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Vacuum Solutions of Five Dimensional Plane Symmetric Metric in $f(R)$ Theory of Gravity

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Abstract: *The present paper is dealt with the study of the exact non vacuum perfect fluid solutions of five dimensional Binachi type I space time in the metric version of $\boldsymbol{f(R)}$ gravity considering stiff matter to obtain energy density and pressure of the universe on the lines of M. Sharif and M. Farasat Shamir (2010). In particular, we obtain two exact solutions which corresponds to two models of the universe in higher five dimensional Binachi type I space time. The function $\boldsymbol{f(R)}$ are also evaluated for both the models. Finally the physical properties of these models have been discussed.*
Keywords: *Higher order theory of gravitation , $\boldsymbol{f(R)}$ theory of gravity, Five dimensional Binachi type I Space time.*

I. Introduction

The $f(R)$ theory of gravity provides a very natural unification of the early time inflation and late time acceleration as proved by Nojiri and Odintsov (2007,2008). Sharif and Shamir (2010) have been obtained non vacuum solutions in Binachi type-I and type-V using perfect fluid in $f(R)$ gravity. Binachi type-III space time with anisotropic fluid in $f(R)$ gravity have been studied by Sharif and Kausar (2011).Aktas et al. (2012) have studied anisotropic models in $f(R)$ theory of gravity. Adhav (2012) discussed the KS string cosmological model in $f(R)$ gravity. Singh et al. (2013) studied functional from of $f(R)$ with power law expansion in anisotropic model. Recently, Reddy et al. (2014) studied vacuum solution of Binachi type-I and type-V models in $f(R)$ gravity with a special form of declaration parameter.

Higher dimensional cosmological models play a vital role in many aspects of early stage of cosmological problems. The study of higher dimensional space time provides an idea that our universe is much smaller at early stage of evolution as observed today. There is nothing in the equation of relativity which restricts them to four dimensions. Many researchers inspired to enter into the field of higher dimension theory to explore the knowledge of the universe. Wesson (1983,1984) andD R K Reddy (1999) have studied several aspects of five dimensional space time in variable mass theory and biometric theory of relativity. Lorentz and Petzold (1985), Ibanez and Verdaguer (1986), Reddy and Venkateswara (2001), Khadekar and Gaikwad (2001). Adhav et al. (2007) alternative theories of gravitation. Jaiswal et al. (2012) have studied the exact vacuum solutions of five dimensional Binachi type-I space time in $f(R)$ theory of gravity.

Here the general solution of the field equations in Binachi type-I space time Kasner form with five dimensions have been obtained using special form of deceleration parameter. The physical aspects of the model are also discussed.

II. Theoty of Gravity and Deceleration Parameter:

The $f(R)$ theory of gravity is nothing but the modifification of general theory of relativity proposed by Einstein. Therefore this theory is the generalization of Einstein's general theory of relativity. In the $f(R)$ theory of gravity there are two approaches to obtain the solutions of modified Einstein's field equations. The first approach is known as metric approach and second one is called Palatini formalism. The $f(R)$ gravity theory is modified by replacing R with $f(R)$ in the standard Einstein's Hilbert action and $f(R)$ is a general function of the Ricci scalar. If we consider R in place of $f(R)$ then the action of standard Einstein's Hilbert can be obtained. The five dimensional field equations in $f(R)$ theory of gravity are given by :

$$F(R)R_{ij} - \frac{1}{2}f(R)g_{ij} - \nabla_i\nabla_j F(R) + g_{ij}\,\Box F(R) = kT_{ij}\ ,(i,j = 1,2,3,4,5) \tag{1}$$

Where $F(R) = \frac{df(R)}{dR}$, $\Box \equiv \nabla^i\nabla_i\ ,\nabla_i$ (2)

is the covariant derivative and T_{ij} is the standard matter energy momentum tensor.
Contracting the above field equations we have

$$F(R)R - \frac{5}{2}f(R) + 4\square F(R) = kT \tag{3}$$

Using this equation in (1) ,the field equations take the form

$$F(R)R_{\mu\nu} - \nabla_\mu\nabla_\nu F(R) - kT_{\mu\nu} = \frac{1}{5}[F(R)R\square F(R) - kT]g_{\mu\nu} \tag{4}$$

In vacuum this field equation (3) reduces to

$$F(R)R - \frac{5}{2}f(R) + 4\square F(R) = 0 \tag{5}$$

This yields a relationship between $f(R)$ and $F(R)$ which can be used to simplify the field equations and to evaluate $f(R)$.

III.Exact vacuum solutions of five dimensional Binachi type-I space-time:

The five dimensional plane symmetric metric of Binachi type-I is

$$ds^2 = -A^2(dx^2 + dy^2 + dz^2) + B^2dm^2 + dt^2 \tag{6}$$

Where A and B are functions of cosmic time t only. The corresponding Ricci scalar becomes
Where the . Denotes ordinary differentiation with respect to t.
We define the average scale factor

$$a = (A^3B)^{\frac{1}{4}} \tag{8}$$

And the volume scale factor is defined as

$$V = a^4 = A^3B \tag{9}$$

The mean Hubble parameter H is defined by

$$H = \frac{1}{4}\sum_{i=1}^{4} H_i \tag{10}$$

Where $H_1 = H_2 = H_3 = \frac{\dot{A}}{A}$ $\quad H_4 = \frac{\dot{B}}{B}$ are the directional Hubble parameters in the directions of x, y, z and m axis respectively. Using equations (8),(9) and (10), we obtain

$$H = \frac{1}{4}\frac{\dot{V}}{V} = \frac{1}{4}\sum_{i=1}^{4} H_i = \frac{\dot{a}}{a} \tag{11}$$

From equation (5) we obtain

$$f(R) = \frac{2}{5}[4\square F(R) + F(R)R] \tag{12}$$

Putting this value of $f(R)$ in the vacuum field equations (3) we obtain

$$\frac{F(R)R_{ij} - \nabla_i\nabla_j F(R)}{g_{ij}} = \frac{1}{5}[F(R)R - \square F(R)] \tag{13}$$

Since the metric (6) depends only on t, one can view equation (13) as the differential equations for $F(t), A, B$. It follows from equation (13) that the combination

$$K_i = \frac{F(R)R_{ii} - \nabla_i\nabla_i F(R)}{g_{ii}} \tag{14}$$

is independent of the index i and j. Consequently $K_i - K_j = 0$ gives

$$-2\frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} + 2\frac{\dot{A}^2}{A^2} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{F}}{F} - \frac{\ddot{F}}{F} = 0 \tag{15}$$

$$-3\frac{\ddot{A}}{A} + 3\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{B}}{B}\frac{\dot{F}}{F} - \frac{\ddot{F}}{F} = 0 \tag{16}$$

Now (15)−(16) we get respectively

$$\left(\frac{\ddot{A}}{A} - \frac{\ddot{B}}{B}\right) + 2\frac{\dot{A}}{A}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right) + \frac{\dot{F}}{F}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right) = 0 \tag{17}$$

This imply

$$\frac{B}{A} = d_1\, exp\left[c_1 \int \frac{dt}{a^4F}\right] \tag{18}$$

$$B = a(d_1)^{\frac{3}{4}}\, exp\left[\frac{3}{4}c_1 \int \frac{dt}{a^4F}\right] \tag{19}$$

$$A = a(d_1)^{-\frac{1}{4}}\, exp\left[-\frac{1}{4}c_1 \int \frac{dt}{a^4F}\right] \tag{20}$$

Now we use the power law assumption to solve the integral part in the above equations as

$$F\ \alpha\ a^m \tag{21}$$

Where m is an arbitrary constant.
The equation (21) implies that

$$F = ka^m \tag{22}$$

Where k is constant of proportionality.
The deceleration parameter q in cosmology is the measure of the cosmic acceleration of the universe expansion and is defined as

$$q = -\frac{a\ddot{a}}{\dot{a}^2} \tag{23}$$

The sign of q plays an important role to identify the behavior of the universe. The positive declaration parameter corresponds to a decelerating model while the negative value provides inflation.

The well known relation between the average Hubble parameter H and average Hubble parameter H and average scale factor a given as

$$H = la^{-n} \tag{24}$$

Where $l > 0$ and $n \geq 0$.

From equation (11) and (24), we have

$$\dot{a} = la^{1-n} \tag{25}$$

And consequently the declaration parameter becomes

$$q = n - 1 \tag{26}$$

Which is a constant. After integrating equation (26), we have

$$a = (nlt + k_1)^{\frac{1}{n}} \qquad n \neq 0 \tag{27}$$

and

$$a = k_2 exp(lt) \qquad n = 0 \tag{28}$$

k_1 and k_2 are constants of integration.

Thus we have two values of the average scale factors which corresponds to two different models of the universe. From the scalar R given in the equation (7), we can check the singularity of the solutions. If we consider $m = -3$ as a special case then from equation (22) we have

$$F = ka^{-3} \tag{29}$$

IV. Five dimensional Model of the Universe when $\boldsymbol{n \neq 0}$

In this section we study the five dimensional model of the universe for $n \neq 0$.

When $n \neq 0$ then we have $a = (nlt + k_1)^{\frac{1}{n}}$ and for $m = -3$ as a special case, F becomes

$$F = k(nlt + k_1)^{\frac{-3}{n}} \tag{30}$$

For this value of F (19),(20) and (30), imply that

$$A = a(d_1)^{-\frac{1}{4}} exp\left[-\frac{1}{4}c_1 \int \frac{dt}{a^4 k(nlt + k_1)^{\frac{-3}{n}}}\right] \tag{31}$$

$$B = a(d_1)^{\frac{3}{4}} exp\left[\frac{3}{4}c_1 \int \frac{dt}{a^4 k(nlt + k_1)^{\frac{-3}{n}}}\right] \tag{32}$$

The mean generalized Hubble parameter becomes

$$H = \frac{1}{4}\sum_{i=1}^{4} H_i = l(nlt + k_1)^{-1} \tag{33}$$

And the volume scale factor becomes

$$V = (nlt + k_1)^{\frac{4}{n}} \tag{34}$$

From equations (12), the function $f(R)$, found as

$$f(R) = \frac{2k}{5}(nlt + k_1)^{-\frac{3}{n}}R - \frac{3}{5}kl^2(1 + 2n)(nlt + k_1)^{-\frac{3}{n}-2} \tag{35}$$

From equation (7) Ricci scalar R becomes

$$R = -2\left[6l^2 a^{2-2n} - \frac{9l^2 c_1 n}{4k}a^{1-n} + 4l^2(1 - n)a^{1-2n} + \frac{9l^2 c_1 na}{4k} - \frac{3l^2 {c_1}^2 n^2}{8k^2}\right] \tag{36}$$

Which clearly indicates that $f(R)$ can not be explicitely written in terms of R. However, by inserting this value of R , $f(R)$ can be written as a function of t, which is true as R depends upon t. for a special case $n = \frac{1}{2}$, $f(R)$ turns out to be

$$f(R) = \frac{2k}{5}\left(\frac{1}{2}lt + k_1\right)^{-6} R \tag{37}$$

This gives $f(R)$ only as a function of R.

V. Five dimensional Model of the Universe when $\boldsymbol{n = 0}$

In this section we study the five dimensional model of the universe for $n = 0$.

For $n = 0$ the average scale factor for the model of the universe $a = k_2 exp(lt)$ and hence F becomes

$$F = k(k_2 exp(lt))^{-3} \tag{38}$$

For this value of F (19),(20) and (38), imply that

$$A = a(d_1)^{-\frac{1}{4}} exp\left[-\frac{1}{4}c_1 \int \frac{dt}{a^4 k(k_2 exp\,(lt))^{-3}}\right] \tag{39}$$

$$B = a(d_1)^{\frac{3}{4}} exp\left[\frac{3}{4}c_1 \int \frac{dt}{a^4 k(k_2 exp\,(lt))^{-3}}\right] \tag{40}$$

The mean generalized Hubble parameter becomes

$$H = l \tag{41}$$

And the volume scale factor becomes

$$V = \left(k_2 exp(lt)\right)^4 \tag{42}$$

From equations (12), the function $f(R)$, found as

$$f(R) = \frac{2k}{5{k_2}^3} exp(-3lt)(R + 24l^2) \quad (43)$$

From equation (7) Ricci scalar R becomes

$$R = -2\left[10l^2 + \frac{{c_1}^2}{16k^2}a^{-2} + \frac{9}{16k^2}a^{-2} + \frac{4l}{k}a^{-1}\right] \quad (44)$$

Which corresponds to the general function $f(R)$

$$f(R) = \sum a_n R^n, \quad (45)$$

Where n may take the values from negative or positive.

VI. Conclusion

In this paper we have investigated two exact vacuum solutions of the five dimensional Binachi type I space time in $f(R)$ theory of gravity by using the variation law of Hubble parameter to discuss the well known phenomenon of the universe expansion on the lines of M.Sharif and M.Farasat Shamir (2009). These five dimensional solutions correspond to two models of the universe (i.e., $n \neq 0$ and $n = 0$). The first solution gives a singular model with power law expansion and positive deceleration parameter while the second solution gives a non singular model with exponential expansion and negative deceleration parameter. The functions $f(R)$ are evaluated for both models.

The physical behavior of these five dimensional models is observed as under :

i. For $n \neq 0$ i.e, Singular model of the universe

For this model average scale factor $a = (nlt + k_1)^{\frac{1}{n}}$.

This model has point of singularity at $t = -\frac{k_1}{nl}$.

The physical parameter H is infinite at this point.

The volume scale factor V is not vanish for this model.

The function of the Ricci scalar, $f(R)$, is also infinite.

The metric functions A and B vanishes at this point of singularity.

ii. For $n = 0$ i.e, Non- singular five dimensional model of the universe

For this model average scale factor $a = k_2 exp(lt)$.

This model of the universe is non singular because exponential function is never zero and hence there does not exist any physical singularity for this five dimensional model of the universe.

The physical parameter H is constant.

The volume scale factor V vanishes at the point.

The function of the Ricci scalar, $f(R)$, is also finite.

The metric functions A and B do not vanish for this model.

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