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Newtons Gravitational Field Equations for a Static Homogeneous Spherical Distribution of Mass in Rotational Spherical Polar Coordinates

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Abstract: *In this paper we formulate and solve Newton's gravitational Field equations for a static homogeneous spherical distribution of mass in rotational spherical polar coordinates to pave the way for applications such as planetary theory in rotational spherical polar coordinates.*
Key Words: *Newton's Gravitational Field Equations, Static Homogeneous Spherical distribution of mass, Rotational Spherical Polar Coordinates*

Theory

It is well established that the Newton's gravitational Field equations for the gravitational scalar potential f due to a distribution of mass density ρ is given by

$$\nabla^2(x^\mu) = 4\pi G\rho_0(x^\mu)^{[1]} \qquad (1)$$

where ∇^2 is the Euclidean Laplacian given by$^{[2]}$

$$\nabla^2 = \frac{1}{h_1h_2h_3}\left\{\frac{\partial}{\partial u}\left[\frac{h_2h_3}{h_1}\frac{\partial}{\partial u}\right] + \frac{\partial}{\partial v}\left[\frac{h_1h_3}{h_2}\frac{\partial}{\partial v}\right] + \frac{\partial}{\partial w}\left[\frac{h_1h_2}{h_3}\frac{\partial}{\partial w}\right]\right\} \qquad (2)$$

$h_1h_2h_3$ are scale factors

It follows immediately that

$$\nabla^2 = \frac{1}{\left[\frac{u^2}{1-v^2}\right]^{\frac{1}{2}}\left[\frac{u(1-v^2)}{1-w^2}\right]^{\frac{1}{2}}}$$

$$\left\{\left[\frac{\partial}{\partial u}\left[\frac{u^2}{1-v^2}\right]^{\frac{1}{2}}\left[\frac{u(1-v^2}{1-w^2}\right]^{\frac{1}{2}}\frac{\partial}{\partial u}\right] + \frac{\partial}{\partial v}\left[\left[\frac{u(1-v^2}{1-w^2}\right]^{\frac{1}{2}}\frac{\partial}{\partial v}\right] + \frac{\partial}{\partial w}\left[\frac{u^2}{(1-v^2)^{\frac{1}{2}}}\left[\frac{1-w^2}{u(1-v^2}\right]^{\frac{1}{2}}\frac{\partial}{\partial w}\right]\right\} \qquad (3)$$

This reduces to

$$\nabla^2 = \frac{1}{u^2}\frac{\partial}{\partial u}(u^2) + \frac{1}{u}\frac{\partial}{\partial v}\left((1-v^2)\frac{\partial}{\partial v}\right) + \frac{1}{u(1-v^2)}\frac{\partial}{\partial w}\left[(1-w^2)\frac{\partial}{\partial w}\right] \qquad (4)$$

I. Research Elaborations

For a Static Homogeneous Spherical distribution of mass in Rotational spherical polar coordinates we consider two conditions following from equation (1)

$$\frac{1}{u^2}\frac{d}{du}\left(u^2\frac{d}{du}\right)f(u) = 4\pi G\rho_0 \;; u < R \qquad (5)$$

$$\frac{1}{u^2}\frac{d}{du}\left(u^2\frac{d}{du}\right)f(u) = 0; u > R \qquad (6)$$

From equation (6)

$$\frac{d}{du}\left(u^2\frac{d}{du}f\right) = 0$$

$$u^2\frac{d}{du}f = A$$

$$\frac{df}{du} = \frac{A}{u^2}$$

$$\Rightarrow f^+ = -\frac{A}{U} + B \qquad (7)$$

From equation (5)

$$\frac{1}{u^2}\frac{d}{du}\left(u^2\frac{d}{du}\right)f(u) = 4\pi G\rho_0$$

$$\frac{d}{du}\left(u^2\frac{d}{du}\right)f(u) = \rho_0 G\rho_0 u^2$$

$$u^2\frac{d}{du}f(u) = \frac{4}{3}\pi G\rho_0 u^3 + A$$

$f(u) = \frac{4}{3}\pi G\rho_0 \int u du$

$f^-{}_p(u) = \frac{2}{3}\pi G\rho_0 u^2$+B (8)

Solving for Complimentary $f^-{}_c$

$\nabla^2 f^-{}_c = o$

$\frac{d}{du}\left(u^2 \frac{d}{du} f^-{}_c\right) = 0$

Setting,$f^-{}_c = DU^m$ (9)

It follows that $m = -1\ or\ 0$

$\Rightarrow f^-{}_c = D$

$f^- = f^-{}_p + f^-{}_c$

$= D + \frac{2}{3}\pi G\rho_0 u^2$

Knowing that $f^+ = \frac{A}{U}; U > R$ and also that $\rho_0 = \frac{3M_o}{4\pi R^3}$

Therefore

$f^- = D + \frac{GM_o u^2}{2R^3}; U < R$ (10)

$f^+ = \frac{A}{U}; U > R$ (11)

Now applying the conditions $f^- = f^+; U = R$ and $\frac{\partial f^-}{\partial u} = \frac{\partial f^+}{\partial u}; U = R$ we have

$\frac{\partial}{\partial u} = \left[D + \frac{GM_o U^2}{2R^3}\right] = \frac{\partial}{\partial u}\left[\frac{A}{U} + B\right]$

$\Rightarrow A = -GM_O$

From $f^- = f^+$ we get

$D + \frac{GM_o U^2}{2R^3} = \frac{A}{U}$

$\therefore D = \frac{GM_O}{2R}$ (12)

The explicit Newton's gravitational Scalar potential field for a static homogeneous spherical distribution of mass is hence given by:

$f^+ = \frac{GM_0}{U}$ (13)

$f^- = \frac{GM_0}{2R}\left[1 + \frac{U^2}{R^2}\right]$ (14)

II. Conclusion

The Scalar potential field so derived can now be applied to derive the Riemannian gravitational intensity for the interior and exterior fields and hence decompose the Riemann's geodesic equation[3] into the Riemann's acceleration part and a pure gravitational intensity or acceleration due to gravity . The scalar potential field plays a fundamental role in establishing a proof for the vanishing of the Riemannian curvature scalar in the Newton's gravitational field exterior to a static homogeneous spherical distribution of mass which has far reaching consequences in the theories of gravitation.

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