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Electromagnetic fields of time-dependent magnetic monopole

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Abstract: *Dirac-Maxwell's equations, retained for magnetic monopoles, are generalized by introducing magnetic scale field. It allows the magnetic monopoles to be time-dependent and the potentials to be Lorentz gauge free. The non-conserved part or the time-dependent part of the magnetic charge density is responsible to produce the magnetic scalar field which further contributes to the magnetic and electric vector fields. This contribution makes possible to create an ideal square wave magnetic field from an exponentially rising and decaying magnetic charge.*
Keywords: *Dirac-Maxwell's equations, Lorentz Gauge, Magnetic scalar field*

I. INTRODUCTION

The Maxwell–Heaviside equations prescribe both open dissipative systems having coefficient of performance (COP) > 1 and equilibrium systems having COP < 1. Imposition of the Lorentz symmetrical regauging reduces the set of Maxwell–Heaviside equations into a subset which discards open dissipative Maxwellian systems and retains only those in equilibrium [1]. However, the discarded class of Maxwellian systems contains all Maxwellian electromagnetic (EM) power systems exhibiting COP > 1, by functioning as open dissipative systems freely receiving and using excess energy from the active vacuum. To study the open dissipative systems the potentials are to be made Lorentz gauge free. While studying such systems, Anastasovski et al [2] obtained equations for vacuum current density and vacuum charge density and proposed to pick up by a receiver and use to generate huge electrical energy. Similar results have been deduced by Lehnert [3–6] and Lehnert and Scheffel [7] from the condition of a nonzero charge density from vacuum fluctuations in combination with the requirement of Lorentz invariance. By another approach Teli and Jadhav [8] removed the Lorentz condiction on potentials by introducing scalar potentials in the Generalized Dirac-Maxwell's equations which made the electrical charges time varying in nature. These charges then did not satisfy the continuity equation. The non-conserved part of the charge density was accommodated in terms of a scalar field. They obtained electromagnetic fields of non-conserving electric charged particle. However, magnetic monopoles, elementary particles with a net magnetic charge, have been a curiosity for physicists and many believe they ought to exist. Our attempt is to find out the fields of non-conserving magnetic monopoles with removing Lorentz gauge on the potentials.

The next section includes the Generalization Maxwell's equations for magnetic monopoles by introducing a magnetic scalar field H_0, which removes the Lorentz condition on the potentials. In section 3, the magnetic scalar field, in addition to the magnetic vector field and the electric vector field of a time-dependent magnetic monopole is obtained. It is found that the magnetic scalar field H_0 is a function of the time rate of change of the magnetic charge on the monopole. In section 4, the magnetic vector field and magnetic scalar field of a stationary time-dependent magnetic monopole are evaluated. Section 5 includes calculation of ideal square wave magnetic field from a rising and decaying magnetic monopole. Section 6 includes the discussion.

II. GENERALIZATION OF MAXWELL'S EQUATIONS FOR MAGNETIC MONOPOLES

Analogous to [8], we may introduce a scalar function H_0 into Dirac-Maxwell's equations retained for magnetic monopoles only to accommodate the time dependent part of the source densities such as

$$\nabla\cdot\boldsymbol{E}=0 \tag{1a}$$

$$\nabla\cdot\boldsymbol{H}+P_1H_0=4\pi\rho^m \tag{1b}$$

$$\nabla\times\boldsymbol{E}+\frac{1}{c}\frac{\partial\boldsymbol{H}}{\partial t}+P_2H_0=-\frac{4\pi}{c}\boldsymbol{j}^m \tag{1c}$$

$$\nabla\times\boldsymbol{H}-\frac{1}{c}\frac{\partial\boldsymbol{E}}{\partial t}=0 \tag{1d}$$

where P_1 and P_2 are operators and H_0 is a magnetic scalar field.
As usual the vector fields can be expressed in terms of the potentials

$$\boldsymbol{E} = -\nabla \times \boldsymbol{A}^m \tag{2a}$$

$$\boldsymbol{H} = -\nabla \phi^m - \frac{1}{c}\frac{\partial A^m}{\partial t} \tag{2b}$$

These potentials satisfy the usual differential equations:

$$\nabla^2 \phi^m - \frac{1}{c^2}\frac{\partial^2 \phi^m}{\partial t^2} = -4\pi\rho^m \tag{3a}$$

$$\nabla^2 \boldsymbol{A}^m - \frac{1}{c^2}\frac{\partial^2 \boldsymbol{A}^m}{\partial t^2} = -\frac{4\pi}{c}\boldsymbol{j}^m \tag{3a}$$

provided that

$$P_1 H_0 = \frac{1}{c}\frac{\partial}{\partial t}\left(\nabla \cdot \boldsymbol{A}_m + \frac{1}{c}\frac{\partial \phi^m}{\partial t}\right) \tag{4a}$$

$$P_2 H_0 = \nabla\left(\nabla \cdot \boldsymbol{A}_m + \frac{1}{c}\frac{\partial \phi^m}{\partial t}\right) \tag{4b}$$

These equations give

$$H_0 = \nabla \cdot \boldsymbol{A}^m + \frac{1}{c}\frac{\partial \varphi^m}{\partial t} \tag{5}$$

with

$$P_1 = \frac{1}{c}\frac{\partial}{\partial t}, \qquad P_2 = \nabla \tag{6}$$

The magnetic scalar field H_0 is the actual replacement of the Lorentz condition. If the potentials satisfy the Lorentz condition, the magnetic scalar field disappears and the Dirac-Maxwell's equations for magnetic monopoles attain their original form. The generalized Dirac-Maxwell's equations for magnetic monopoles are then:

$$\nabla \cdot \boldsymbol{E} = 0 \tag{7a}$$

$$\nabla \cdot \boldsymbol{H} + \frac{1}{c}\frac{\partial H_0}{\partial t} = 4\pi\rho^m \tag{7b}$$

$$\nabla \times \boldsymbol{E} + \frac{1}{c}\frac{\partial \boldsymbol{H}}{\partial t} + \nabla H_0 = -\frac{4\pi}{c}\boldsymbol{j}^m \tag{7c}$$

$$\nabla \times \boldsymbol{H} - \frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t} = 0 \tag{7d}$$

Equations (2a), (2b) and (5) are solutions of Generalized Dirac-Maxwell's equations.

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\boldsymbol{E} = \frac{4\pi}{c}\nabla \times \boldsymbol{j}^m \tag{8a}$$

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\boldsymbol{H} = -\frac{4\pi}{c}\left(c\nabla\rho^m + \frac{1}{c}\frac{\partial \boldsymbol{j}^m}{\partial t}\right) \tag{8b}$$

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)H_0 = -\frac{4\pi}{c}\left(\nabla \cdot \boldsymbol{j}^m + \frac{1}{c}\frac{\partial \rho^m}{\partial t}\right) \tag{8c}$$

In the absence of the sources, all of them propagate with velocity c in vacuum. Equation (8c) shows that the charges satisfy the continuity equation in the absence of the scalar field.

III. FIELDS OF A TIME-DEPENDENT MAGNETIC MONOPOLE

For a time-dependent point magnetic monopole, the equations (3) have usual solutions as given by Panofsky and Philip [9]:

$$\phi^m(r,t) = \frac{q^m(t')}{S} \tag{9a}$$

$$\boldsymbol{A}^m(r,t) = \frac{q^m(t')\boldsymbol{v}(t')}{S} \tag{9b}$$

where $S = R - \frac{\boldsymbol{R}\cdot\boldsymbol{v}}{c}$, $\boldsymbol{R} = \boldsymbol{r} - \boldsymbol{r}'$, $\boldsymbol{r}$ is the field point vector, $\boldsymbol{r}'$ is the source point vector at the retarded time t' defined by $t' = t - R/c$ and $\boldsymbol{v}$ is the instantaneous velocity of the monopole.
The EM fields for such a monopole in arbitrary motion, using equations (2), (5) and (9), are

$$\boldsymbol{H} = \frac{H_0\left(\boldsymbol{r} - \frac{r\boldsymbol{v}}{c}\right)}{S} + q^m\left[\frac{\left(\boldsymbol{r} - \frac{r\boldsymbol{v}}{c}\right)\left(1-\beta^2\right)}{S^3} + \frac{\boldsymbol{r}\times\left[\left(\boldsymbol{r} - \frac{r\boldsymbol{v}}{c}\right)\times\frac{d\boldsymbol{v}}{dt'}\right]}{S^3c^2}\right] \tag{10a}$$

$$\boldsymbol{E} = -\frac{H_0\left(\boldsymbol{v}\times\boldsymbol{r}\right)}{cS} - q^m\left[\frac{\left(\boldsymbol{v}\times\boldsymbol{r}\right)\left(1-\beta^2\right)}{cS^3} + \frac{\boldsymbol{r}\times\left\{\boldsymbol{r}\times\left[\left(\boldsymbol{r} - \frac{r\boldsymbol{v}}{c}\right)\times\frac{d\boldsymbol{v}}{dt'}\right]\right\}}{rS^3c^2}\right] \tag{10b}$$

$$H_0 = \frac{1}{cS}\frac{dq^m}{dt'} \tag{10c}$$

Both the fields $\boldsymbol{H}$ and $\boldsymbol{E}$ receive contributions from the instantaneous value of the magnetic charge on the monopole as well as the rate of the change of the charge with time. Clearly, the magnetic scalar field H_0 is a function of the rate of change of the magnetic charge on the monopole with time. For time-independent magnetic charge on the monopole, the scalar field disappears and the above field equations reduce to their usual forms.

IV. MAGNETIC FIELD OF A STATIONARY MAGNETIC MONOPOLE WHOSE CHARGE IS CHANGING WITH TIME

The net magnetic field due to a magnetic monopole when its velocity is zero, using equation (10a) and with $dt' = dt$, is

$$\boldsymbol{H}(r) = \frac{\boldsymbol{n}}{r^2}\left[q^m + \frac{r}{c}\frac{dq^m}{dt}\right] \tag{11a}$$

$$\boldsymbol{E} = 0 \tag{11b}$$

and

$$H_0 = \frac{1}{cr}\frac{dq^m}{dt} \tag{11c}$$

Thus it is possible that $\boldsymbol{H}$ vanishes at $r = r_0$, if

$$q^m + \frac{r_0}{c}\frac{dq^m}{dt} = 0 \tag{12}$$

which gives

$$r_0 = \frac{-cq^m}{dq^m/dt} \tag{13}$$

Thus, if r_0 to be positive, dq^m/dt should be negative, i.e. the magnetic charge should decrease with time.
For decaying magnetic charge on the monopole, equation (11) with respect of equation (13) gives

$$\boldsymbol{H}(r) = \frac{\boldsymbol{n}q^m}{r^2}\left[1 - \frac{r}{r_0}\right] \tag{14}$$

It suggests that, if the magnetic charge on the monopole is positive and decaying, then at $r = r_0$, $\boldsymbol{H}$ is zero, for $r < r_0$, $\boldsymbol{H}$ is positive, and at $r > r_0$, $\boldsymbol{H}$ is negative, where r_0 is given by equation (13).

V. IDEAL SQUARE WAVE MAGNETIC FIELD FROM A RISING AND DECAYING MAGNETIC MONOPOLE

Developing an ideal square wave magnetic field at any point in space from conserved magnetic monopoles is impossible. But if the charge on the magnetic monopole is not conserved then the property of vanishing field from a decaying magnetic charge and maximum saturation value from an exponentially rising charge makes it possible to construct an ideal square wave magnetic vector field.

Suppose that a charge q^m is initially at q_1^m and it rises to q_2^m in time T according to the following equation

$$q^m = q_0^m\left(1 - e^{-t/\tau}\right) + q_1^m \qquad \text{for } 0 \le t \le T \tag{15}$$

At $t = T$, it reaches a maximum value q_2^m given by

$$q_2^m = q_0^m\left(1 - e^{-T/\tau}\right) + q_1^m \tag{16}$$

The field produced in this time interval is

$$\boldsymbol{H} = \frac{\boldsymbol{n} q_0^m}{r^2}\left[1 + \left(\frac{r}{c\tau} - 1\right)e^{-t/\tau}\right] + \frac{\boldsymbol{n} q_1^m}{r^2} \tag{17}$$

Thus the magnetic field at $r = c\tau$ is independent on the instantaneous value of the magnetic charge on the monopole.

In the further time interval T to $2T$, suppose, the charge decays exponentially according to the equation

$$q^m = q_2^m e^{(T - t/\tau)} \qquad \text{for } T \le t \le 2T \tag{18}$$

The field produced in this time interval as computed from (11) is

$$\boldsymbol{H} = \frac{\boldsymbol{n} q_2^m e^{(T - t/\tau)}}{r^2}\left[1 - \frac{r}{c\tau}\right] \tag{19}$$

Thus the field at $r = c\tau$ is zero and is again independent on the instantaneous value of the magnetic charge on the monopole.

The requirement of $q^m(0) = q^m(2T)$ gives

$$q_1^m = q_2^m e^{-T/\tau} \tag{20}$$

Equations (19) and (23) give $q_2^m = q_0^m$.

We thus see that for the decaying interval the magnetic field at $r = c\tau$, by equation (19), is zero, while during the charging interval the field at $r = c\tau$, by equation (17), is

$$\boldsymbol{H} = \frac{\boldsymbol{n}\left(q_0^m + q_1^m\right)}{r^2} = \boldsymbol{H}_{const} \tag{21}$$

Thus periodically the field $\boldsymbol{H}$ attains only two states at $r = c\tau$ viz. 0 and $\boldsymbol{H}_{const}$, which is an ideal square wave magnetic field.

VI. DISCUSSION

Dirac-Maxwell's theory under the Lorentz gauge does not include a time-dependent term of the magnetic charge in the fields. To see the effect of the time-dependence of magnetic charges on their fields, Dirac-Maxwell's equations are made Lorentz gauge free. It shows the contribution of the rate of change of charge with time to the fields. Outcome of this proposition is that two magnetic charges of the same type can attract each other if they are decaying with time then $\frac{dq^m}{dt}$ is negative for both and are placed apart by the distance $r > \frac{cq^m}{dq^m/dt}$ (equation (11)), where q^m is the instantaneous value of both the charges, since beyond the distance $\frac{cq^m}{dq^m/dt}$ the field gets inverted. It is clear that if the magnetic charges are time-independent then the scalar field H_0 disappears and the generalized Dirac-Maxwell's equations reduce to their usual form.

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