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Bulk Viscous Fluid in Bianchi Type VI_0 Universe with Electromagnetic Field

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Abstract: *We have investigated Bianchi type VI_0 cosmological model with the matter bulk viscous fluid coupled with electromagnetic field in general theory of relativity. To obtain the deterministic solution of Einstein's field equation, by assuming a relation between metric potential $A = C^n$ further some physical and geometrical properties of the model are also discussed.*
Key Words: *Bianchi type VI_0, Bulk viscous fluid, electromagnetic field, general theory of relativity.*

I. Introducation

Bianchi type cosmological models are important in the sense that these models are homogeneous and anisotropic, from which the process of isotropization of the universe is studied through the passage of time. Moreover, from the theoretical point of view, anisotropic universe has a greater generality than isotropic models. The simplicity of the field equations made Bianchi space time useful in constructing models of spatially homogeneous and anisotropic cosmologies. Hence, these models are to be known as very much suitable models of our universe. Therefore, study of these models creates much more interest.

Bulk viscosity is supposed to play a very important role in the early evolution of the universe. There are many circumstances during the evolution of the universe in which bulk viscosity could arise. The bulk viscosity coefficients determine the magnitude of viscous stress relative to the expansion.

Barrow [1] has pointed out that Bianchi type VI_0 model of the universe give a better explanation of some of the cosmological problem. Dunn and Tupper [2] investigated a class of Bianchi type VI_0 perfect fluid cosmological model with electromagnetic field. Bali [3] studied a Bianchi type VI_0 magnetized barotropic bulk viscous fluid massive string universe in general relativity. R. Bali, R. Banerjee and S. K. Banerjee [4] have studied Bianchi type VI_0 Bulk viscous massive string cosmological models in general theory of relativity. G. P. Singh and A. Y. Kale, [5] investigated Anisotropic Bulk viscous cosmological models with variable G and Λ. L. K. Patel and S.S. Koppar, [6] have studied Some Bianchi type VI_0 viscous fluid cosmological models. S. D. Deo and Roughe A K [7] studied Non Existence of Bianchi type I, V and VI_0 cosmic string in Bimetric relativity. M. K. Verma and Ram [8] investigated Bianchi type VI_0 bulk viscous fluid models with variable Gravitational and Cosmological Constants. B. M. Ribeiro and A. K. Sanyal, [9] have studied Bianchi VI_0 viscous fluid cosmology with magnetic field.

In this paper, we have investigated Bianchi type VI_0 bulk viscous fluid coupled with electromagnetic field in general theory of relativity and obtained the solution of field equations. Further we have discussed the physical and geometrical behaviour of bianchi type VI_0 cosmological model.

II. The Metric And Field Equations

Here we consider the spatially homogeneous Bianchi type-VI_0 metric in the form

$$ds^2 = -dt^2 + A^2dx^2 + B^2e^{-2mx}dy^2 + C^2e^{2mx}dz^2$$

(2.1)

Where m is non -zero constant and A, B and C are function of t only.

The energy momentum tensor of the source bulk viscous fluid coupled with electromagnetic field is denoted by

$$T_i^j = (\rho + \bar{p})v_iv^j + \bar{p}g_i^j + E_i^j$$

(2.2)

Where, $\bar{p} = p - \xi v^i_{;i}$

Here ρ , p , $\overline{p}$ and ξ are respective energy density of matter, thermodynamic pressure, effective pressure and bulk viscosity coefficient. The four velocity vector of the fluid satisfies $v_i v^j = -1$.

Electromagnetic field is defined as

$$E_i^j = -F_{ir}F^{jr} + \frac{1}{4}F_{ab}F^{ab}g_i^j$$

(2.3)

Where, E_i^j is electromagnetic energy tensor and F_i^j is the electromagnetic field tensor.

We assume that F_{13} is the only non-vanishing component of F_{ij} which corresponds to the presence of magnetic field along y-direction.

The Einstein field equation in the general theory of relativity is given by

$$R_i^j - \frac{1}{2}Rg_i^j = -8\pi k T_i^j$$

(2.4)

Where, R_i^j is known as Ricci tensor and $R = g^{ij}R_{ij}$ is the Ricci scalar and T_i^j is the energy momentum tensor for matter.

The field equations (2.4) together with the line element (2.1) with equations (2.2) and (2.3) we get

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} + \frac{m^2}{A^2} = -8\pi G\left[\overline{p} - \frac{(F_{13})^2 e^{-2mx}}{2A^2C^2}\right]$$

(2.5)

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{m^2}{A^2} = -8\pi G\left[\overline{p} + \frac{(F_{13})^2 e^{-2mx}}{2A^2C^2}\right]$$

(2.6)

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{m^2}{A^2} = -8\pi G\left[\overline{p} - \frac{(F_{13})^2 e^{-2mx}}{2A^2C^2}\right] \quad (2.7)$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} - \frac{m^2}{A^2} = -8\pi G\left[-\rho + \frac{(F_{13})^2 e^{-2mx}}{2A^2C^2}\right]$$

(2.8)

$$m\left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right) = 0$$

(2.9)

From equation (2.9) we get

$$\frac{\dot{B}}{B} = \frac{\dot{C}}{C}$$

(2.10)

After integration, we have

$$B = kC$$

(2.11)

Where k is a constant of integration.

Here the sake of simplicity, we consider $B = C$, by taking $k = 1$.

III. Solution Of The Field Equations

We assume that the expansion is proportional to the shear which is physical conditions. This condition leads to

$A = C^n$, where n is a constant and $B = C$ (3.1)

From equation (2.5) and (2.7) with (3.1) we obtain

$$\frac{\ddot{C}}{C}-\frac{\ddot{A}}{A}+\frac{\dot{C}^2}{C^2}-\frac{\dot{A}\dot{C}}{AC}+\frac{2m^2}{A^2}=0$$
(3.2)

Using the equation (3.1) in equation (3.2) we get

$$\ddot{C}+(n+1)\frac{\dot{C}^2}{C}=\frac{2m^2}{(n-1)}C^{-2n+1}$$
(3.3)

Now assuming $\dot{C}=f(C)$, equation (3.3) takes the from

$$\frac{d}{dC}\left(f^2\right)+\frac{2(n+1)}{C}f^2=\frac{4m^2}{(n-1)}C^{-2n+1}$$
(3.4)

Equation (3.4) has the general solution

$$f^2=\left(\frac{dC}{dt}\right)^2=\frac{m^2\left(C^4+k_1\right)}{(n-1)C^{2n+2}}$$
(3.5)

Where k_1 is the constant of integration.

The Bianchi type $-VI_0$ model in this case reduces to the form

$$ds^2=-\left(\frac{dt}{dC}\right)^2dC^2+C^{2n}dx^2+C^2\left[e^{-2mx}dy^2+e^{2mx}dz^2\right]$$
(3.6)

Without loss of generality, after substituting C=T, x=X, y=Y, z=Z in (3.6) we have

$$ds^2=-\left[\frac{dT^2}{\frac{m^2\left(C^4+k_1\right)}{(n-1)C^{2n+2}}}\right]+T^{2n}dX^2+T^2\left[e^{-2mX}dY^2+e^{2mX}dZ^2\right]$$
(3.7)

Physical And Geometrical Behaviour Of The Model

It is clear that, we can find the physical and kinematical parameters with the metric (3.7). The effect of bulk viscosity is to produce a change in the cosmic fluid and therefore exhibits essential change character of the solution. The bulk viscosity is assumed to be a simple power function of the energy density and defined as

$$\rho=\frac{m^2}{8\pi G}\left[\frac{(n+2)T^4+(2n+1)k_1}{(n-1)T^{2n+4}}\right]$$
(3.8)

We also assume that the fluid obeys the equation of state $\bar{p}=\gamma\rho$ which gives us

$$\bar{p}=\frac{\gamma m^2}{8\pi G}\left[\frac{(n+2)T^4+(2n+1)k_1}{(n-1)T^{2n+4}}\right]$$
(3.9)

The expansion scalar and shear scalar are given by

$$\theta=(n+2)\left[\frac{m^2\left(T^4+k_1\right)}{(n-1)T^{2n+4}}\right]^{\frac{1}{2}}$$
(3.10)

$$\sigma^2 = \left[\frac{(n-1)m^2\left(T^4 + k_1\right)}{3T^{2n+4}} \right]$$

(3.11)

Where

$$\frac{\sigma^2}{\theta^2} = \frac{(n-1)^2}{3(n+2)^2}$$

(3.12)

For the model (3.7) we observe that the spatial volume increases with time T and it become infinite for large value of T and ρ, $\bar{p}$, σ, θ all are infinite but vanish for large T. Thus, the model has a big-bang singularity at the finite time T. The physical and kinematical parameters are all well behaved for $k < T < \infty$ and the bulk viscosity coefficients are infinite and tend to zero for large.

Since $\lim_{T\to\infty} \frac{\sigma}{\theta} =$ constant, the anisotropy is maintained for all time. It can be seen that the model is irrotational. Therefore, the model describes a continuously expanding, shearing and non rotating universe with big-bang.

IV. Conclusion

In the present study, we have investigated the effect of bulk viscous fluid with electromagnetic field in bianchi type VI_0 Universe. Einstein's field equations have been solved exactly suitable physical and kinematical parameter. Here we have observed that the spatial volume increases with time T and it become infinite for large value of T and ρ, $\bar{p}$, σ, θ all are infinite but vanish for large T. Thus, the model has a big-bang singularity at the finite time T and further since $\lim_{T\to\infty} \frac{\sigma}{\theta} =$ constant, which gives us the anisotropy is maintained for all time and it can be seen that the model is irrotational. Therefore, the model describes a continuously expanding, shearing and non rotating universe with big-bang in general theory of relativity.

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