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Stokes first problem for an unsteady MHD fourth-grade fluid in a non-porous half space with Hall currents

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Abstract : *The well-known problem of unidirectional plane flow of a fluid in a non-porous half-space due to the impulsive motion of the rigid plane wall it rests upon is discussed in the context of an unsteady MHD fourth-grade fluid in presence of Hall currents. The governing non-linear partial differential equations describing the problem are converted to a system of non-linear ordinary differential equations by using the similarity transformations. The complex analytical solution is found by using the homotopy analysis method (HAM). The existing literature on the topic shows that it is the first study regarding the effects of Hall current on flow of an unsteady MHD fourth-grade fluid over an impulsively moving plane wall. The convergence of the obtained complex series solutions is carefully analyzed. The effects of dimensionless parameters on the velocity are illustrated through plots and the effects of the pertinent parameters on the local skin friction coefficient at the surface of the wall are presented numerically in tabular form.*
Keywords: *Fourth-grade fluid, HAM, Hall currents, Stokes first problem, Unsteady.*

I. INTRODUCTION

Almost every student of fluid mechanics is familiar with Stokes' first problem [1], the flows over a plane wall which is initially at rest and is suddenly set into motion in its own plane with a constant velocity is termed as Stokes' first (or Rayleigh-type) problem [2-3]. Teipel [4] discussed impulsive motion of a flat plate in a viscoelastic fluid. Puri [5] investigated impulsive motion of a flat plate in a Rivlin-Ericksen fluid. Erdogan [6] analyzed plane surface suddenly set in motion in a non-Newtonian Fluid. Zeng and Weinbaum [7] investigated Stokes problems for moving half-planes. Tan and Xu [8-9] investigated Stokes' first problem not only for generalized second grade fluid but also for generalized Maxwell fluid. Erdogan [10] analyzed the unsteady unidirectional flows generated by impulsive motion of a boundary or sudden application of a pressure gradient. Fetecau and Fetecau [11] solved Stokes' first problem for ordinary Oldroyd-B fluid by sine transform. Stokes' first problem for Oldroyd-B and second grade fluid in a porous half space is studied by Tan and Masuoka [12-13]. Zierep and Fetecau [14] given energetic balance for the Rayleigh--Stokes problem of a second grade fluid. Zierep et al. [15] discussed Rayleigh--Stokes problem for non-Newtonian medium with memory. Vieru et al. [16] found exact solution corresponding to the first problem of Stokes for Oldroyd-B fluid. Stokes' first problem for the rotating flow of a third grade fluid is numerically solved by Shahzad et al. [17]. Hayat et al. [18] presented numerical solution of Stokes' first problem for a third grade fluid in a porous half space. Fakhari et al. [19] presented a note on the interplay between symmetries, reduction and conservation laws of Stokes' first problem for third-grade rotating fluids. Sajid et al. [20] discussed Stokes' first problem for a MHD third grade fluid in a porous half space. The theoretical study of the Effects of Hall current on flow of non-Newtonian fluids has been a subject of great interest to researchers because of its various applications in power generators and pumps, Hall accelerators, refrigeration coils, electric transformers, in flight MHD, solar physics involved in the sunspot development, the solar cycle, the structure of magnetic stars, electronic system cooling, cool combustors, fiber and granular insulation, oil extraction, thermal energy storage and flow through filtering devices and porous material regenerative heat exchangers. In presence of a strong magnetic field in an ionized gas of low density, the conductivity normal to the magnetic field is decreased by free spiraling of electrons and ions about the magnetic lines of force before suffering collisions. The phenomenon, well known in the literature, is called the Hall effect and a current induced in a direction normal to the electric and magnetic fields is called Hall current [21]. The Hall term representing the Hall current was ignored most of the time while applying Ohm's law, because it has no markable effect for small and moderate values of the magnetic field. The effects of Hall current are very important if the strong magnetic field is applied [22], because for strong magnetic field electromagnetic force is noticeable. The recent investigation for the applications of MHD is towards a strong magnetic field, due to which study of Hall current is important [23-30].

The present investigation is to analyze the Stokes first problem for an unsteady MHD fourth-grade fluid in a non-porous half space with Hall currents. The arising non-linear problem is solved by the homotopy analysis method (HAM) [31-34]. The method gives complex analytic solution which is uniformly valid for all values of the dimensionless time. The convergence region for the complex series solution is found with the help of $\hbar - curve$. The effect of the material parameters of the fourth-grade fluid, Hall parameter, Hartmann number and homotopy parameter on the velocity and its time series are investigated for the impulsive motion of the wall.

II. FORMULATION OF THE PROBLEM AND ITS ANALYTIC SOLUTION

We consider the unsteady flow of an electrically conducting incompressible fourth-grade fluid past a rigid plane wall coinciding with the plane $y=0$. The fluid over the plane wall is initially at rest and it sets in motion due to the sudden jerked of the wall subjected to a uniform transverse magnetic field. It is assumed that the flow takes place in the upper half plane $y>0$ with the wall on the $x-axis$. The Cauchy stress tensor $\mathbf{T}$ for a fourth-grade fluid is given as [35]

$$\begin{aligned}\mathbf{T} = & -p\mathbf{I} + \mu\mathbf{A}_1 + \alpha_1\mathbf{A}_2 + \alpha_2\mathbf{A}_1^2 + \beta_1\mathbf{A}_3 + \beta_2(\mathbf{A}_1\mathbf{A}_2 + \mathbf{A}_2\mathbf{A}_1) \\ & + \beta_3(tr\mathbf{A}_1^2)\mathbf{A}_1 + \gamma_1\mathbf{A}_4 + \gamma_2(\mathbf{A}_3\mathbf{A}_1 + \mathbf{A}_1\mathbf{A}_3) + \gamma_3\mathbf{A}_2^2 \\ & + \gamma_4(\mathbf{A}_2\mathbf{A}_1^2 + \mathbf{A}_1^2\mathbf{A}_2) + \gamma_5(tr\mathbf{A}_2)\mathbf{A}_2 \\ & + \gamma_6(tr\mathbf{A}_2)\mathbf{A}_1^2 + [\gamma_7(tr\mathbf{A}_3) + \gamma_8(tr\mathbf{A}_2\mathbf{A}_1)]\mathbf{A}_1, \end{aligned} \quad (1)$$

where p is the scalar pressure, $\mathbf{I}$ is the identity tensor, μ is the coefficient of viscosity, α_i, β_j, γ_k $((i=1,2),(j=1,2,3) \text{ and } (k=1,2,...,8))$ are the material parameters of fourth-grade fluid, and $\mathbf{A}_i$ $(i=1,2,3,4)$ are the first four Rivlin-Ericksen tensors defined by [35]

$$\mathbf{A}_1 = (\text{grad } \mathbf{V}) + (\text{grad } \mathbf{V})^T, \quad (2)$$

$$\mathbf{A}_{n+1} = \frac{d\mathbf{A}_n}{dt} + \mathbf{A}_n(\text{grad } \mathbf{V}) + (\text{grad } \mathbf{V})^T \mathbf{A}_n, \; n=1,2,3. \quad (3)$$

The equations governing the magnetohydrodynamic flow with Hall effect are:

$$\begin{aligned}\frac{\partial u}{\partial t} = & \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2(1+i\in)}{\rho(1+\in^2)}u + \frac{\alpha_1}{\rho}\frac{\partial^3 u}{\partial y^2 \partial t} + \frac{\beta_1}{\rho}\frac{\partial^4 u}{\partial y^2 \partial t^2} + \frac{6(\beta_2+\beta_3)}{\rho}\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^2 u}{\partial y^2} \\ & + \frac{\gamma_1}{\rho}\frac{\partial^5 u}{\partial y^2 \partial t^3} + \frac{2(3\gamma_2+\gamma_3+\gamma_4+\gamma_5+3\gamma_7+\gamma_8)}{\rho}\left[2\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}\frac{\partial^2 u}{\partial y \partial t} + \left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^3 u}{\partial y^2 \partial t}\right], \end{aligned} \quad (4)$$

The boundary and initial conditions are

$$\begin{aligned} & u(y,t) = u_w = U, \quad \text{at } y=0, \text{ for } t>0, \\ & u(y,t) \to 0 \quad \text{as } \; y \to \infty, \\ & u(y,t) = 0, \text{ at } t=0, \text{ for } y>0, \end{aligned} \quad (5)$$

where $u(y,t)$ is the velocity component in the $x-direction$, t is time, ν is the kinematic viscosity, ρ is the fluid density, σ is the electrical conductivity of the fluid, $\mathbf{B}_0$ is the applied magnetic field, $\in (= w_e\tau_e)$ is the Hall parameter, w_e and τ_e are the cyclotron frequency and collision time of the electrons respectively, and U is the velocity of the wall. In order to non-dimensionalize the problem let us introduce the similarity transformations

$$u = Uf(\eta,\xi), \; \eta = \frac{U}{\nu}y, \; \xi = \frac{U^2}{\nu}t, \quad (6)$$

where $f(\eta,\xi)$ is the dimensionless velocity function, η is the dimensionless distance from the wall and ξ is the dimensionless time. Equations (4) and (5) become

$$\begin{aligned} & f'' + \alpha\frac{\partial f''}{\partial \xi} + \beta\frac{\partial^2 f''}{\partial \xi^2} + \zeta f'^2 f'' + \Gamma_1\frac{\partial^3 f''}{\partial \xi^3} + 4\Gamma_2 f'f''\frac{\partial f'}{\partial \xi} \\ & + 2\Gamma_2 f'^2\frac{\partial f''}{\partial \xi} - \frac{N(1+i\in)}{(1+\in^2)}f - \frac{\partial f}{\partial \xi} = 0, \end{aligned} \quad (7)$$

$$f(0,\xi)=1,\ f(\infty,\xi)=0, \text{ for } \xi>0,$$
$$f(\eta,0)=0,\ \text{for } \eta>0 \qquad (8)$$

where prime denotes differentiation with respect to η, $\alpha\left(=\alpha_1 U^2/\rho\nu^2\right)$ is dimensionless second-grade parameter, $\beta\left(=\beta_1 U^4/\rho\nu^3\right)$, $\zeta\left(=6(\beta_2+\beta_3)U^4/\rho\nu^3\right)$ are dimensionless third-grade parameters, $\Gamma_1\left(=\gamma_1 U^6/\rho\nu^4\right)$, $\Gamma_2\left(=(3\gamma_2+\gamma_3+\gamma_4+\gamma_5+3\gamma_7+\gamma_8)U^6/\rho\nu^4\right)$ are dimensionless fourth-grade parameters and $N\left(=\sigma B_0^2\nu/\rho U^2\right)$ is the dimensionless modified Hartmann number [36]. The local skin friction coefficient or fractional drag coefficient on the surface of the moving wall is

$$C_f=\frac{2\tau_{xy}\big|_{y=0}}{\rho u_w^2}, \qquad (9)$$

Now using equations (1), (2), (3) and (6) the equation (9) can be written in dimensionless variables as

$$\frac{1}{2}\times C_f=f'(0,\xi)+\alpha\frac{\partial}{\partial\xi}f'(0,\xi)+\beta\frac{\partial^2 f'(0,\xi)}{\partial\xi^2}$$
$$+\frac{1}{3}\zeta\left(f'(0,\xi)\right)^3+\Gamma_1\frac{\partial^3 f'(0,\xi)}{\partial\xi^3}+2\Gamma_2\left(f'(0,\xi)\right)^2\frac{\partial f'(0,\xi)}{\partial\xi}, \qquad (10)$$

The boundary conditions (8) lead us to take base functions for the velocity $f(\eta,\xi)$ as

$$\left\{\eta^i\xi^j e^{-n\eta}\ /\ n\geq 0, i\geq 0,\ j\geq 0\right\} \qquad (11)$$

The velocity $f(\eta,\xi)$ can be expressed in terms of base functions as

$$f(\eta,\xi)=\sum_{n=0}^{\infty}\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}A_{n,i}^{j}\eta^i\xi^j e^{-n\eta}, \qquad (12)$$

To start with the homotopy analysis method, due to the boundary conditions (8) it is reasonable to choose the initial guess approximation

$$f_0(\eta,\xi)=(1-\xi\eta)e^{-\eta}, \qquad (13)$$

and the auxiliary linear operator

$$\mathbf{L}(f)=\frac{\partial^2 f(\eta,\xi)}{\partial\eta^2}-f(\eta,\xi), \qquad (14)$$

with the property:

$$\mathbf{L}\left[a_1e^{-\eta}+a_2e^{\eta}\right]=0, \qquad (15)$$

where a_1 and a_2 are arbitrary constants. Following the HAM and trying higher iterations with the unique and proper assignment of the results converge to the exact solution:

$$f(\eta,\xi)\approx f_0(\eta,\xi)+f_1(\eta,\xi)+f_2(\eta,\xi)+\ldots+f_m(\eta,\xi), \qquad (16)$$

At $\in=0$, $N=0.1$, $\alpha=0.1$, $\beta=0.1$, $\zeta=0.1$, $\Gamma_1=0.1$, $\Gamma_2=0.1$, $\hbar=-0.5$, using the symbolic computation software such as MATLAB, MAPLE, MATHEMATICA to successively obtain

$$f_1(\eta,\xi)=-\frac{9}{320}e^{-3\eta}+\frac{9e^{-\eta}}{320}+\frac{3\eta e^{-3\eta}}{80}+\frac{31\eta e^{-\eta}}{80}+\frac{9\eta^2e^{-\eta}}{80}$$
$$-\frac{33}{640}e^{-3\eta}\xi+\frac{33}{640}e^{-\eta}\xi+\frac{17}{160}e^{-3\eta}\eta\xi+\frac{31}{80}e^{-\eta}\eta\xi-\frac{3}{40}e^{-3\eta}\eta^2\xi$$
$$-\frac{9}{80}e^{-\eta}\eta^2\xi-\frac{59}{2560}e^{-3\eta}\xi^2+\frac{59}{2560}e^{-\eta}\xi^2+\frac{53}{640}e^{-3\eta}\eta\xi^2$$
$$-\frac{27}{320}e^{-3\eta}\eta^2\xi^2+\frac{3}{80}e^{-3\eta}\eta^3\xi^2-\frac{11}{5120}e^{-3\eta}\xi^3+\frac{11}{5120}e^{-\eta}\xi^3$$

$$+\frac{13}{1280}e^{-3\eta}\eta\xi^3-\frac{7}{640}e^{-3\eta}\eta^2\xi^3+\frac{1}{160}e^{-3\eta}\eta^3\xi^3+e^{-\eta}(1-\eta\xi), \quad (17)$$

similarly $f_2(\eta,\xi)$, $f_3(\eta,\xi)$, $f_4(\eta,\xi)$ and so on are calculated. The obtained values of $f_0(\eta,\xi)$, $f_1(\eta,\xi)$, $f_2(\eta,\xi)$, - - - -leads us to write

$$f_m(\eta,\xi)=\sum_{n=0}^{2m+1}\sum_{i=0}^{2m+2-n}\sum_{j=0}^{2m+1}A_{m,n}^{i,j}\eta^i\xi^j e^{-n\eta}. \quad (18)$$

By avoiding the detailed recurrence relations which we calculated but not presenting here due to space, the total complex analytic solution in compact form is

$$f(\eta,\xi)=\sum_{m=0}^{\infty}f_m(\eta,\xi)=\lim_{L\to\infty}\sum_{m=0}^{L}A_{m,0}^{0,0}$$
$$+\lim_{L\to\infty}\sum_{n=1}^{L+1}e^{-n\eta}\left(\sum_{m=n-1}^{L}\sum_{i=0}^{2m+2-n}\sum_{j=0}^{m}A_{m,n}^{i,j}\eta^i\xi^j\right), \quad (19)$$

where from initial guess in equation (13) we obtain

$$A_{0,1}^{0,0}=1,\ A_{0,1}^{1,1}=-1 \text{ and all } A_{0,n}^{i,j}=0, \text{ for } (i=0,1,2), (j,n=0,1). \quad (20)$$

We know that the auxiliary parameter $\hbar$ gives the convergence region and rate of approximation for the homotopy analysis method. From $\hbar-curve$ in Fig. 1 we note that the range for the admissible value for $\hbar$ is $-0.8<\hbar<0$. Our calculations depict that the series of the dimensionless velocity in equation (19) converges in the whole region of η and ξ for $\hbar=-0.5$.

III. GRAPHS, TABLE AND DISCUSSION

The discussion of emerging parameters on the dimensionless velocity $f(\eta,\xi)$ is as follows:
Figs. 2 to 9 are plotted in absence of Hall currents and in Fig. 10 Hall current is taken into account. Fig. 2 displays the velocity f for various values of η. This figure describes that as we move away from the moving wall the velocity decreases and also the fluid near the wall moves together with the wall with the same velocity as of the wall. Fig. 3 presents the velocity profile f for various values of ξ. This figure shows that with the passage of time the velocity of the fluid decreases as we go in the increasing direction of η. Fig. 4 elucidates the variation of Hartmann number on the velocity. It is found that the velocity and boundary layer thickness decreases with an increase in N. When magnetic field is applied transverse to the fluid velocity then it gives rise to a drag-like or resistive force which slow down or suppress the motion of the fluid on the wall. This leads to a reduction in the velocity of the fluid as seen in Fig. 4. This means that the magnetic force provides a mechanism to the control of boundary layer thickness. Fig. 5 illustrates the influence of second-grade parameter on the velocity profile f. It is evident from the figure that an increase in α results in the increase of the velocity, here boundary layer thickness increases and shear thickening is observed. In Figs. 6 and 7 the velocity distribution is presented for the various values of third-grade parameters β and ζ. It is observed that the velocity decreases by increasing the influence of β and ζ. Figs. 8 and 9 show that the velocity f decreases for fourth-grade parameters Γ_1 and Γ_2. With the inclusion of Hall term velocity field becomes complex, so we plot absolute value of the velocity profile f in Fig. 10. We observe that with increase in Hall parameter $\in$ absolute value of the velocity and boundary layer thickness increases.

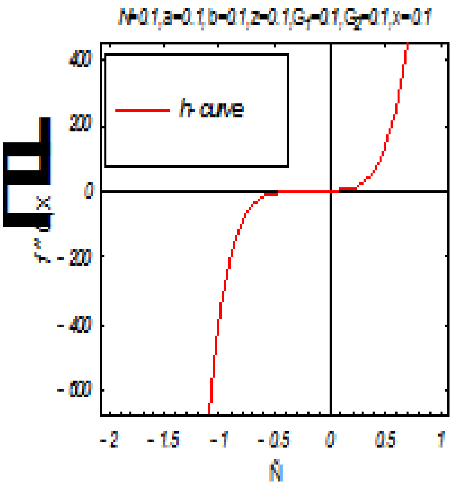

Fig. 1 $\hbar - curve$ for $f(\eta,\xi)$.

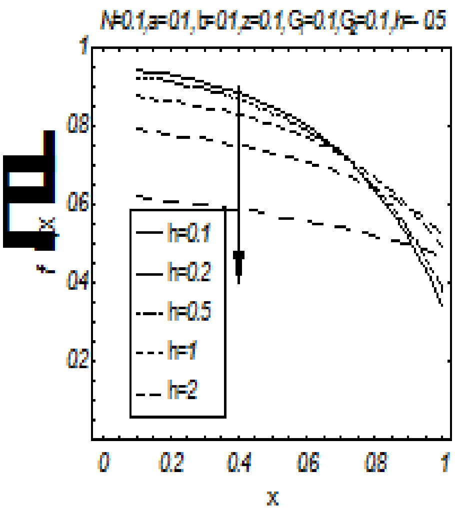

Fig. 2 Influence of η on $f(\eta,\xi)$.

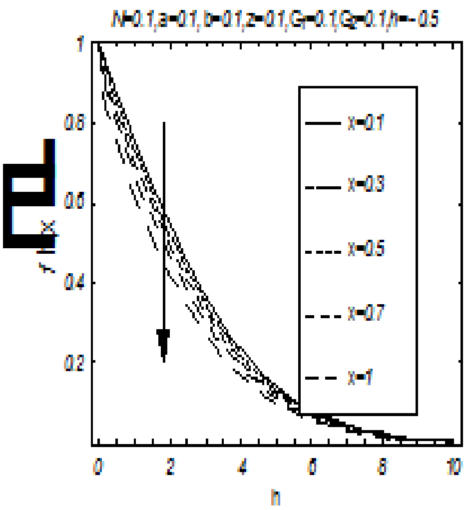

Fig. 3 Influence of ξ on $f(\eta,\xi)$.

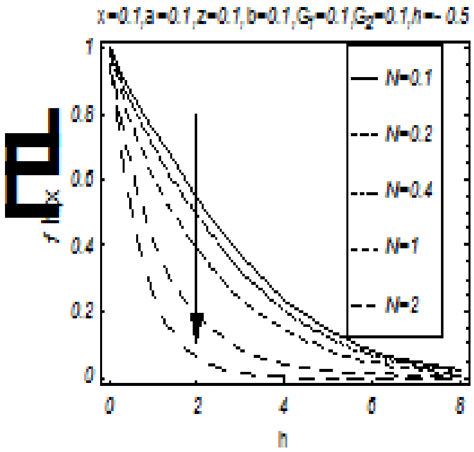

Fig. 4 Influence of N on $f(\eta,\xi)$.

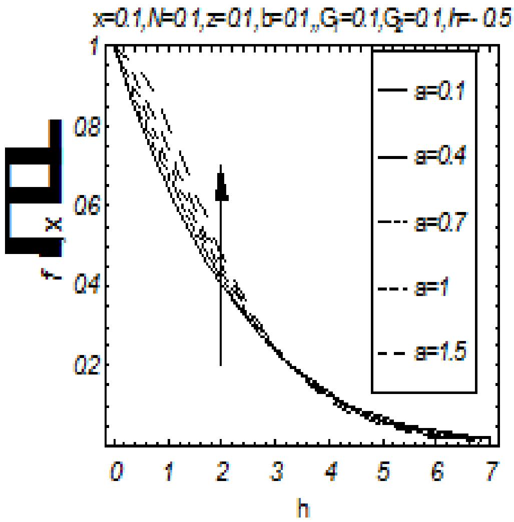

Fig. 5 Influence of α on $f(\eta,\xi)$.

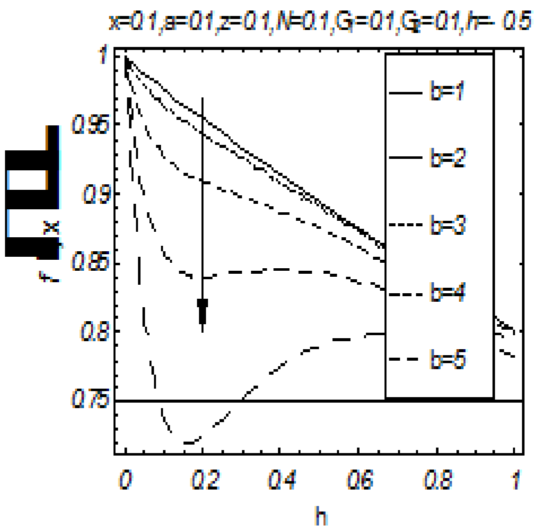

Fig. 6 Influence of β on $f(\eta,\xi)$.

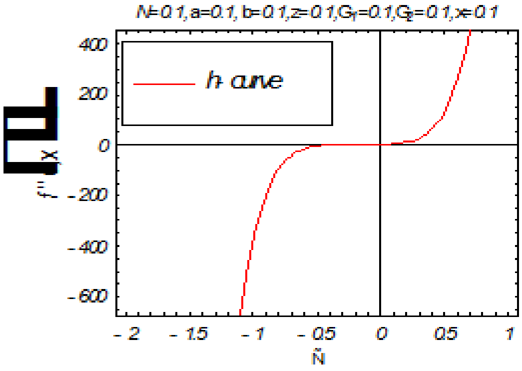

Fig. 1 $\hbar-curve$ for $f(\eta,\xi)$.

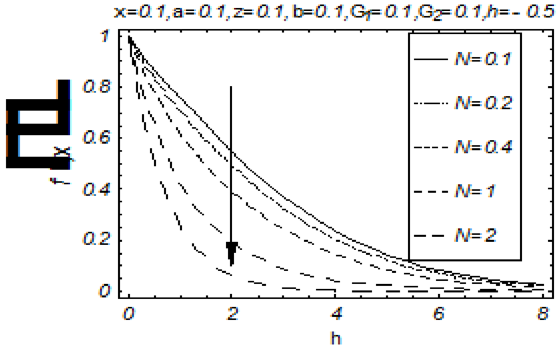

Fig. 4 Influence of N on $f(\eta,\xi)$.

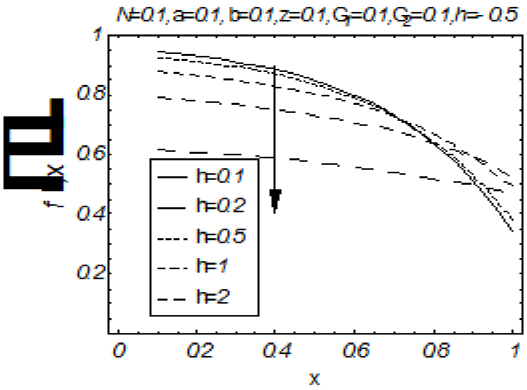

Fig. 2 Influence of η on $f(\eta,\xi)$.

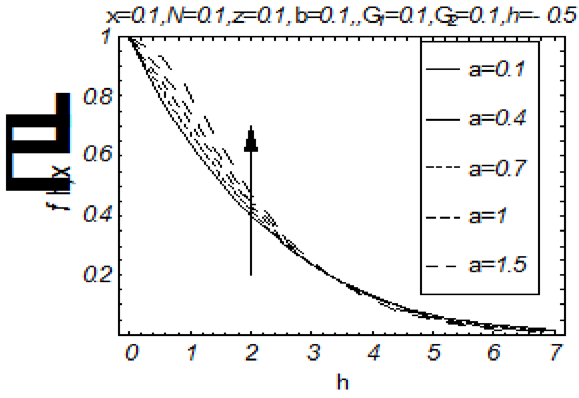

Fig. 5 Influence of α on $f(\eta,\xi)$.

Table shows the influence of the Hartmann number N , Hall parameter $\in$ and fourth-grade parameters Γ_1, Γ_2 on local skin friction coefficient C_f given by equation (10). It is observed from table that with increase in Hartmann number N absolute value of the skin friction coefficient C_f increases and with increase in Hall current $\in$ absolute value of the skin friction coefficient decreases. When magnetic field is applied normal to the fluid velocity then it gives rise to a drag-like or resistive force which slow down or suppress the motion of the fluid on the moving surface. This leads to a reduction in the velocity of the fluid and flow rates. With the increase in the strength of the magnetic field the motion of the particulate suspension on the surface reduces due to which shear stress at the wall reduces with increase in Hall current $\in$, as observed in the table. Table illustrates that increase in the fourth-grade material parameters Γ_1 and Γ_2 give an increase in the value of the shear stress at the moving wall.

Table. Absolute Values of the skin friction coefficient C_f with $\alpha=0.1$, $\beta=0.1$, $\zeta=0.1$, $\xi=0.1$, $\hbar=-0.5$.

Γ_1	Γ_2	$\in=0.1$ $N=0.1$	$\in=0.1$ $N=0.3$	$\in=0.1$ $N=0.5$	$N=0.1$ $\in=0.2$	$N=0.1$ $\in=0.3$	$N=0.1$ $\in=0.5$
0.1	0.1	1.31797	1.50744	1.69454	1.31532	1.31123	1.30026
0.3	0.1	1.53032	1.72276	1.9128	1.52762	1.52345	1.51226
0.5	0.1	1.74271	1.93812	2.13112	1.73996	1.73571	1.72432
0.1	0.2	2.16588	2.39382	2.62091	2.16267	2.15771	2.14446
0.1	0.3	4.07518	4.42794	4.78267	4.07021	4.06253	4.04201
0.1	0.5	20.1533	21.4182	22.6789	20.1354	20.1076	20.0334

IV. CONCLUSION

The Stokes' first problem of an unsteady MHD fourth-grade fluid in a non-porous half space is discussed by taking Hall currents in to account. Analytical solution for the non-linear problem is given and convergence of the solution is appropriately discussed. The non-linear effects on the velocity is shown and discussed. This reveal that plots for shear thickening/shear thinning behavior of a fluid are dependent upon the rheological properties of the fluid. The numerical results for Hall parameter $\in$, Hartmann number N and fourth-grade material parameters Γ_1, Γ_2 reveal that Hall parameter, Hartmann number and fourth-grade material parameters have significant influence on the local skin-friction coefficient on the surface of the impulsively moving wall.

REFERENCES

[1] G. G. Stokes, On the effect of the internal friction of fluids on the motion of pendulums, *Trans. Cambridge Phil. Soc, (Part II), 9(2)*, 1851, 8-106.

[2] R. I. Tanner, Note on the Rayleigh problem for a visco-elastic fluid, *Z. angew. Math. Phys, (ZAMP), 13*, 1962, 573-580.

[3] V.M. Soundalgekar, Stokes problem for elastico-viscous fluid, *Rheol. Acta, 13(2)*, 1974, 177-179.

[4] I. Teipel, The impulsive motion of a flat plate in a viscoelastic fluid, *Acta Mech, 39(3-4)*, 1981, 277-279.

[5] P. Puri, Impulsive motion of a flat plate in a Rivlin--Ericksen fluid, *Rheol. Acta, 23(4)*, 1984, 451-453.

[6] M.E. Erdogan, Plane surface suddenly set in motion in a non-Newtonian Fluid, *Acta Mech, 108(1-4)*, 1995, 179-187.

[7] Y. Zeng and S. Weinbaum, Stokes problems for moving half-planes, *J. Fluid Mech, 287*, 1995, 59-74.

[8] W. C. Tan and M. Y. Xu, The impulsive motion of flat plate in a generalized second grade fluid, *Mech. Res. Comm, 29*, 2002, 3-9.

[9] W. C. Tan and M. Y. Xu, Plane surface suddenly set in motion in a viscoelastic fluid with fractional Maxwell model, *Acta Mech. Sin, 18(4)*, 2002, 342-349.

[10] M. E. Erdogan, On the unsteady unidirectional flows generated by impulsive motion of a boundary or sudden application of a pressure gradient, *Int. J. Non-Linear Mech, 37(6)*, 2002, 1091-1106.

[11] C. Fetecau and C. Fetecau, The first problem of Stokes for an Oldroyd-B fluid, *Int. J. Non-Linear Mech, 38(10)*, 2003, 1539-1544.

[12] W. C. Tan and T. Masuoka, Stokes' first problem for an Oldroyd-B fluid in a porous half space, *Phys. Fluids, 17*, 2005, 023101-023107.

[13] W. C. Tan and T. Masuoka, Stokes' first problem for a second grade fluid in a porous half-space with heated boundary, *Int. J. Non-Linear Mech, 40*, 2005, 515-522.

[14] J. Zierep and C. Fetecau, Energetic balance for the Rayleigh--Stokes problem of a second grade fluid, *Int. J. Engng Sci, 45*, 2007, 155-162.

[15] J. Zierep, R. Bohning and C. Fetecau, Rayleigh--Stokes problem for non-Newtonian medium with memory, *Z. Angew. Math. Mech. (ZAMM), 87*, 2007, 462-467.

[16] D. Vieru, M. Nazar, C. Fetecau and C. Fetecau, New exact solutions corresponding to the first problem of Stokes for Oldroyd-B fluids, *Comput. Math. Appl, 55*, 2008, 1644-1652.

[17] F. Shahzad, T. Hayat and M. Ayub, Stokes' first problem for the rotating flow of a third grade fluid, *Nonlinear Analysis: Real World Applications, 9(4)*, 2008, 1794-1799.

[18] T. Hayat, F. Shahzad, M. Ayub and S. Asghar, Stokes' first problem for a third grade fluid in a porous half space, *Comm. Non-linear Sci. Numer. Simm, 13(9)*, 2008, 1801-1807.

[19] K. Fakhari, A. A. Zainal and A. H. Kara, A note on the interplay between symmetries, reduction and conservation laws of Stokes' first problem for third-grade rotating fluids, *Pramana J. Phys, 77(3)*, 2011, 439-445.

[20] M. Sajid, N. Ali, T. Javed and Z. Abbas, Stokes' first problem for a MHD third grade fluid in a porous half space, *J. Porous Media, 1(3)*, 2010, 279-284.

[21] H. Sato, The Hall effects in the viscous flow of ionized gas between parallel plates under transverse magnetic field, *J. Phys. Soc. Japan, 16(7)*, 1961, 1427-1433.

[22] K. Cramer, S. Pai, *Magnetofluid Dynamics for Engineers and Applied Physicists*, (McGraw - Hill, New York, 1973).

[23] M. Ayub, H. Zaman and M. Ahmad, Series solution of hydromagnetic flow and heat transfer with Hall effect in a second grade fluid over a stretching sheet, *Cent. Eur. J. Phys, 8(1)*, 2010, 135-149.

[24] M. Ahmad, H. Zaman and N. Rehman, Effects of Hall current on unsteady MHD flows of a second grade fluid, *Cent. Eur. J. Phys*, 8(3), 2010, 422-431.

[25] H. Zaman, Hall effects on the unsteady incompressible MHD fluid flow with slip conditions and porous walls, *Applied Mathematics and Physics, 1(2)*, 2013, 31-38.

[26] T. Hayat, H. Zaman and M. Ayub, Analytic solution of hydromagnetic flow with Hall effect over a surface stretching with a power law velocity, *Numerical Methods for Partial Differential Equations, 27(4)*, 2010, 937-959.

[27] I. Pop and V. M. Soundalgekar, Effects of Hall current on hydromagnetic flow near a porous plate, *Acta Mech, 20(3-4)*, 1974, 315-318.

[28] A. S. Gupta, Hydromagnetic flow past a porous flate plate with Hall effects, *Acta Mech, 22(3-4)*, 1975, 281-287.

[29] L. Debnath, S. C. Ray and A. K. Chatterjee, Effects of Hall current on unsteady hydromagnetic flow past a porous plate in a rotating fluid system, *Z. Angew. Math. Mech, 59(9)*, 1979, 469-471.

[30] M. Katagiri, The effect of Hall currents on the magnetohydrodynamic boundary layer flow past a semi-infinite flate plate, *J. Phys. Soc. Japan, 27(4)*, 1969, 1051-1059.

[31] S. J. Liao, *Beyond Perturbation: Introduction to Homotopy Analysis Method*, Chapman and Hall, CRC Press, Florida 2003.
[32] M. Ayub, H. Zaman, M. Sajid and T. Hayat, Analytical solution of stagnation-point flow of a viscoelastic fluid towards a stretching surface, *Comm. Nonlinear Sci. and Numer. Simul, 13(9),* 2008, 1822-1835.
[33] H. Zaman and M. Ayub, Series solution of unsteady free convection flow with mass transfer along an accelerated vertical porous plate with suction, *Cent. Eur. J. Phys, 8(6),* 2010, 931-939.
[34] H. Zaman, T. Hayat, M. Ayub and R. S. R. Gorla, Series solution for heat transfer from a continuous surface in a parallel free stream of viscoelastic fluid, *Numerical Methods for Partial Differential Equations, 27(6)*, 2011, 1511-1524.
[35] R. S. Rivlin, J. L. Ericksen, Stress deformation relations for isotropic materials, *J. Rational Mech. Anal. 4(11)*, 1955, 323-425.
[36] G. W. Sutton, *A. Sherman, Engineering Magnetohydrodynamics*, McGraw-Hill, New York, 1965.