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Discussion for Invalid Algebraic Revisions

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Abstract: *Leung (2010) provided two comments to criticize Teng (2009). This paper examines Leung (2010) to discuss his challenge to Teng (2009). We show that two comments proposed by Leung (2010) contained questionable results such that this paper presents further discussions for Teng (2009) and Leung (2010).*
Keywords: *Inventory models; Linear backordered cost; Algebraic method*

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I. Introduction

This paper discusses Leung [11] that was published in European Journal of Operational Research which is an important journal. Up to now, there are two papers, Wee et al. [18] and Cárdenas-Barrón [3] that had referred to Leung [11] in their References. We examine Wee et al. [18] and Cárdenas-Barrón [3] to find out that Wee et al. [18] and Cárdenas-Barrón [3] only mentioned Leung [11] in their Introduction without any detailed discussion for the two comments proposed by Leung [11]. So few citations usually indicates two possibilities: (a) This paper contains some material that researchers did not understand such that to avoid trouble, practitioners are not willing to take risk to cite an unknown paper, (b) The research topic is out of date that does not arouse attention any more. After 2010, there are still more than fifty papers that discussed inventory models solving by algebraic methods such that Leung [11] must contain some contents that confuse researchers and then results in only two papers are daring to cite Leung [11] in their References. The purpose of this paper is to point out several questionable findings in Leung [11] to reflect researchers' hesitancy is reasonable.

II. Notation And Assumptions

To be compatible with Teng [15] and Leung [11], we use the same notation and assumptions as their papers.
Notation

A the ordering cost per replenishment
h the holding cost per unit and per unit of time
d the constant demand per unit of time
Q the order quantity per replenishment
Q^* the optimal order quantity
r the fill rate, that is the ratio between the inventory period and one replenishment cycle
v the backorder cost per unit per unit of time

Assumptions

(1) The lead time is zero so that replenishment is instantaneous.
(2) There is no quantity discount.
(3) Both the initial and the ending inventory levels are zero so that there is no salvage value.
(4) $TC(Q)$ is the EOQ model with backorders.

III. Review Of The First Comment Proposed By Leung

We review two comments proposed by Leung [11] to criticize Teng [15]. The first comment is that Teng [15] did not finish his solution for the EOQ model with linear backordered cost,

$$TC(Q)=\frac{Ad}{Q}+\frac{Q}{2}\left[hr^2+v(1-r)^2\right]. \tag{1}$$

Teng [15] applied the Arithmetic-Geometric-Mean-Inequality (AGMI),

$$\frac{a+b}{2}\geq\sqrt{ab}\ , \tag{2}$$

for any two real positive numbers a and b, and the equality holds if and only if $a=b$, to obtain

$$TC(Q) \geq \sqrt{2Ad\left[hr^2+v(1-r)^2\right]}, \tag{3}$$

and when $\dfrac{Ad}{Q}=\dfrac{Q}{2}\left[hr^2+v(1-r)^2\right]$, that is, the optimal order quantity,

$$Q^*=\sqrt{\frac{2Ad}{hr^2+v(1-r)^2}}, \tag{4}$$

to imply the inventory attains its minimum

$$TC\left(Q^*\right)=\sqrt{2Ad\left[hr^2+v(1-r)^2\right]}. \tag{5}$$

Leung [11] mentioned that "if the value of r is not fixed and is taken to be a decision variable, Teng's approach cannot completely solve the EOQ model with complete backorders"

Hence, Leung [11] assumed an auxiliary function, denoted as $\eta(r)$ with

$$\eta(r)= hr^2+v(1-r)^2. \tag{6}$$

Then Leung [11] derived

$$\eta(r)=(h+v)\left(r-\frac{v}{h+v}\right)^2-\frac{v^2}{h+v}+v, \tag{7}$$

to imply that

$$r^*=\frac{v}{h+v}. \tag{8}$$

IV. Our Discussion For The First Comment

We must point out that Equation (7) should be further simplified as

$$\eta(r)=(h+v)\left(r-\frac{v}{h+v}\right)^2+\frac{hv}{h+v}, \tag{9}$$

to show the positivity of $\eta(r)$.

Moreover, we are motivated by Cárdenas-Barrón [2], by the Cauchy-Schwarz inequality, that is, if $a_1, a_2,..., a_n$ and $b_1, b_2,..., b_n$ are two sets of real numbers, then

$$\sum_{k=1}^{n} a_k^2 \sum_{k=1}^{n} b_k^2 \geq \left(\sum_{k=1}^{n} a_k b_k\right)^2, \tag{10}$$

with equality if and only if the two sets of numbers are proportional:

$$\frac{a_1}{b_1}=\frac{a_2}{b_2}=...=\frac{a_n}{b_n}, \tag{11}$$

to derive the result of Equation (9) directly,

$$\begin{aligned}\eta(r)&=\left\{\left[\sqrt{h}r\right]^2+\left[\sqrt{v}(1-r)\right]^2\right\}\left\{\left(\frac{\sqrt{v}}{\sqrt{h+v}}\right)^2+\left(\frac{\sqrt{h}}{\sqrt{h+v}}\right)^2\right\}\\ &\geq\left(\sqrt{h}r\frac{\sqrt{v}}{\sqrt{h+v}}+\sqrt{v}(1-r)\frac{\sqrt{h}}{\sqrt{h+v}}\right)^2\\ &=\frac{hv}{h+v},\end{aligned} \tag{12}$$

and the minimum attains when

$$\frac{\sqrt{h}r}{\sqrt{v}/\sqrt{h+v}}=\frac{\sqrt{v}(1-r)}{\sqrt{h}/\sqrt{h+v}} \tag{13}$$

holds, then $hr = v(1-r)$ that is $r^* = v/(h+v)$ as the same result of Leung [11].
We must point out that in Teng [15], he claimed that "In the EOQ model with backorders, we know from Wee et al. [17] that the total relevant cost is

$$TC(Q) = \frac{Ad}{Q} + \frac{Q}{2}\left[hr^2 + v(1-r)^2\right]." \tag{14}$$

In Wee et al. [17], they applied the Cost-Difference-Comparison Method (CDCM) to derive

$$r^* = \frac{v}{h+v}. \tag{15}$$

When Teng [15] referred to the findings of Wee et al. [17], Teng [15] already accept that $r^* = v/(h+v)$ and then Teng [15] simplify the inventory model from $TC(Q,r)$ to $TC(Q)$. Hence, in Teng [15], he treated r as a constant.
In the first comment, Leung [11] neglected the model construction of Teng [15] to challenge that Teng [15] did not know $r^* = v/(h+v)$.
Consequently, Leung [11] used algebraic method to obtain Equations (6-8). In fact, $r^* = v/(h+v)$ already derived in Wee [17] by CDCM.
We conclude that Leung [11] overlooked Teng [15] already cited the findings of Wee [17] with $r^* = v/(h+v)$ to reduce his inventory model from $TC(Q,r)$ to $TC(Q)$ such that in Case (2) of Teng [15], he already referred $r^* = v/(h+v)$. Consequently, the patchwork proposed by Leung [11] finds an already known result of $r^* = v/(h+v)$, derived by Wee et al. [17], and used by Teng [15], that is unnecessary.

V. Review Of The Second Comment Proposed By Leung

For the second comment of Leung [11], Leung [11] claimed that "in order to determine which is the global minimum of $TC(Q,r)$, we must compare the following three situations:
Situation (a): The existence of both positive inventory and negative inventory (i.e. complete backorders) implies that $0 < r < 1$. The optimal solution (Q^*, r^*) is given by Equations (1) and (2) and the resulting local minimum cost is given by Equation (3) and denoted by

$$TC^*_{(a)} = TC(Q^*, r^*) = \sqrt{\frac{2Adhv}{h+v}} , \tag{16}$$

Situation (b): The existence of no positive but all negative inventory implies that $r = 0$. Setting $r = 0$ in Equations (10) and (11) of Teng [15], or alternatively assigning $h = \infty$ (which implies that holding a unit is extremely expensive and thus we should never hold any) in Equations (2) and (3) yields

$$Q^*_{(b)} = \sqrt{\frac{2Ad}{v}} , \tag{17}$$

and

$$TC^*_{(b)} = TC\left(Q^*_{(b)}\right) = \sqrt{2Adv}. \tag{18}$$

Situation (c): The existence of no negative but all positive inventory implies that $r = 1$. Setting $r = 1$ in Equations (10) and (11) of Teng [15], or alternatively assigning $v = \infty$ (which implies that incurring a backorder is extremely expensive and thus we should never incur any) in Equations (2) and (3) yields

$$Q^*_{(c)} = \sqrt{\frac{2Ad}{h}} , \tag{19}$$

and

$$TC^*_{(c)} = TC\left(Q^*_{(c)}\right) = \sqrt{2Adh}. \tag{20}$$

Comparing the local minima in Situations (a)–(c) through Equations (6), (8) and (10), we have $TC^*_{(a)} < TC^*_{(b)}$ and $TC^*_{(a)} < TC^*_{(c)}$. Hence, the global minimum of $TC(Q,r)$ occurs when Q^* and r^* are given by Equations (1) and (2).

In short, it is apparent from Equation (6) in the form of $TC^*_{(a)} = \sqrt{2Adv\frac{h}{h+v}} = \sqrt{2Adh\frac{v}{h+v}}$ that it will be optimal to incur some backorders towards the end of an order cycle if neither $h=\infty$ nor $v=\infty$ occurs.

An immediate remark follows: When solving an objective function with two decision variables without derivatives, we have to check the global minimum solution by comparing the above-mentioned three situations, in line with checking the Hessian matrix with derivatives. Hence, this checking is also necessary for such papers as Grubbström and Erdem [7], Cárdenas-Barrón [1], Huang [8], Wu and Ouyang [19], Ronald et al. [13], Chang et al. [4], Chiu et al. [5], Sphicas [14], Minner [12], Chung and Wee [6], Wee and Chung [16], Leung [9, 10] and Wee et al. [17]."

In the next section, we will show that the assertion proposed by Leung [11] contained severe questionable results.

VI. Our Discussion For The Second Comment

We begin to show that the second Comment 2 Leung [11] is redundant.

To be fully understand the description proposed by Leung [11], we cited Equations (1-3) of Leung [11] as follows,

$$r^* = \frac{v}{h+v}, \tag{21}$$

$$Q^* = \sqrt{\frac{2Ad(h+v)}{hv}}, \tag{22}$$

and

$$TC^* = \sqrt{\frac{2Adhv}{h+v}}. \tag{23}$$

We also cited Equations (10-11) of Teng [15] in the following,

$$Q^* = \sqrt{\frac{2Ad}{hr^2 + v(1-r)^2}}, \tag{24}$$

and

$$TC(Q^*) = \sqrt{2Ad\left[hr^2 + v(1-r)^2\right]}. \tag{25}$$

Leung [11] divided $0 \le r \le 1$ into three situations: In Situation (a), $0 < r < 1$, in Situation (b), $r = 0$, and in Situation (c), $r = 1$. We will show that for $0 \le r \le 1$, then the optimal fill rate is $r^* = v/(h+v)$ to reveal three situations proposed by Leung [11] is meaningless.

From algebraic method, researchers can directly handle $0 \le r \le 1$ to derive

$$r^* = \frac{v}{h+v}. \tag{26}$$

We list four possible methods to obtain Equation (26).

The simplest way to derive Equation (26) is mentioned by Sphicas [14] that used the balance of hold cost and shortage cost. Wee et al. [17] provided another approach by CDCM that was motivated by Minner [12]. The third approach is referred to Leung [11] that was cited as Equations (7) and (8) of this paper. The fourth method is proposed by this paper by the Cauchy-Schwarz inequality.

Consequently, to divide the solution into three different situations is tedious and useless.

At last, we point out that to divide problem into three situations is workable and necessary for analytical approach, calculus, because calculus is only suitable for interior points such that by calculus, there will be three different objective functions as mentioned by Equations (6, 8, 10) in Leung [11].

VII. Conclusion

We show that Leung [11] misunderstood the difference between algebraic and differential methods. Leung [11] did not fully realized his own derivation for r^* is workable for the entire domain, $0 \le r \le 1$. Hence, his comparison for three situations: (a) $0 < r < 1$, (b) $r = 0$, and (c) $r = 1$, becomes unnecessary.

Conflicts Of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

Finding

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