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Five-Dimensional Cosmological Model with Time-Dependent G and Lamda for Constant and Variable Deceleration Parameter

Gitumani Sarma
Department Of Mathematics,Gauhati University. Guwahati-781014. Assam, India

Abstract: *In this paper we have consider five-dimensional cosmological model in presence of perfect fluid source with time dependent* G *and* Λ*.The Einstein field equations are solvable with the help of constant deceleration parameter. Physical and kinematical properties of this model are investigated. It has been shown that the solutions are comparable with recent observations. The behavior of gravitational constant, cosmological constant, density, critical density and pressure is discussed for dust, radiation dominated and stiff matter of the Universe. It is also examined the behavior of gravitational constant and cosmological constant for expansion law and exponential law for stiff matter.*
***Keywords*: Cosmological parameters, Deceleration parameter, Cosmology**

I. Introduction

Recent cosmological observations shown that an accelerating Universe with variable G and Λ.The generalized Einstein's theory of gravitation with time dependent G and Λ has been proposed by Lau [1].The possibility of variable G and Λ in Einstein's theory has also been studied by Dersarkissian [2].Berman [3] and Sistero [4] have considered the Einstein's field equations with perfect fluid and variable G for Robertson-walker metric. On the other hand variation of the gravitational constant was first suggested by Dirac [6].Beesham [5] has studied the creation with variable G and pointed out the variation of the form G, originally proposed by Dirac [6]. Variable Λ was introduced such that Λ was large in the early universe and then decayed with evolution. On the other hand numerous modifications of general relativity, variable G based on different arguments have been proposed by many authors have shown that G-varying cosmology is consistent with whatsoever cosmological observations available at present. A modification linking the variation of G with that of Λ has been considered within the framework of general relativity by a number of workers. The cosmological models with variable G and Λ have been recently studied by several authors. The cosmological constant Λ is the most favored candidate of dark energy representing energy density of vacuum. Some author have studied its significance and suggested that cosmological term corresponds to a very small value of the order 10^{-22} when applied to Friedman universe. Linde has investigated that Λ is a function of temperature. A dynamic cosmological term Λ(t) remains a focal point of interest in modern cosmological theories as it solves the per year with redshift up to $z = 1.7$.

The nature of dark energy lies in ω, the equation of state parameter which is nothing but the ratio of fluid pressure and matter-energy density of dark energy.viz, $\omega = \frac{p}{\rho}$.This parameter ω has been selected in different ways indifferent models. . The equation of state ω, which is the ratio of the dark energy density to its pressure which is not necessarily constant. Recently the parameter ω has been calculated with some reasoning which reduced to some simple parameterization of the dependences by some authors (Huterer and Turner, Weller and Albrecht, Linden and Virey, Krauss and Usmani, and Chen). The present day observation indicates that the Universe at large scale is homogeneous and isotropy and the universe is accelerating. According to Johri, the cosmic expansion of the models with constant deceleration parameter is given by big-bang theory. Spatially homogeneous and anisotropic cosmological models play a significant role in the description of large scale behavior of universe and such models have been widely studied in framework of general Relativity in search of a realistic picture of the universe in its early stages. The supernova cosmology project and High-z supernova team presented evidence that the expansion of the universe is accelerating. The simplest and most theoretically appealing possibility for dark energy is the energy density stored on the vacuum state of all existing fields in the universe,i,e $\rho_v = \frac{\Lambda}{8\pi G}$,where Λ is the cosmological constant. However a constant Λ cannot explain the huge difference between the cosmological constant inferred from observations and the vacuum energy density resulting from quantum field theories.

Law of variation for Hubble's parameter which was proposed by Berman and Cimento. It is interesting to observe that this law yields a constant value for the deceleration parameter. Forms for the deceleration parameter which are variable have been investigated by Beesham. In earlier literature cosmological models with constant deceleration parameter have been studied by Singh and Baghel[24], Maharaja and Naidoo[23] whereas pradhan et al[22] have been studied FRW model with variable deceleration parameter. There are significant observational evidence that the expansion of the universe is undergoing a late time acceleration.

II. The Einstein Field Equation For The Cosmological Model

We consider five-dimensional Kaluza-Klein space-time given by

$$ds^2 = dt^2 - A^2(dx^2 + dy^2 + dz^2) - B^2 du^2 \quad (1)$$

Where A and B are function of t only. The universe is assumed to be filled with distribution of matter represented by energy-momentum tensor for a perfect fluid by

$$T_{ij} = (\rho + p)v_i v_j - p g_{ij} \quad (2)$$

Where ρ is the energy density of cosmic matter, p is pressure, v^i is the unit flow vector such that $v_i v^i = 1$

We assume that the matter content obeys an equation of state

$$p = \omega\rho, 0 \le \omega \le 1 \quad (3)$$

The Einstein's field equation with variable G and Λ,

$$R^{ij} - \frac{1}{2} R g^{ij} = 8\pi G T^{ij} - \Lambda g^{ij} \quad (4)$$

The spatial average scale factor $R(t)$ is given by

$$R^4 = A^3 B$$

The average volume scale factor $V = R^4$

The average Hubble parameter H may be generalized in anisotropic cosmological model as

$$H = \frac{1}{4}\left(\frac{3\dot{A}}{A} + \frac{\dot{B}}{B}\right)$$

The field equations are

$$\frac{3\dot{A}^2}{A^2} + \frac{3\dot{A}\dot{B}}{AB} = 8\pi G\rho + \Lambda \quad (5)$$

$$\frac{2\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{2\dot{A}\dot{B}}{AB} + \frac{\dot{A}^2}{A^2} = -8\pi G p + \Lambda \quad (6)$$

$$\frac{3\ddot{A}}{A} + \frac{3\dot{A}^2}{A^2} = -8\pi G p + \Lambda \quad (7)$$

Covariant derivative of Einstein's field equation,

$$\dot{\rho} + (\rho + p)\left(\frac{3\dot{A}}{A} + \frac{\dot{B}}{B}\right) + \frac{\dot{G}}{G}\rho + \frac{\dot{\Lambda}}{8\pi G} = 0 \quad (8)$$

Conservation equation gives,

$$\dot{\rho} + (\rho + p)\left(\frac{3\dot{A}}{A} + \frac{\dot{B}}{B}\right) = 0 \quad (9)$$

So,

$$\dot{G} = -\frac{\dot{\Lambda}}{8\pi\rho} \quad (10)$$

It shows that when G increases or decreases Λ, increases or decreases. Also it shows that when Λ is constant or zero G turns out to be constant.

III. General Solutions To The Field Equation:

The equation (3), (8)-(10) and (11) are five independent equations with six unknown $A, B, p, G, \rho, \Lambda$

Hence to get a realistic solution we assume that deceleration parameter constant.i, e $q = -\frac{R\ddot{R}}{\dot{R}^2} = b$,where b is constant.

From this we get,

$$R = (k_1 t + k_2)^{1/(1+b)}, b \neq -1 \qquad (11)$$

Where k_2 is integration of constant.

The expansion scalar $\theta = 3H = \frac{3u_0}{k_1 t + k_2}$ (12)

,where $u_0 = \frac{k_1}{1+b}$ and $H = \frac{\dot{R}}{R}$

Again, $3H = \frac{3}{4}\left(\frac{3\dot{A}}{A} + \frac{\dot{B}}{B}\right)$ (13)

So,

$$\frac{3\dot{A}}{A} + \frac{\dot{B}}{B} = \frac{4u_0}{k_1 t + k_2} \qquad (14)$$

From field equation (9) and (10),we get

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = \frac{c_3}{A^3 B} \qquad (15)$$

Solving (14) and (15),we get,

$$A = (k_1 t + k_2)^l e^{n(k_1 t + k_2)^{\frac{b-3}{1+b}}}, b > 3, n = \frac{k_5}{4} \qquad (16)$$

$$B = (k_1 t + k_2)^l e^{-m(k_1 t + k_2)^{\frac{b-3}{1+b}}}, b > 3, l = \frac{k_4}{4}, m = 3k_5 \qquad (17)$$

From equation (12)

$$\rho = \frac{k_6}{(k_1 t + k_2)^{\frac{4(1+\omega)}{1+b}}} \qquad (18)$$

Where k_6 is integration of constant.Again using equation (3),we get

$$p = \omega\rho = \frac{\omega k_6}{(k_1 t + k_2)^{\frac{4(1+\omega)}{1+b}}} \qquad (19)$$

Again subtracting (10) from (8)

$$G = \frac{1}{8\pi\omega k_6}\left[\frac{a_0}{(k_1 t + k_2)^{\frac{(1+b)-4\omega}{1+b}}} - \frac{b_0}{(k_1 t + k_2)^{\frac{4(1-\omega)}{1+b}}} - \frac{k_7}{(k_1 t + k_2)^{\frac{2b-2-2\omega}{1+b}}}\right] \qquad (20)$$

,where

$$a_0 = -\left(k_8 + \frac{3u_0 c_3}{2} + \frac{24u_0 c_3}{16}\right), \quad b_0 = \frac{12c_3^{\ 2}}{16}$$

From (10) we get

$$\Lambda = \frac{d_0}{(k_1t+k_2)^2} + \frac{e_0}{(k_1t+k_2)^{-\frac{8}{1+b}}} + \frac{f_0}{(k_1t+k_2)^{\frac{5+b}{1+b}}} +$$

$$+\frac{1}{(k_1t+k_2)^{\frac{4(1+\omega)}{1+b}}}\left[\frac{a_0}{(k_1t+k_2)^{\frac{1+b-4\omega}{1+b}}} - \frac{b_0}{(k_1t+k_2)^{\frac{4(1-\omega)}{1+b}}} - \frac{k_7}{(k_1t+k_2)^{\frac{2b-2-2\omega}{1+b}}}\right] \qquad (21)$$

The critical density is given by

$$\rho_c = \frac{3H^2}{8\pi G}$$

$$= \frac{\omega I_0}{\left[\frac{a_0}{(k_1t+k_2)^{-(1+\frac{4\omega}{1+b})}} - \frac{b_0}{(k_1t+k_2)^{-(2+\frac{4-4\omega}{1+b})}} - \frac{k_7}{(k_1t+k_2)^{-2(1-\frac{b-1-\omega}{1+b})}}\right]} \qquad (22)$$

IV. The physical behavior of the model:

The Hubble parameter H is related to the average scale factor by the relation

$$H = l\,\mathrm{R}^{-n} = 1\left(\mathrm{A}^3 B\right)^{-\frac{n}{4}} \qquad (23)$$

Where $l(>0)$ and $n(\geq 0)$ are constant. Pradhan and Jotania have studied flat FRW and bianchi type models by using the special law for hubble parameter that yields constant value of deceleration parameter.

$$\dot{R} = 1\,\mathrm{R}^{-n+1} \qquad \mathrm{i,e}\ 0 \leq \mathrm{n} < 1 \qquad (24)$$

$$\ddot{R} = -l^2(n-1)R^{-2n+1} \qquad (25)$$

Using (24) and (25) in expression of q ,we get

$$q = n-1 \qquad (26)$$

We observed that the relation (26) gives q as a constant. The sign of q indicated whether the model inflates or not. The positive sign of q i,e $\mathrm{n} > 1$ correspond to decelerating model whereas the –ve sign of q i,e $0 \leq \mathrm{n} < 1$ indicates inflation. Current observation of SNe Ia and CMBR favors accelerating model $(q < 0)$.

$$R = (nlt + c_1)^{\frac{1}{n}} \quad ,n\neq 0 \qquad (27)$$

$$R = c_2 lt \quad ,n=0 \qquad (28)$$

Thus the law (23) provides two types of expansion in the stiff matter universe. i,e (i) power law $(n \neq 0)$ and (ii) exponential law $(n = 0)$.

Case (i): $n \neq 0$, i,e the model for power law expansion

From equation (20),using (27)

$$G = \frac{1}{8\pi\omega k_6}\left[\frac{a_0}{(nlt+c_1)^{\frac{1+b-4\omega}{n}}} - \frac{b_0}{(nlt+c_1)^{\frac{4(1-\omega)}{n}}} - \frac{k_7}{(nlt+c_1)^{\frac{2b-2-2\omega}{n}}}\right] \qquad (29)$$

$$\Lambda = \frac{d_0}{(nlt+c_1)^{\frac{1+b}{n}}} + \frac{e_0}{(nlt+c_1)^{\frac{8}{n}}} + \frac{f_0}{(nlt+c_1)^{\frac{5+b}{n}}} + \frac{1}{(nlt+c_1)^{\frac{4(1+\omega)}{n}}}\left[\frac{a_0}{(nlt+c_1)^{\frac{1+b-4\omega}{n}}} - \frac{b_0}{(nlt+c_1)^{4(1-\omega)/n}} - \frac{k_7}{(nlt+c_1)^{\frac{2b-2-2\omega}{n}}}\right] \quad (30)$$

$$\rho = \frac{k_7}{(nlt+c_1)^{\frac{4(1+\omega)}{n}}} \quad (31)$$

$$p = \frac{\omega k_7}{(nlt+c_1)^{\frac{4(1+\omega)}{n}}} \quad (32)$$

From equation (30) ,using (29)

$$\Lambda = \frac{d_0}{(nlt+c_1)^{\frac{1+b}{n}}} + \frac{e_0}{(nlt+c_1)^{\frac{8}{n}}} + \frac{f_0}{(nlt+c_1)^{\frac{5+b}{n}}} + \frac{1}{(nlt+c_1)^{\frac{4(1+\omega)}{n}}} 8\pi\omega k_6 G \quad (33)$$

Case (ii): $n = 0$**,i,e the model for exponential law**

From equation (20), using (28)

$$G = \frac{1}{8\pi\omega k_6}\left[\frac{a_0}{(c_2lt)^{1+b-4\omega}} - \frac{b_0}{(c_2lt)^{4(1-\omega)}} - \frac{k_7}{(c_2lt)^{2b-2-2\omega}}\right] \quad (34)$$

$$\Lambda = \frac{d_0}{(c_2lt)^{1+b}} + \frac{e_0}{(c_2lt)^{8}} + \frac{f_0}{(c_2lt)^{5+b}} + \frac{1}{(c_2lt)^{4(1+\omega)}}\left[\frac{a_0}{(c_2lt)^{1+b-4\omega}} - \frac{b_0}{(c_2lt)^{4(1-\omega)}} - \frac{k_7}{(c_2lt)^{2b-2-2\omega}}\right] \quad (35)$$

$$\rho = \frac{k_7}{(c_2lt)^{4(1+\omega)}} \quad (36)$$

$$p = \frac{\omega k_7}{(c_2lt)^{4(1+\omega)}} \quad (37)$$

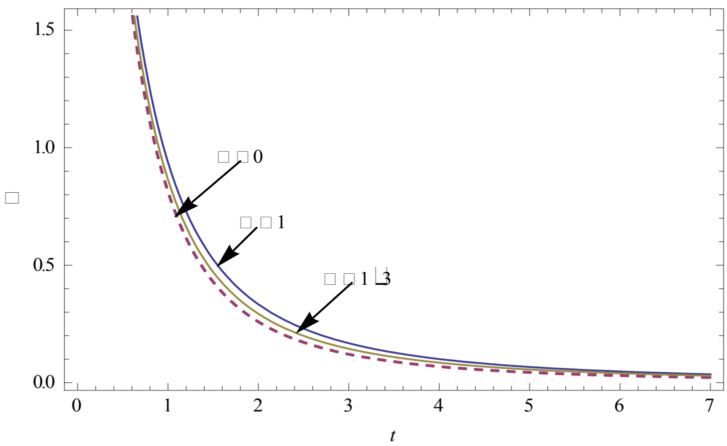

Fig-The plot of Λ vs.time t . Here $d_0 = 2$, $e_0 = 2$, $f_0 = 2$, $k_1 = 1, k_2 = 1, b = 1$, $a_0 = 1, b_0 = 1, k_7 = 1$

Fig-1 shows the variation of cosmological constant and time for dust, radiation and stiff matter of the universe. It is observed that in three stages of the universe cosmological constant decreases as the time increases.

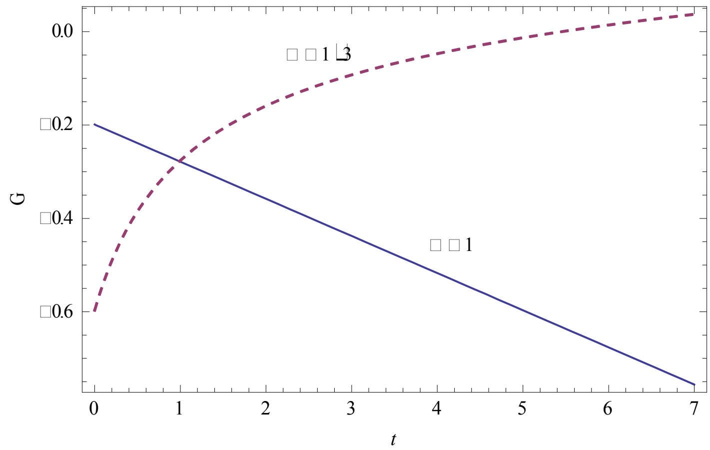

Fig-2The plot of G vs. t. Here $a_0 = 3, b_0 = -3, k_7 = 1,\ k_1 = 1, k_2 = 1, k_6 = -1, b = 4$

Fig-2 shows the variation of G and t for radiation and stiff matter. For dust matter G is undefined.Fig-2 shows that in radiation dominated universe gravitational constant increases with time whereas in stiff matter G decreases with time.

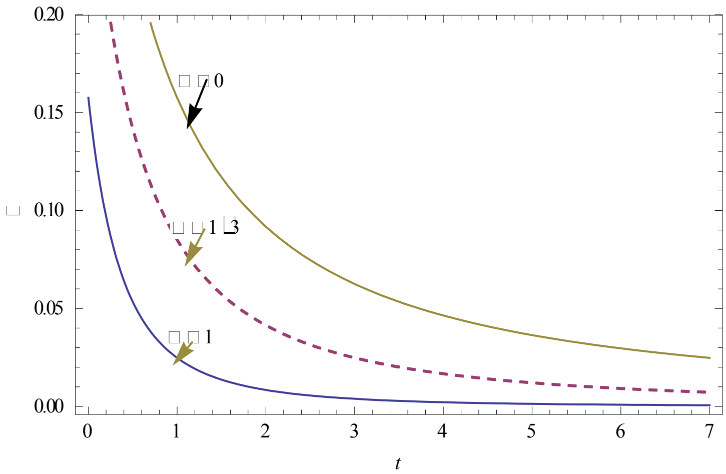

Fig-3The plot of energy density ρ vs. time. Here $k_6 = 1, k_1 = 2, k_2 = 2, b = 4$

Fig-3 shows the variation of density and time for radiation, dust and stiff matter of the universe. From this fig it is examined that energy density decreases with time.

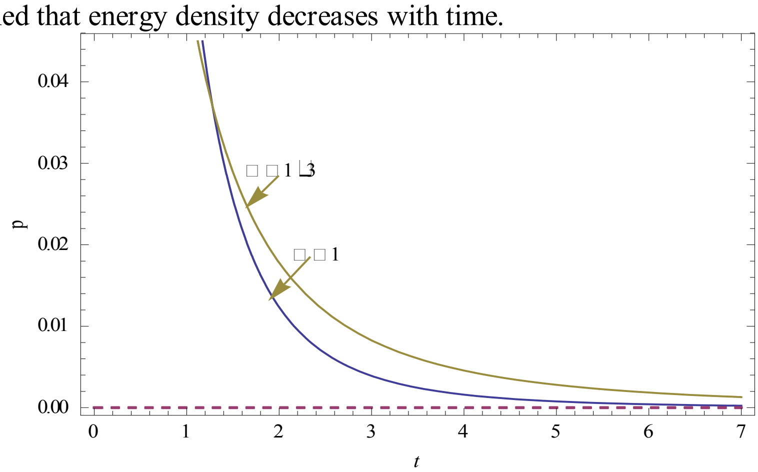

Fig-4The plot of pressure p vs. time. Here $k_1 = 1, k_2 = 1, k_6 = 1, b = 4$

Fig-4 shows the variation of pressure and time for radiation and stiff matter of the universe. For dust matter pressure is zero

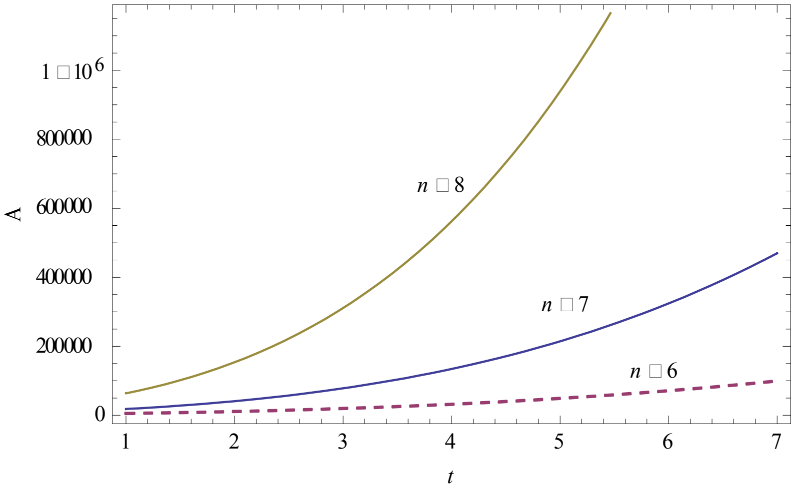

Fig-5 The plot of scale factor A vs. time(t). Here $k_1 = 1, k_2 = 2, l = 1, b = 4$

From Fig-5 it is observed that scale factor A increases as the time increases.Here $b > 3$ .Also A depend on constant n also.We observed that $n > 5$.

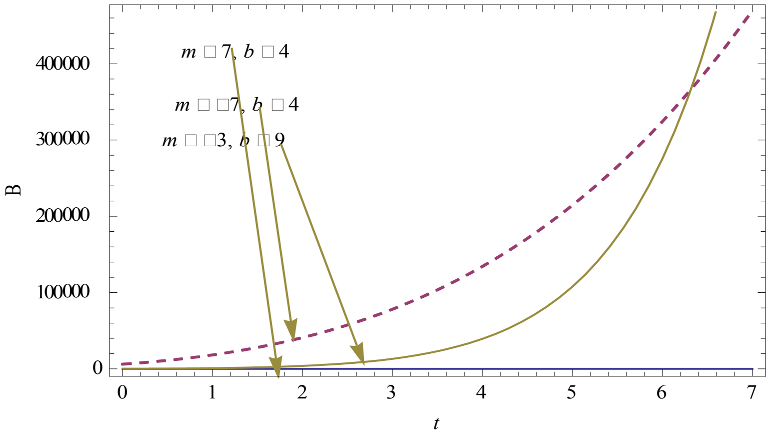

Fig-6 The plot of scale factor B vs. time(t). Here $k_1 = 1, k_2 = 2, l = 1, b = 4$

From Fig-6 it is observed that B depends on constant m. We observed that whem m>3, B decreases with time. Whem $m < 3$, B increases as the time increases.

For expansion law:

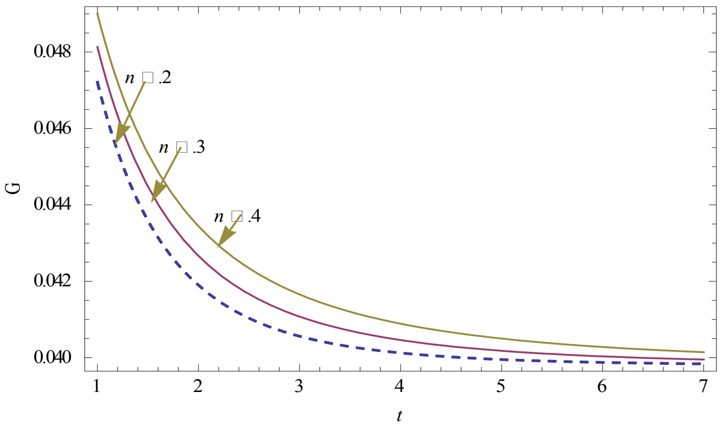

Fig-7 The plot of gravitational constant G vs. time for expansion law. Here $k_6 = 2, a_0 = 2, b_0 = -2, k_7 = -1, l = 2, c_1 = 1, b = 4$

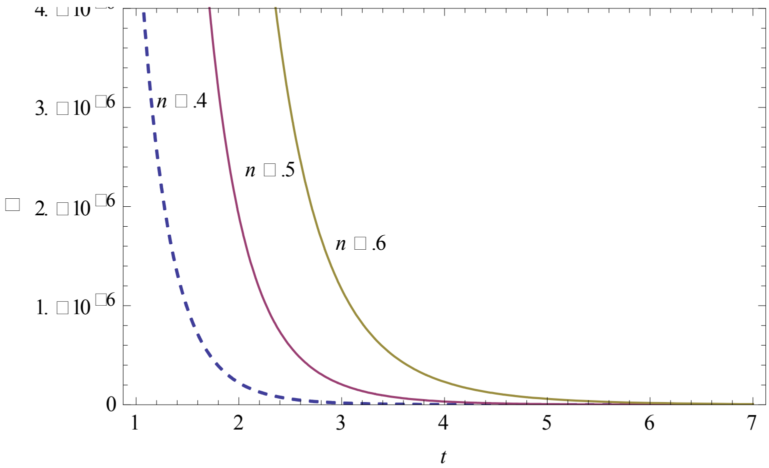

Fig-8 The plot of Λ vs. time. Here $d_0 = 2$,
$e_0 = 2, f_0 = 1, a_0 = 3, b_0 = 2, k_7 = 1, l = 2, c_1 = 2, b = 4$

For exponential law

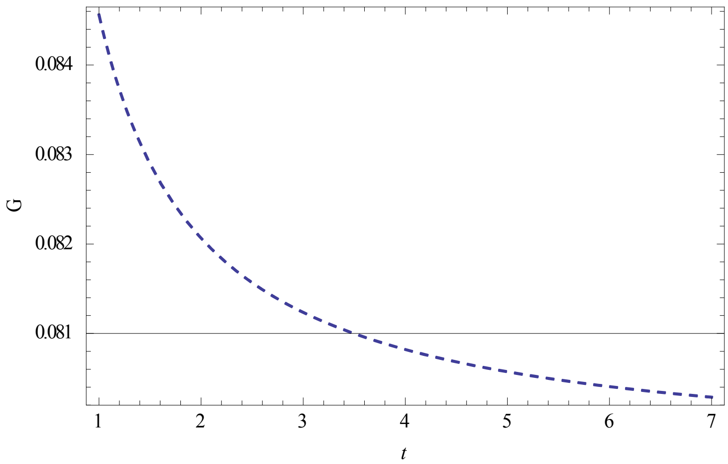

Fig-9 The plot of G vs. time. Here $a_0 = 1, b_0 = -2, k_7 = -1, k_6 = 1, c_2 = 2, l = 3, b = 4$

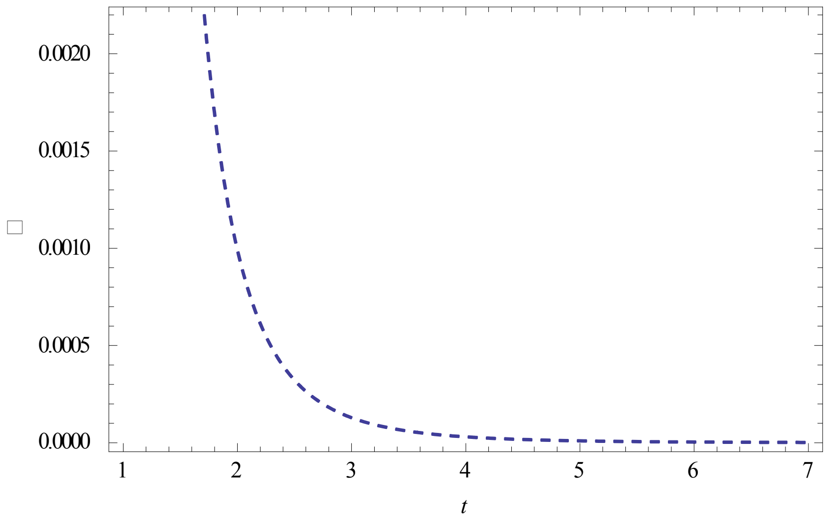

Fig-10 The plot of Λ vs. time.
,Here $a_0 = 2, b_0 = 2, k_7 = 1, d_0 = 1, e_0 = 2, f_0 = 2, c_2 = 1, l = 2, b = 4$

V. Discussion:

In this paper we have obtained the gravitational constant, cosmological constant, density and pressure of the Kaluza-Klein five-dimensional cosmological model.The equation of state ω lies between 0 and 1.It is graphically shown that variation of G with time for special three cases as we discussed above whereas the cosmological constant also decreases as the time increases. The spatial volume V is zero at $t = t_0$,where $t_0 = -\frac{k_2}{k_1}$.And the expansion scalar θ is infinite at $t = t_0$,which shows that universe starts evolving with zero volume and infinite rate of expansion at $t = t_0$ The scale factor also convert to a single number 1 at $t = t_0$ .At $t = t_0$,the parameter G and Λ are infinite in value,whereas they become zero as $t \to \infty$ .It is also observed that the energy density and pressure decreases as the time increases.We also observed gravitation for $\omega = 0,1/3,1$. Moreover it is observed that due to the combined effect of time variable Λ and G the Universe evolved with deceleration as well as acceleration.The model shows that G varies with time as suggested earlier by Dirac. The Einstein's field equations has two parameters, the gravitational constants G and cosmological constants Λ.The Newtonian constant of gravitation G plays the role of a coupling constant between geometry and matter in the Einstein's field equation. In an evolving universe it appears to look at this constant as a function of time. There are significant observational evidence evidence that the expansion of the universe is undergoing a late time acceleration [7-21] cosmological models with a cosmological constant are currently serious to candidates describe the expansion history of the universe. The huge difference between the small cosmological constant inferred from observations and vacuum energy density resulting from quantum field theories has been for a long time a difficult and fascinating problem (Carneiro, 2003; Crauss, 1995; Sahni and Stravinsky, 2000; Weinberg, 1989) for cosmologists and field theory researches.Assuming a relation between hubble parameter and average scale factor I have examined the universe provides two types of expansion.It is expansion law and exponential law. In both cases I have observed the behaviour of gravitational constant and cosmological constant.

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