

Constrained Engineering Design Optimization using Average Differential Evolution

Burhanettin Durmuş

(Department of Electrical and Electronics Engineering/ Kütahya Dumlupınar University, Kütahya, Turkey)

Corresponding Author: Burhanettin Durmuş

Abstract: The use of metaheuristics has a growing interest in solving constrained optimization problems due to the computational disadvantages of numerical methods. Metaheuristics are a powerful tool in reaching the global optimum. In this work, the Average Differential Evolution (ADE) algorithm, which is one of the newly proposed metaheuristics, has been adapted to the constrained engineering design problems. The ADE algorithm is a population-based approach with a high convergence rate. It uses a mutation operator with collective diversity in the production of candidates. The results show the robustness and effectiveness of the proposed algorithm compared to state-of-the-art algorithms in literature.

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I. Introduction

Constrained engineering design problems (CEDP) are considered as real-world problems with many constraints. Constraints are very important for engineering design problems because they make searching difficult and search method ineffective. Many researchers investigate the solution of these problems and offer different approaches [1-3]. However, because of the complex and nonlinear characterization of the problems, classical optimization techniques may be inadequate to achieve the global optimum. To overcome this, the interest in the use of metaheuristics in this area has been increasing in recent years.

Metaheuristics such as Genetic algorithm (GA) [4], artificial bee colony (ABC) [5], bat algorithm (BA) [6], crow search algorithm (CSA) [7] and particle swarm optimization (PSO) [8] provide remarkable performance in solving engineering design problems. Including natural phenomena, these algorithms essentially maintain a population of solutions that are evolved through random alterations and selection. The differences between these operations lie in the type of alterations used for generating new solutions, and the mechanism employed for selecting new members.

The ADE algorithm is one of the newly proposed metaheuristics and has been introduced as a search method with rapid convergence rate. This metaheuristic, in which new individual production is performed using the average value of the solutions in the population, collectively provides the evolution of the candidate solution [9]. In this study, the ADE algorithm has been applied to well-known CEDP's. The obtained results have been compared with the results reported in the literature.

II. The ADE Algorithm

The ADE algorithm is a newly proposed metaheuristic algorithm based on population [9]. This metaheuristic, which has a rapid convergence, provides considerable success in solving the system identification problems. It has six computational phases, including initialization, evaluation, improvising of new trial solution, handling of bound, selection and termination.

2.1 Initialization

In initialization phase, the algorithm parameters are initialized and the initial population is randomly generated within the range of boundaries of variables as follows:

$$x_{i,G}^j = x_{i,\min} + rand * (x_{i,\max} - x_{i,\min}) \quad i = 1, 2, \dots, NP \text{ and } j = 1, 2, \dots, D \quad (1)$$

where, x is the set of solution vector, NP is the population size or the number of solution vectors, D is the number of variables, $x_{i,\max}$ and $x_{i,\min}$ are the maximum and minimum allowable values for the D_{th} variable, $rand$ is a random number in the interval $[0, 1]$, and G is the generation number.

2.2 Evaluation of solution vectors

The fitness values of the solution vectors are determined at this stage. Fitness values actually represent the quality of vectors. Therefore, the value of each solution vector in the objective function of the problem is taken as the fitness value of that vector.

2.3 Improvising of new trial solution

At this phase, the candidate vector for next generation is created. Firstly, the average vector in the present generation is computed. This vector is calculated by taking the average of the solution vectors in the present population, as in the following:

$$\bar{A}_G = \frac{1}{NP} \sum_{i=1}^{NP} \bar{x}_{i,G} \quad (2)$$

Here, \bar{A}_G shows the average vector of the generation G , NP shows the solution number in the population, \bar{x}_i is the current solution vector, and G shows the current generation. Then, a mutant vector is created by the following equation for each solution vector.

$$\bar{u}_{i,G+1} = \bar{x}_{best,G} + \gamma * rand_i[-1,1] * (\bar{A}_G - \bar{x}_{i,G}) \quad (3)$$

where, $\bar{u}_{i,G+1}$ is the mutant vector, $\bar{x}_{best,G}$ is the best solution vector in generation G , \bar{A}_G is the average vector in generation G , $\bar{x}_{i,G}$ is the original solution vector in generation G , γ is the scaling factor, and $rand_i[-1,1]$ is the random number in interval between $[-1, 1]$.

Finally, In order to form the trial vector, $\hat{x}_{i,G+1}$, the mutant vector $\bar{u}_{i,G+1}$ is put on a crossover with Cr (crossover rate) possibility together with the original solution vector $\bar{x}_{i,G}$ as done in the DE algorithm. Each variable belonging to the trial vector is selected with Cr possibility from the mutant vector and with $1-Cr$ possibility from the original solution vector.

2.4 Handling of bound violations

Constraint violations are checked for candidate solutions produced in the previous stage. If any variable of the trial vector is found to be outside the boundaries defined in initialization, then this variable is assigned the nearest limit value.

2.5 Selection

The decision of transferring the candidate solution to the next generation is decided in this process step. As expressed in the following equation, the vector with better fitness function is transferred to the next generation.

$$\bar{x}_{i,G+1} = \begin{cases} \hat{x}_{i,G+1} & \text{if } f(\hat{x}_{i,G+1}) < f(\bar{x}_{i,G}) \\ \bar{x}_{i,G} & \text{otherwise} \end{cases} \quad (4)$$

where, $f(\hat{x}_{i,G+1})$ and $f(\bar{x}_{i,G})$ represent the fitness function of $\hat{x}_{i,G+1}$ and $\bar{x}_{i,G}$, respectively.

2.6 Termination

The five stages described above are maintained until the termination criteria are met. When the number of predefined generations is reached, the computation is stopped and the best vector is considered as the global optimum.

III. Results and Discussions

In this section, simulation studies based on some well-known constrained engineering design problems are carried out for investigating the performance of the proposed ADE algorithm. The selected problems are well-known benchmarks studied by various approaches [4, 8, 10-12]. For an accurate comparison, ADE has been run 30 times independently and it has been taken as $FES = 5000$. In all cases, parameters of the ADE are set as follows: $NP = 25$, $Cr = 0.9$ and $\gamma = 2$.

3.1 The design of a tension/compression spring

The tension/compression spring structure, shown in Figure 1, is a design problem. This problem consists of minimizing the weight ($f(x)$) of a tension/compression spring subject to constraints on shear stress, minimum deflection, and surge frequency. It can be stated as following with three design variables such as the wire diameter, $d(= x_1)$ the mean coil diameter, $D(= x_2)$ and the number of active coils $N(= x_3)$ [10].

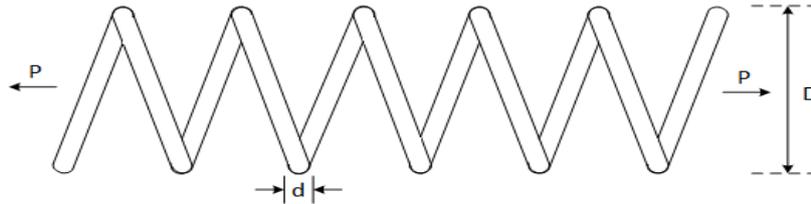


Fig. 1: A tension/compression spring

$$f_{\text{cost}}(x) = (x_3 + 2)x_2x_1^2 \tag{5}$$

Subject to

$$g_1(x) = 1 - \frac{x_2^3x_3}{71785x_1^4} \leq 0$$

$$g_2(x) = \frac{4x_2^2 - x_1x_2}{12566(x_2x_1^3 - x_1^4)} + \frac{1}{5108x_1^2} - 1 \leq 0 \tag{6}$$

$$g_3(x) = 1 - \frac{140.45x_1}{x_2^2x_3} \leq 0$$

$$g_4(x) = \frac{x_1 + x_2}{1.5} - 1 \leq 0$$

The variable regions are limited by $0.05 \leq x_1 \leq 2, 0.25 \leq x_2 \leq 1.3, 2 \leq x_3 \leq 15$. Further, Arora [10] has also provided a solution to this problem using nonlinear programming (NP) technique. Also, Coello [4] has solved this problem using a GA-based method. He and Wang [8] have proposed the co-evolutionary PSO for solving this problem. In addition, some researchers have used newer improved DE algorithms and other metaheuristic algorithms to solve this problem [5-7, 11, 13-18].

Table 1 presents statistical results of ADE. And, results of ADE are compared with solutions reported by other researchers, as shown in Table 2. From Table 2, it can be seen that the best solution obtained by ADE is better than those of the other methods.

Table 1: Statistical results of tension/compression spring problem by ADE.

Best	Mean	Worst	S. D.
0.009873	0.009906	0.010164	0.000073
$g_1(x)$	$g_2(x)$	$g_3(x)$	$g_4(x)$
-3.43197E-05	-4.80553E-05	-4.860240	-0.717057

It is observed that the number of coils $N(=x_3)$ is not an integer at the optimum design point. This problem was solved by methods in literature as a continuous case study. If the problem is assumed as a discrete case study, the number of active coils should be integer values [10]. In this case, the problem can be re-optimized for discrete values. Thus, best solution of ADE is $f_{\text{cost}}(x) = 0.00998$ at $(x_1 = 0.050634, x_2 = 0.389267, x_3 = 8)$ with $(g_1(x) = -6.93044 \text{E}-05, g_2(x) = -1.53117 \text{E}-04, g_3(x) = -4.8665, g_4(x) = -0.706733)$.

Table 2: Comparison of results for the tension/compression spring problem.

Methods	Design parameters			$f_{\text{cost}}(x)$	
	$d(=x_1)$	$D(=x_2)$	$N(=x_3)$		
IHS [13]	0.051154	0.349871	12.076432	0.012670	
NP [10]	Continuous	0.051680	0.356532	11.313501	0.012677
	Discrete	0.051200	0.345400	12	0.012680
GA [4]	0.051480	0.351661	11.632201	0.012704	
CPSO [8]	0.051728	0.357644	11.244543	0.012674	
IACO [11]	0.051865	0.361500	11.000000	0.012643	
rank-iMDDE [14]	0.051689	0.356717	11.288998	0.012665	
MAL-DE [15]	0.051689	0.356717	11.288955	0.012665	
BA [6]	0.051690	0.356730	11.2885	0.012670	
ABC [5]	0.051749	0.358179	11.203763	0.012665	
WCA [16]	0.051680	0.356522	11.300410	0.012665	
CSA [7]	0.051689	0.3567169	11.289011	0.012665	
IAPSO [17]	0.051685	0.356629	11.294175	0.012665	

CS [18]		0.051680	0.356522	11.300410	0.012665
ADE	Continuous	0.050000	0.374414	8.548155	0.009873
	Discrete	0.050634	0.389267	8	0.009980

3.2 The design of a pressure vessel

The pressure vessel design problem consists of minimization of the cost of the pressure vessel as shown in Figure 2. The main purpose is to decrease the total cost [19]. There are totally four different design variables namely, T_s is the thickness of the shell ($= x_1$), T_h is the thickness of the head ($= x_2$), R is the inner radius ($= x_3$), and L is the length of the cylindrical section of the vessel except head ($= x_4$). T_s and T_h show the available thickness of rolled steel plates and these parameters are also the definition of integer multiples of 0.0625 inch in scale. Moreover, R and L are the continuous parameters in a regular pressure vessel designs. The problem can be based on the same explanation by Coello [19] in below;

$$f_{cost}(x) = 0.6224x_1x_3x_4 + 1.7781x_2x_3^2 + 3.1661x_1^2x_4 + 19.84x_1^2x_3 \tag{7}$$

Subject to

$$\begin{aligned} g_1(x) &= -x_1 + 0.0193x_3 \leq 0 \\ g_2(x) &= -x_2 + 0.00954x_3 \leq 0 \\ g_3(x) &= -\pi x_3^2x_4 - \frac{4}{3}\pi x_3^3 + 1,296,000 \leq 0 \\ g_4(x) &= x_4 - 240 \leq 0 \end{aligned} \tag{8}$$

The design space is bounded by $1 \times 0.0625 \leq x_1, x_2 \leq 99 \times 0.0625, 10 \leq x_3, x_4 \leq 200$.

Various approaches such as GA [12], HSA [13], and discrete programming (DP) [3] were applied to solve this problem. Also, a detailed mathematical analysis of this problem is provided that proves that 6,059.714335 is the global minimum [20].

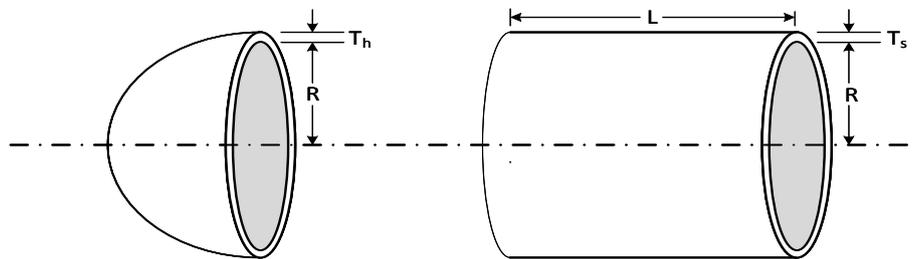


Fig. 2: A pressure vessel

The statistical results of ADE and the comparison of results are presented in Tables 3 and 4, respectively. The results show that ADE algorithm reached to global optimum. It is worth mentioning that the best objective value obtained by Eskandar et al. [16] is not feasible since design variables x_1 and x_2 are not integer multiples of 0.0625.

Table 3: Statistical results of pressure vessel problem by ADE.

Best	Mean	Worst	S. D.
6059.714362	6547.574957	10544.421824	1132.581152
$g_1(x)$	$g_2(x)$	$g_3(x)$	$g_4(x)$
-0.000000000677	-0.035880829350	-0.005417838693	-63.363402749637

Table 4: Comparison of results for the pressure vessel problem.

Methods	Design parameters				$f_{cost}(x)$
	$T_s(= x_1)$	$T_h(= x_2)$	$R(= x_3)$	$L(= x_4)$	
IHS [13]	1.1250	0.6250	58.2901	43.6926	7,197.7300
GA [4]	0.8125	0.4375	40.3239	200.00	6,288.7445
CPSO [8]	0.8125	0.4375	42.0912	176.7465	6,061.0777
IACO [11]	0.8125	0.4375	42.0983	176.6377	6,059.7258
meta-GA [12]	1.1250	0.6250	58.1978	44.2930	7,207.4940
rank-iMDDE [14]	13.0	7.0	42.0984	176.6365	6,059.7143
MAL-DE [15]	0.8125	0.4375	42.0984	176.6365	6,059.7143
BA [6]	0.8125	0.4375	42.0984	176.6365	6,059.7143
ABC [5]	0.8125	0.4375	42.0984	176.6365	6,059.7143

WCA [16]	0.7781	0.3846	40.3196	-200.00	5,885.3327
CSA [7]	0.8125	0.4375	42.098445	176.636598	6,059.7143
IAPSO [17]	0.8125	0.4375	42.0984	176.6366	6,059.7143
CS [18]	0.8125	0.4375	42.098445	176.636595	6,059.7143
DP [3]	1.1250	0.6250	48.9700	106.72	7,980.8940
ADE	0.8125	0.4375	42.098445	176.636595	6,059.7143

3.3 The design of a welded beam

As it can be seen in Figure 3, the welded beam structure is a practical design problem [4]. The objective is to carry out the minimum fabrication cost of the welded beam subject into the constraints on bending stress, (σ), shear stress, (τ), end deflection, (δ), buckling load, (P_c), and side constraint. There are four design variables: $h(=x_1)$, $l(=x_2)$, $t(=x_3)$, and $b(=x_4)$. The cost function is stated in below:

$$f_{cost}(x) = 1.10471x_1^2x_2 + 0.04811x_3x_4(14.0 + x_2) \tag{9}$$

Subject to

$$\begin{aligned} g_1(x) &= \tau(x) - \tau_{max} \leq 0 \\ g_2(x) &= \sigma(x) - \sigma_{max} \leq 0 \\ g_3(x) &= x_1 - x_4 \leq 0 \\ g_4(x) &= 0.10471x_1^2 + 0.04811x_3x_4(14.0 + x_2) - 5.0 \leq 0 \\ g_5(x) &= 0.125 - x_1 \leq 0 \\ g_6(x) &= \delta(x) - \delta_{max} \leq 0 \\ g_7(x) &= P - P_c(x) \leq 0 \end{aligned} \tag{10}$$

The variable regions are limited by $0.1 \leq x_1 \leq 2$, $0.1 \leq x_2 \leq 10$, $0.1 \leq x_3 \leq 10$, $0.1 \leq x_4 \leq 2$. Coello [4] and Deb [21] have provided a possible solution to find out the GA based method problems. On the other hand, some other researchers have used metaheuristic methods to solve this problem [5-8, 11, 13, 14, 16-18]. The statistical results of ADE and the comparison of results are presented in Tables 5 and 6, respectively.

where,
$$\tau(x) = \sqrt{(\tau')^2 + 2\tau'\tau''\frac{x_2}{2R} + (\tau'')^2}, \tau' = \frac{P}{\sqrt{2}x_1x_2}, \tau'' = \frac{MR}{J}, M = P\left(L + \frac{x_2}{2}\right), R = \sqrt{\frac{x_2^2}{4} + \left(\frac{x_1 + x_3}{2}\right)^2},$$

$$J = 2\left\{\sqrt{2}x_1x_2\left[\frac{x_2^2}{4} + \left(\frac{x_1 + x_3}{2}\right)^2\right]\right\}, \sigma(x) = \frac{6PL}{x_4x_3^2}, \delta(x) = \frac{6PL^3}{Ex_3^2x_4}, P_c(x) = \frac{4.013E\sqrt{x_3^2x_4^6}}{L^2} \left(1 - \frac{x_3}{2L}\sqrt{\frac{E}{4G}}\right),$$

$P = 6,000$ lb, $L = 14$ in, $\delta_{max} = 0.25$ in, $E = 30 \times 10^6$ psi, $G = 12 \times 10^6$ psi, $\tau_{max} = 13,600$ psi, $\sigma_{max} = 30,000$ psi.

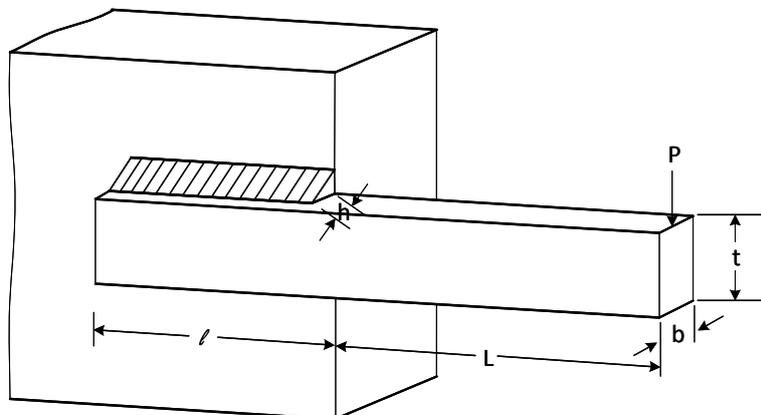


Fig. 3: Welded beam structure

Table 5: Statistical results of welded beam problem by ADE.

Best	Mean	Worst	S. D.
1.699214	1.701265	1.727508	0.005645
$g_1(x)$	$g_2(x)$	$g_3(x)$	$g_4(x)$
-16.25887716834	-0.210643521565	-0.004255039403	-3.445807411616
$g_5(x)$	$g_6(x)$	$g_7(x)$	
-0.076500000000	-0.054001376204	-1.992842957436	

Table 6: Comparison of results for the welded beam problem.

Methods	Design parameters				$f_{cost}(x)$
	$h(=x_1)$	$l(=x_2)$	$t(=x_3)$	$b(=x_4)$	
IHS [13]	0.2057	3.4704	9.0366	0.2057	1.7248
GA [4]	0.2088	3.4205	8.9975	0.2100	1.7483
CPSO [8]	0.2023	3.5442	9.0482	0.2057	1.7280
IACO [11]	0.2057	3.4711	9.0366	0.2057	1.7249
rank-iMDE [14]	0.2057	3.4704	9.0366	0.2057	1.7248
BA [6]	0.2015	3.562	9.0414	0.2057	1.7312
ABC [5]	0.2057	3.4704	9.0366	0.2057	1.7248
WCA [16]	0.205728	3.470522	9.03662	0.205729	1.724856
CSA [7]	0.2057296	3.470488	9.036623	0.205729	1.724852
IAPSO [17]	0.2057296	3.470488	9.036623	0.205729	1.724852
CS [18]	0.205728	3.470522	9.036620	0.205729	1.724856
GA [21]	0.2489	6.1730	8.1789	0.2533	2.4328
ADE	0.2015	3.3280	9.036098	0.205755	1.699214

As it can be seen in Table 6, the best solution found by the ADE algorithm is better than the other solutions utilizing by other techniques.

IV. Conclusions

Optimization is a significant issue in the design process of engineering optimization problems. An optimizer aims at achieving the optimal solution for design problems that are encountered in several areas. Over the last two decades, metaheuristic algorithms have been successfully applied as an optimum utilizer for solving complicated real-world optimization problems. These algorithms have carried out conventional numerical methods and also provide the optimal solution. Thus, researchers have focused on improving these metaheuristic algorithms.

In this study, an effective metaheuristic algorithm (ADE) has been successfully applied to design problems. A comparative study has been carried out to show the effectiveness of the ADE over other methods. The results indicate that the proposed method provides successful results in general. The proposed method is promising for future works especially for the solution of complex real-world problems including optimum design.

References

- [1]. Cha J, Mayne R. Optimization with discrete variables via recursive quadratic programming. *Journal of Mechanisms Transmissions and Automation in Design*. 1989;111(1):130-136
- [2]. Fu J, Fenton R, Cleghorn W. A mixed integer discrete-continuous programming method and its applications to engineering design optimization. *Engineering Optimization*. 1991;17(4):263-280
- [3]. Sandgren E. Nonlinear integer and discrete programming in mechanical design optimization. *ASME Journal of Mechanical Design*. 1990;112(2):223-229
- [4]. Coello, CAC. Use of a self-adaptive penalty approach for engineering optimization problems. *Computers in Industry*. 2000;41(2):113-127
- [5]. Akay B, Karaboga D. Artificial bee colony algorithm for large-scale problems and engineering design optimization. *Journal of Intelligent Manufacturing*. 2012;23(4):1001-1014
- [6]. Gandomi AH, Yang XS, Alavi AH, et al. Bat algorithm for constrained optimization tasks. *Neural Computing and Applications*. 2013;22(6):1239-1255
- [7]. Askarzadeh A. A novel metaheuristic method for solving constrained engineering optimization problems: crow search algorithm. *Computers & Structures*. 2016;169:1-12
- [8]. He Q, Wang L. An effective co-evolutionary particle swarm optimization for constrained engineering design problems. *Engineering Applications of Artificial Intelligence*. 2007;20(1):89-99
- [9]. Durmuş B. Optimal components selection for active filter design with average differential evolution algorithm. *International Journal of Electronics and Communications*. 2018;94:293-302
- [10]. Arora JS. *Introduction to optimum design*. 2012; Academic Press, Massachusetts.
- [11]. Kaveh A, Talatahari S. An improved ant colony optimization for constrained engineering design problems. *Engineering Computations*. 2010;27(1):155-182
- [12]. Wu SJ, Chow PT. Genetic algorithms for nonlinear mixed discrete-integer optimization problems via meta-genetic parameter optimization. *Engineering Optimization*. 1995;24(2):137-159

- [13]. Mahdavi M, Fesanghary M, Damangir E. An improved harmony search algorithm for solving optimization problems. *Applied Mathematics and Computation*. 2007;188(2):1567-1579
- [14]. Gong W, Cai Z, Liang D. Engineering optimization by means of an improved constrained differential evolution. *Computer Methods in Applied Mechanics and Engineering*. 2014;268:884-904
- [15]. Long W, Liang X, Huang Y, et al. A hybrid differential evolution augmented Lagrangian method for constrained numerical and engineering optimization. *Computer-Aided Design*. 2013;45(12):1562-1574
- [16]. Eskandar H, Sadollah A, Bahreininejad A, et al. Water cycle algorithm – A novel metaheuristic optimization method for solving constrained engineering optimization problems. *Computers & Structures*. 2012;110-111:151-166
- [17]. GuedriaNB. Improved accelerated PSO algorithm for mechanical engineering optimization problems. *Applied Soft Computing*. 2016;40:455-467
- [18]. Gandomi AH, Yang XS, Alavi AH. Cuckoo search algorithm: a metaheuristic approach to solve structural optimization problems. *Engineering Computations*. 2013;29(1):17-35
- [19]. Coello CAC. Theoretical and numerical constraint-handling techniques used with evolutionary algorithms: a survey of the state of the art. *Computer Methods in Applied Mechanics and Engineering*. 2002;191(11-12):1245-1287
- [20]. Yang XS, Huyck C, Karamanoglu M, et al. True global optimality of the pressure vessel design problem: a benchmark for bio-inspired optimisation algorithms. *International Journal of Bio-Inspired Computation*. 2013;5(6):329-335
- [21]. Deb K. Optimal design of a welded beam via genetic algorithms. *American Institute Aeronautics and Astronautics Journal*. 1991; 29(11):2013-2015

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