambda}{}^{3}H$ hypernucleus using different combinations of realistic N-N and quark model Λ- N interactions are presented in table (2). It is shown form table (2) that the binding energies results for the combination of MI and V^x interactions with quark model QM1 interactions indicate that a bound state is obtained for the ${}_{\Lambda}{}^{3}H$ hypernucleus . It is also shown from Table (2) that the separation energy B_Λ indicate that the Λ particle is very close to the nucleon- nucleon pair (core). This result is supported by figures(1.a,1.b) for which a bound state wave function is generated using these potentials. As for the N-N interactions with the QM2 interaction it is clear from the table that the results unbound the ${}_{\Lambda}{}^{3}H$ hypernucleus and the separation energy values B_Λ show that the ${}_{\Lambda}{}^{3}H$ is loosely bound and the Λ particle is far from the nucleon-nucleon cluster, a result which is also supported by Figures (2.a,2.b) for which the generated wave functions indicate unbound state for the ${}_{\Lambda}{}^{3}H$ hypernucleus. Finally in reviewing and studying the properties of the ${}_{\Lambda}{}^{3}H$ using different quark model Λ- N interactions we conclude that the different types of the Λ- N interactions affect the binding energies E and separation energy B_Λ in the case of the ${}_{\Lambda}{}^{3}H$ trinucleon system. In other words the Λ- N interactions is effective to bound or unbound the bound state of the ${}_{\Lambda}{}^{3}H$ hypernucleus a result which is expected due to the quark construction of the Λ particle.

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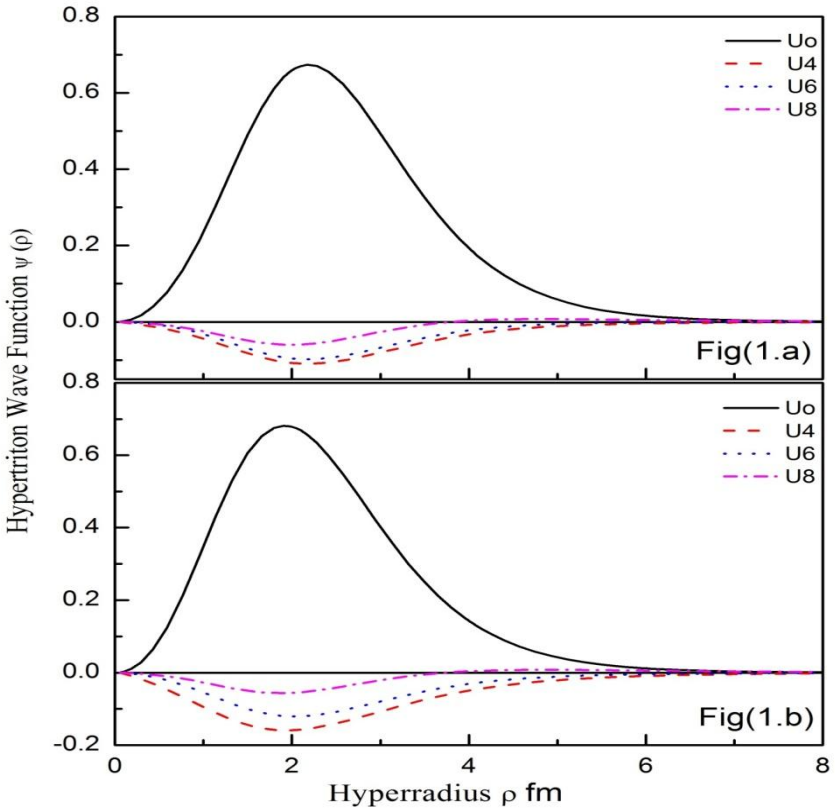

Fig.(1) First four hyperradial partial waves $U_{(2K)}$ (with K = 0,,2,3,4) for the ${}_{\Lambda}{}^{3}H$ hypernucleus: generated by MI[33] N-N and QM1[36] Λ-N interactions Fig. (1.a), V^{x} [35] N-N and QM1[36] Λ-N interactions Fig.(1.b) respectively.

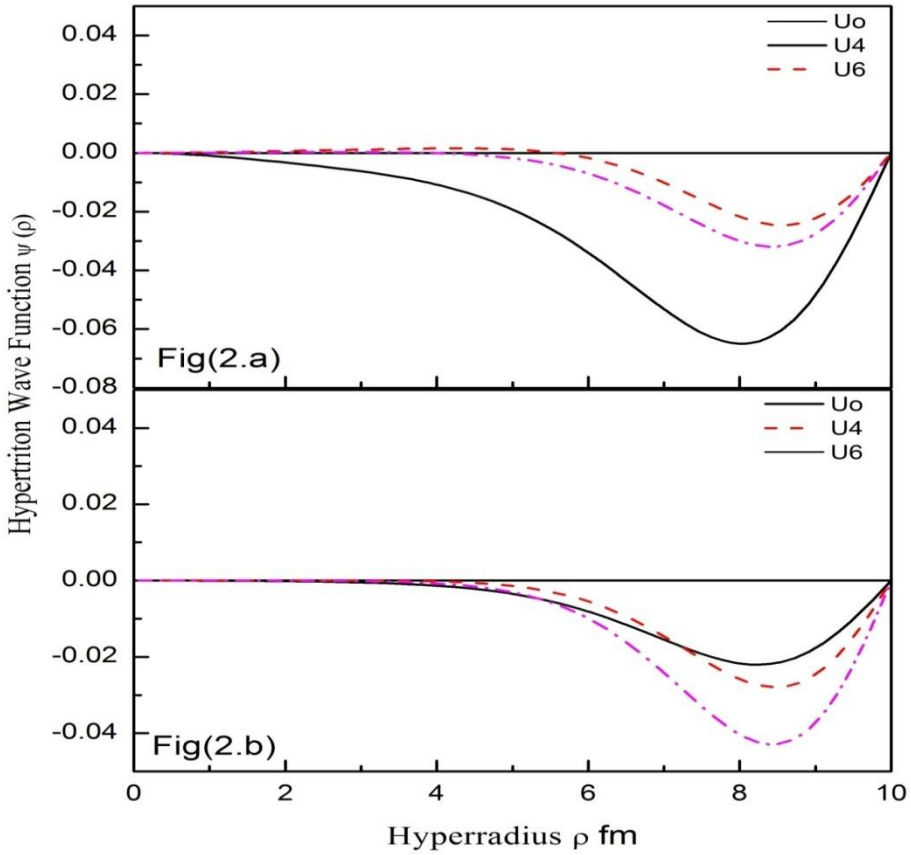

Fig.(2) First three hyperradial partial waves $U_{(2K)}$ (with K = 0,,2,3) for the ${}_{\Lambda}{}^{3}H$ hypernucleus: generated by MI[33] N-N and QM2[37] Λ-N interactions Fig.(2.a), V^{x} [35] N-N and QM2[37] Λ-N interactions Fig.(2.b) respectively.