IOSR Journal of Business and Management (IOSR-JBM)
e-ISSN: 2278-487X, p-ISSN: 2319-7668. Volume 24, Issue 2. Ser. IV (February. 2022), PP 59-64
www.iosrjournals.org

Improvement on Inventory Model with Stock Dependent Selling Rate

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Abstract: *In this paper, we constructed a new inventory model with stock dependent selling rate for deteriorated items with a partially backlogged rate. We derived a criterion to obtain the best-replenished policy. Our model is a generalization for Chuang (2012) that was an extension of Wu et al. (2008). Our findings will help the researcher develop more realized inventory models.*

Key words: *Inventory models; Stock dependent selling rate; Ramp type demand*

Date of Submission: 08-02-2022 Date of Acceptance: 22-02-2022

I. Introduction

Hill (1995) was the pioneer in the development of inventory models with ramp-type demands. Many papers have been devoted to generalizing inventory models with ramp-type demands. Such papers include Mandal and Pal (1998) with deterioration items, Wu et al. (1999) with back-ordered rate relative to the waiting time, Wu and Ouyang (2000) with two replenishment policies, Wu (2001) with two parameters Weibull distributed deterioration items, Giri et al. (2003) with three parameters Weibull distribution, and Manna and Chaudhuri (2006) with varied deteriorated rate. There have also been a few attempts to improve on previous papers. For example, Deng (2005) revised Wu et al. (1999) and Deng et al. (2007) rectified Mandal and Pal (1998) and Wu and Ouyang (2000).

Wu et al. (2008) developed a maximum profit inventory model with ramp type demands and stock-dependent selling rates. Chuang (2012) generalized Wu et al. (2008) from two different approaches. In the first approach, he devised linear differential equations to describe the inventory level, to solve the inventory level. He constructed a maximum profit inventory model and used its first partial derivative to determine the maximums of interior and boundary points. For the second approach, he tried to solve the maximum problem without the total profit function that helps researchers comprehend inventory models with simplified mathematical analytical work. In this article, we will first generalize the inventory model in which the backlogged rate is proportional to the waiting time. Next, we point out that there are some problems in the second approach of Chuang (2012), and then we have provided a patchwork that will help practitioners without a firm grasp of calculus study stochastic inventory models with stock-dependent selling rates.

II. Notation and Assumptions

Chuang (2012) studied an inventory model proposed by Wu et al. (2008), which is a finite planning horizon inventory system starting with stock with deteriorated items and a complete backlog for shortages. There is only one decision variable for this kind of model, that is, the time when the inventory level drops to zero, which is denoted as t_1. The initial replenishment occurs at $t = 0$, and then the inventory level gradually decreases to zero at t_1, due to the selling and deterioration. The second replenishment happens at $t = T$, such that all shortages accumulated during $[t_1, T]$ are completely back-ordered. For a selling item, there are two policies to fulfill it: (a) from stock, and (b) from backlogging.

In the development of a maximum profit inventory model under stock-dependent selling rate, we have used the same notation as in Chuang (2012).

(1) The replenishment rate is infinite so that replenishments are instantaneous and the lead time is zero.
(2) T is a finite time horizon.
(3) A is the set up cost.
(4) s is the selling price per unit.
(5) C_h is the inventory holding cost per unit per unit of time.

(6) C_d is the deterioration cost per unit. We extend the model of Wu et al. (2008) to include the deterioration cost. Wu et al. (2008) is a special case in our proposed model with (a) $C_d = 0$, and (b) ramp type demand.

(7) C_s is the shortage cost per unit per unit of time.

(8) c is the purchasing cost per unit.

(9) θ is the constant fraction of the on-hand inventory deterioration per unit of time.

(10) $I(t)$ is the on-hand inventory level at the time t over the ordering cycle $[0, T]$.

(11) The shortage is allowed and partially backlogged which the back-ordered rate is inversely linearly proportional to the waiting time.

(12) The theoretical demand, $D(t)$, is a positive continuous function.

(13) $R(t)$ is the actual selling rate, that is stock dependent, where α is a positive constant, with

$$R(x) = \begin{cases} D(x) + \alpha I(x), & I(x) > 0, \\ D(x), & I(x) \le 0. \end{cases}$$

(14) t_1 is the time when the inventory level drops zero.

(15) t_1^* is the optimal solution for t_1.

(16) $f(t_1)$ is an auxiliary function defined as $f(t_1) = \left(\frac{\Omega}{\theta+\alpha}\right)\left(e^{(\theta+\alpha)t_1} - 1\right) + C_s(T - t_1)$ where $\Omega = \alpha(s-c) - (C_h + \theta c + \theta C_d)$.

(17) $Z(t_1)$ is the total profit that consists of the selling price, set up cost, purchasing cost, holding cost, deterioration cost, and shortage cost.

(18) $g(t_1) = \frac{\Omega}{\theta+\alpha}\left(e^{(\theta+\alpha)t_1} - 1\right) + \frac{T - t_1}{1 + a(T - t_1)}\left(a(s-c) + C_s\right)$ is the second auxiliary function for our new inventory model where the back-ordered rate is inversely linearly proportional to the waiting time.

(19) $W(t_1)$ is the total profit for our new inventory model.

Under the same assumptions as Chuang (2012), replenishment occurs at the time $t = 0$ when the inventory level attains its maximum. From $t = 0$ to t_1, the inventory level reduces due to the two effects of selling rate, $R(t)$ and deterioration. At t_1, the inventory level drops to zero, after which shortages are allowed during the time interval (t_1, T), and is completely back-ordered at T.

III. Our generalization

In our generalized inventory model, the shortage is partially backlogged with the back ordered rate $B(t) = \frac{1}{1 + a(T - t)}$ with $a \ge 0$. For the special case with $a = 0$, $B(t) \equiv 1$ that is the fully backlogged case of Deng et al. (2007), Wu et al. (2008), and Chuang (2012). The differentiation system controlling the inventory level is described by the following two equations

$$\frac{d}{dt} I(t) + (\theta + \alpha) I(t) = -R(t), \tag{1}$$

for $0 < t < t_1$, and

$$\frac{d}{dt} I(t) = -\frac{R(t)}{1 + a(T - t)}, \tag{2}$$

for $t_1 < t < T$.

We directly solve Equations (1) and (2) to imply that

$$I(t) = \int_{t}^{t_1} R(x) e^{(\theta+\alpha)(x-t)} dx \ , \tag{3}$$

for $0 \le t \le t_1$, and

$$I(t) = \int_{t}^{t_1} \frac{R(x)}{1 + a(T - x)} dx \ , \tag{4}$$

for $t_1 \le t \le T$.

The amount of deteriorated items during $[0, t_1]$ is evaluated as

$$I(0) - \int_{0}^{t_1} (R(x) + \alpha\, I(t)) dx \ = \frac{\theta}{\theta + \alpha} \int_{0}^{t_1} R(x) \left(e^{(\theta+\alpha)x} - 1\right) dx \ . \tag{5}$$

Using integration by part, the holding cost during $[0, t_1]$ is evaluated as

$$C_h \int_{0}^{t_1} I(t) dt \ = C_h \int_{x=0}^{t_1} \frac{R(x)}{\theta + \alpha} \left(e^{(\theta+\alpha)x} - 1\right) dx \ . \tag{6}$$

The shortage cost during $[t_1, T]$ is evaluated through integration by part

$$C_s \int_{t_1}^{T} (-1) I(t) dt \ = C_s \int_{x=t_1}^{T} \frac{R(x)}{1 + a(T - x)} (T - x) dx \ . \tag{7}$$

Therefore, the total profit, say $W(t_1)$, is the difference between the revenue of accumulated selling items and four types of costs: purchasing cost, holding cost, deterioration cost, and shortage cost. It is then expressed as follows

$$\begin{aligned} W(t_1) = {} & s\left(\frac{\alpha}{\theta + \alpha} \int_{0}^{t_1} R(x) e^{(\theta+\alpha)x} dx + \frac{\theta}{\theta + \alpha} \int_{0}^{t_1} R(x) dx + \int_{t_1}^{T} \frac{R(x)}{1 + a(T - x)} dx \right) \\ & - c\left(\int_{0}^{t_1} R(x) e^{(\theta+\alpha)x} dx + \int_{t_1}^{T} \frac{R(x)}{1 + a(T - x)} dx \right) - C_h \int_{0}^{t_1} \frac{R(x)}{\theta + \alpha} \left(e^{(\theta+\alpha)x} - 1\right) dx \\ & - C_d \frac{\theta}{\theta + \alpha} \int_{0}^{t_1} R(x) \left(e^{(\theta+\alpha)x} - 1\right) dx - C_s \int_{x=t_1}^{T} \frac{R(x)}{1 + a(T - x)} (T - x) dx \ . \end{aligned} \tag{8}$$

We take the first derivation of $W(t_1)$ and then try to solve $W(t_1) = 0$ to imply that

$$R(t_1) \left[\frac{\Omega}{\theta + \alpha} \left(e^{(\theta+\alpha)t_1} - 1\right) + \frac{T - t_1}{1 + a(T - t_1)} \left(a(s - c) + C_s\right) \right] = 0 \ , \tag{9}$$

where we adopt an abbreviation Ω with

$$\Omega = \alpha\,(s - c) - (C_h + \theta c + \theta C_d) . \tag{10}$$

Motivated by Equation (9), we assume an auxiliary function, say $g(t_1)$, as follows

$$g(t_1) = \frac{\Omega}{\theta + \alpha} \left(e^{(\theta+\alpha)t_1} - 1\right) + \frac{T - t_1}{1 + a(T - t_1)} \left(a(s - c) + C_s\right) . \tag{11}$$

We derive that

$$g'(t_1) = \Omega\, e^{(\theta+\alpha)t_1} - \frac{a(s - c) + C_s}{\left[1 + a(T - t_1)\right]^2} . \tag{12}$$

Based on Equation (11), we will divide the maximum profit problem into two cases: (a) Case A: $\Omega < 0$, and (b) Case B: $\Omega \ge 0$.

For Case A, based on $g'(t_1) < 0$, we derive that $g(t_1)$ decreases from

$$g(0) = \frac{T}{1 + aT}\left(a(s - c) + C_s\right) > 0 \;, \tag{13}$$

to

$$g(T) = \frac{\Omega}{\theta + \alpha}\left(e^{(\theta+\alpha)T} - 1\right) < 0 \;, \tag{14}$$

such that there is a unique point, say $t_1^{\#}$ that satisfies

$$g\left(t_1^{\#}\right) = 0 \;, \tag{15}$$

and then $W'(t_1) > 0$ for $0 < t_1 < t_1^{\#}$ to imply that $W(t_1)$ increases in $\left[0, t_1^{\#}\right]$.
On the other hand, $W'(t_1) < 0$ for $t_1^{\#} < t_1 <, T$ to imply that $W(t_1)$ decreases in $\left[t_1^{\#}, T\right]$ to obtain that $t_1^{\#}$ is the maximum point.
For Case B, under the condition of $\Omega \geq 0$, from Equation (11), we know that $g(t_1) > 0$ to yield $W'(t_1) > 0$. Hence, $W(t_1)$ is an increasing function that attains its maximum at T. We summarize our generalization in the next theorem.

Theorem. For an inventory model with deterioration, the consumption rate is proportional to the inventory level and the backlogged rate is inversely proportional to the waiting time, and the optimal solution for the zero inventory point, t_1^*, is decided by the following rules.
Case A: When $\Omega < 0$, $t_1^* = t_1^{\#}$, where $g\left(t_1^{\#}\right) = 0$
Case B: When $\Omega \geq 0$, $t_1^* = T$

Remark. When $a = 0$, our new inventory model will degenerate to the inventory model proposed by Chuang (2012). Therefore, our new inventory model is an extension of Chuang (2012).

IV. Review of Chuang's Approach

Chuang (2012) assumed that if there is an item with demand Q, that takes place at the time t, then there are two replenishment policies: (a) fulfill the demand from the stock, or (b) satisfy the demand from the backorder.
Firstly, we consider his approach to fulfilling the demand from the stock. From the linear differential equation, $\frac{d}{dt}I(t) + (\theta + \alpha)\, I(t) = 0$, the inventory level is $I(x) = I(0)e^{-(\theta+\alpha)x}$. Under the requirement of $I(t) = Q$, Chuang (2012) derived that $I(0) = Q\, e^{(\theta+\alpha)t}$ and so the inventory level becomes $I(x) = Q\, e^{(\theta+\alpha)(t-x)}$ for $0 \leq x \leq t$.
Chuang (2012) computed the holding cost as

$$C_h \int_0^t Q\, e^{(\theta+\alpha)(t-x)}dx = C_h \frac{Q}{\theta + \alpha}\left(e^{(\theta+\alpha)t} - 1\right). \tag{16}$$

He derived the deterioration of items as

$$\theta \int_0^t Q\, e^{(\theta+\alpha)(t-x)}dx = \frac{\theta\, Q}{\theta + \alpha}\left(e^{(\theta+\alpha)t} - 1\right). \tag{17}$$

He also found the stock dependent selling items to be

$$\alpha \int_0^t Q\, e^{(\theta+\alpha)(t-x)}dx = \frac{\alpha\, Q}{\theta + \alpha}\left(e^{(\theta+\alpha)t} - 1\right). \tag{18}$$

The total profit when the items are provided from stock is computed as the sum of selling price for items with demand Q; plus the selling price for stock dependent demand minus the holding cost, minus the deterioration cost, and minus the purchasing cost:

$$sQ + s\frac{\alpha Q}{\theta+\alpha}\left(e^{(\theta+\alpha)t}-1\right)-\frac{C_h Q}{\theta+\alpha}\left(e^{(\theta+\alpha)t}-1\right)$$
$$-C_d\frac{\theta Q}{\theta+\alpha}\left(e^{(\theta+\alpha)t}-1\right)-cQ\left(e^{(\theta+\alpha)t}-1\right). \tag{19}$$

On the other hand, if the backlogged policy is applied, the total profit is expressed as

$$sQ - C_s Q(T-t). \tag{20}$$

Chuang (2012) took the difference of equations (16) and (17) to obtain that

$$Q\left(\frac{\Omega}{\theta+\alpha}\right)\left(e^{(\theta+\alpha)t}-1\right)+C_s Q(T-t)=Q\ f(t). \tag{21}$$

Depending on the sign of $f(t)$, he derived the optimal replenishment policy.

V. Our Improvement

First, we must point out that equations (19) and (20), contained questionable results. The corrected result of the purchasing cost in equation (19) should be denoted as $cQ\ e^{(\theta+\alpha)t}$, therefore the improved equation should be as follows

$$sQ + s\frac{\alpha Q}{\theta+\alpha}\left(e^{(\theta+\alpha)t}-1\right)-C_h\frac{Q}{\theta+\alpha}\left(e^{(\theta+\alpha)t}-1\right)$$
$$-C_d\frac{\theta Q}{\theta+\alpha}\left(e^{(\theta+\alpha)t}-1\right)-cQ\ e^{(\theta+\alpha)t}. \tag{22}$$

As such, the error in the last term of equation (19) proposed by Chuang (2012) has been amended by us.
Moreover, the total profit from backlogging should be computed as the sum of the selling price for items with demand Q minus the shortage cost and minus the purchasing cost

$$sQ - C_s Q(T-t) - cQ\ . \tag{23}$$

Based on our corrected expressions, which are equations (22) and (23), we subtract Equation (20) from Equation (19) to derive that $Q\ f(t)$ is the same, as expected. Hence, the rest of the proof in Chuang (2012) for replenishment policy is accidentally corrected. Consequently, we will not discuss his replenishment policies in Chuang (2012).

Next, we will point out that the derivations of equations (16-18) are too complicated. We will provide a simple solution as follows.

The initial stock is $I(0)=Q\ e^{(\theta+\alpha)t}$ and at the time t, the remaining stock is Q, such that the sum of (a) the deteriorated item, and (b) the stock dependent selling, can be directly computed as follows

$$Q\ e^{(\theta+\alpha)t}-Q=Q\left(e^{(\theta+\alpha)t}-1\right). \tag{24}$$

The ratio between the deteriorated item and the stock dependent selling is $\theta:\alpha$ such that the equation for deteriorated items is

$$\frac{\theta}{\theta+\alpha}Q\left(e^{(\theta+\alpha)t}-1\right), \tag{25}$$

which is essentially equation (17). The stock depending selling is

$$\frac{\alpha}{\theta+\alpha}Q\left(e^{(\theta+\alpha)t}-1\right), \tag{26}$$

which is the same as equation (18). The holding cost is $C_h\int_0^t I(x)dx$ and the deteriorated item is $\theta\int_0^t I(x)dx$.

Based on equation (25), we know that the holding cost is

$$\frac{C_h}{\theta+\alpha}Q\left(e^{(\theta+\alpha)t}-1\right), \tag{27}$$

which is the same as equation (16).
Hence, we came up with an elementary method to replace the tedious integration of Chuang (2012) for equation s (16-18).

VI. Conclusion

We developed a new inventory model that generalized Chuang (2012) and Wu et al. (2008). Our generalization will help researchers to develop advanced inventory models with effective practical applications. Moreover, we examined Chuang (2012) for his second approach, in which the objective profit function was not constructed from differential calculus. Chuang (2012) claimed that his alternative approach will provide an intuitive point of view to help more researchers study stochastic inventory models. In this article, we not only point out two errors in his derivations but also simplify three derivations. We believe that this will help readers without any extensive background in mathematics to comprehend maximum profit inventory models from ramp type demand to arbitrary positive demand.

References

[1]. Chuang, J. P. C., Inventory model with stock dependent selling rate, African Journal of Business Management, Vol. 6, No. 3, pp. 780-786, 2012.
[2]. Deng, P. S., Improved inventory models with ramp type demand and Weibull deterioration, International Journal of Information Management Sciences, Vol. 16, No. 4, pp. 79-86, 2005.
[3]. Deng, P. S., Lin, R. and Chu, P., A note on the inventory models for deteriorating items with ramp type demand rate, European Journal of Operational Research, Vol. 178, pp. 112-120, 2007.
[4]. Giri, B. C., Jalan, A. K. and Chaudhuri, K. S., Economic Order Quantity model with Weibull deterioration distribution, shortage and ramp-type demand, International Journal of System Science, Vol. 34, No. 4, pp. 237-243, 2003.
[5]. Hill, R. M., Inventory models for increasing demand followed by level demand, Journal of Operations Research Society, Vol. 46, No. 10, pp. 1250-1259, 1995.
[6]. Mandal, B. and Pal, A. K., Order level inventory system with ramp type demand rate for deteriorating items, Journal of Interdisciplinary Mathematics, Vol. 1, pp. 49-66, 1998.
[7]. Manna, S. K. and Chaudhuri, K. S., An EOQ model with ramp type demand rate, time dependent deterioration rate, unit production cost and shortages, European Journal of Operational Research, Vol. 171, No. 2, pp. 557-566, 2006.
[8]. Wu, J. W., Lin, C. Tan, B. and Lee, W. C., An EOQ model with ramp type demand rate for items with Weibull deterioration, International Journal of Information Management Sciences, Vol. 10, pp. 41-51, 1999.
[9]. Wu, K. S. and Ouyang, L. Y., A replenishment policy for deteriorating items with ramp type demand rate, Proceeding of National. Science Council ROC (A), Vol. 24, pp. 279-286, 2000.
[10]. Wu, K. S., An EOQ inventory model for items with Weibull distribution deterioration, ramp type demand rate and partial backlogging, Production Planning & Control, Vol. 12, pp. 787-793, 2001.
[11]. Wu, K. S. Ouyang, L. Y. and Yang, C. T., Retailer's optimal ordering policy for deteriorating items with ramp-type demand under stock-dependent consumption rate, International Journal of Information Management Sciences, Vol. 19, pp. 245-262, 2008.

Yung-Ning Cheng, et. al. "Improvement on Inventory Model with Stock Dependent Selling Rate." *IOSR Journal of Business and Management (IOSR-JBM)*, 24(02), 2022, pp. 59-64.