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Stability Analysis of Time-delay Interconnected Systems Based on Lyapunov Function Method

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Abstract: *In this paper, The stability of a class of infinite-dimensional nonlinear interconnected vehicle following systems was studied. Under the assumption that the systems are globally Lipschitz, from the conditions of stability of isolated subsystems decomposed from the interconnected nonlinear systems, by applying vector Lyapunov function method and comparison principle, the sufficient conditions of exponential string stability for this class of interconnected systems was obtained. The sufficient criterion obtained is delay-independent, thus it is easy to test in practice.*

Keywords - *nonlinear interconnected systems, string stability, global asymptotic stability, vector Lyapunov function, time delays*

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I. Introduction

The purpose of vehicle following is to keep the distance between the controlled vehicle and the vehicle ahead within the set range by automatically adjusting the speed of the controlled vehicle [1-3]. Since stability is the basic requirement of the control system, the design of the controller should be based on the stability analysis of the system. Especially when the vehicles are running in a platoon, the design of the control law should ensure the string stability of the vehicles platoon. In reference [4-7], the stability of vehicle following system is studied by transfer function method. We know that the transfer function method is an effective tool to study the stability of linear systems, but it is not ideal to study the stability of nonlinear vehicle following systems.

In the automated highway system, the number of vehicles in the platoon changes with the vehicles leaving or entering the queue. It is necessary to study the stability of vehicle string independent of the number of vehicles. The definition of string stability of infinite dimensional interconnected systems is given in reference [8]. The stability of several kinds of infinite dimensional interconnected systems is studied by using scalar Lyapunov function method, and the results obtained were applied to the stability analysis of vehicle platoon. By using the method of vector Lyapunov function, the stability of vehicle following system with finite dimension and infinite dimension is studied in reference [9], and the stability region of system parameters is larger than that in reference [8]. Because the information transmission and vehicle dynamic behavior have time delay to control command, but the time delay factor is not considered in the above literature, in this paper, we study the string stability of a kind of infinite dimensional interconnected systems with time delay by using vector Lyapunov function method.

II. System Model

In reference [8], a class of nonlinear infinite dimensional interconnected systems related to the vehicle following system model based on comprehensive information of preceding and following vehicles was studied. The system can be expressed as a differential equation:

$$\begin{cases} \dot{x}_1 = f_1(x_1, 0, x_2) \\ \dot{x}_i = f_i(x_i, x_{i-1}, x_{i+1}), i = 2,3,\cdots \end{cases} \tag{1}$$

where $i \in N$, $x_i \in R^n$, $f_i : R^n \times R^n \times R^n \to R^n$, and $f_i(0,0,0) = 0$. For $\forall i \le 0$, $x_i = 0$.

The string exponential stability of this kind of system is studied in reference [10]. Since system (1) does not contain the time delay factor, we introduce the time delay term and obtain

$$\begin{cases} \dot{x}_1 = f_1(x_1, 0, x_2) \\ \dot{x}_i = f_i(x_i, x_{i-1}(t-\tau), x_{i+1}(t-\tau)), \quad i = 2,3,\cdots \end{cases} \tag{2}$$

Where, $\tau \in [0, +\infty)$ represents the fixed time delay.

Suppose that the initial condition of system (2) is $x_i(s)=\phi_i(s), -\infty<s\le 0, \phi_i$ is bounded and continuous in interval $(-\infty,0]$. In this paper, it is assumed that the system (2) satisfies the existence and uniqueness conditions of the equilibrium point.

III. Notation and Preliminaries

In this paper, $\| f \|$ denotes Euclidean norm of f , $\| f_i(t)\|_\infty$ is defined as $\sup_{t\ge 0}\| f_i(t)\|$ and $\| f_i(s)\|_\infty$ is defined as $\sup\limits_{s\in(-\infty,0]} \| f_i(s)\|$.

Definition 1 The origin $x_i=0$ （$i\in N$）of system (2) is string stable, if for any given $\varepsilon>0$, there exists $\delta>0$, such that $\sup_i \| x_i(s)\|_\infty<\delta \Rightarrow \sup_i \| x_i(t)\|_\infty<\varepsilon$.

Definition 2 The origin $x_i=0\,(i\in N)$ of system (2) is exponentially string stable, if the origin is string stable, and there exist constants $\lambda>0$ and $M>0$, for all $t\ge 0$, satisfies $\sup_i \| x_i(t)\| \le M\| \Phi_i(s)\| e^{-\lambda t}$, where $\| \Phi_i(s)\| = \sup\limits_{s\in(-\infty,0]} |\phi_i(s)|$.

Lemma 1 Assume that $v_i(t)\ge 0$ （$\forall t\ge 0, i\in N$）, and satisfies the differential inequality

$$\dot{v}_i \le g_i(v_i, v_{i-1}(t-\tau_1),\cdots,v_1(t-\tau_{i-1}),t)\,[-\beta_{i0}v_i^m+\sum_{j=1}^{\infty}\beta_{ij}v_{i-j}^m(t-\tau_j)], i=1,2,\cdots \quad (3)$$

For $v_i>0$, has $g_i(\cdot)>0$, $\beta_{i0}>0$, $\beta_{ij}\ge 0$, $\beta_{ij}=0$ （$j\ge i$）. Where $\tau_j\in[0,+\infty)$ are constants, and $i,j=1,2,\cdots$. If there exists $v=(v_{10},v_{20},\cdots)$, such that

$$-\beta_{i0}v_{i,0}^m+\sum_{j=1}^{\infty}\beta_{ij}v_{i-j,0}^m<0, i=1,2,\cdots \quad (4)$$

and $\inf_i\{v_{i0}\}=\alpha>0$, $\sup_i\{v_{i0}\}=\beta>0$, then for any given $\varepsilon>0$, exists $\delta>0$, such that $\sup_i \| v_i(s)\|_\infty<\delta \Rightarrow \sup_i \| v_i(t)\|_\infty<\varepsilon$.

Proof. Introduced a set

$$\gamma=\{z(l): z_i=lv_{i0}, l>0, i=1,2,\cdots\}, \Omega(z)=\{u: 0\le u\le z, z\in\gamma\}$$

For any given $\varepsilon>0$, there exists a $l_0>0$, such that $\varepsilon=\sup_i\{l_0v_{i0}\}$, here take $\delta=\inf_i\{l_0v_{i0}\}$.
Define another set

$$O=\{u: u_i=l_0v_{i0}, u_j\le l_0v_{j0}, j\ne i, i,j=1,2,\cdots\}$$

It is not hard to see that $O\subset\Omega(z(l_0))$. If $\sup_i \| v_i(s)\|_\infty<\delta$, then we have $v(s)\in\Omega(z(l_0))$, and $v(s)\notin O$.

The following we prove that when $t>0, \sup_i \| v_i(t)\|_\infty<\varepsilon$. Here we use the method of reduction to absurdity.

Suppose there exists some time $t_0>0$, such that $\sup_i \| v_i(t_0)\|_\infty=\varepsilon$, then exist at some point $t_1\in[0,t_0]$, and a valve of i, such that $v(t_1)\in O$, namely, $v_i(t_1)=l_0v_{i0}$, $v_j(t)\le l_0v_{j0}$, where $t\in(-\infty,t_1]$, $j=1,2,\cdots, j\ne i$. This means that in the time $t=t_1$, there are $\dot{v}_i\ge 0$. However, from（3）, we get

$$\dot{v}_i \le g_i(v_i, v_{i-1}(t_1-\tau_1),\cdots,v_1(t_1-\tau_{i-1}),t)\,[-\beta_{i0}v_i^m+\sum_{j=1}^{\infty}\beta_{ij}v_{i-j}^m(t_1-\tau_j)]$$

$$\le g_i(\cdot)[-\beta_{i0}v_i^m+\sum_{j=1}^{\infty}\beta_{ij}\sup_{s\in(-\infty,t_1]}v_{i-j}^m(s)] \quad (5)$$

Since $g_i(\cdot)>0$, according to（4）and（5）, we have

$$\dot{v}_i \le g_i(\cdot)[-\beta_{i0}(l_0 v_{i0})^m + \sum_{j=1}^{\infty}\beta_{ij} \sup_{s\in(-\infty,t_1]} v_{i-j}^m(s)]$$
$$\le g_i(\cdot)[-\beta_{i0}(l_0 v_{i0})^m + \sum_{j=1}^{\infty}\beta_{ij}(l_0 v_{i-j,0})^m] < 0$$

This is contradictory to $\dot{v}_i \ge 0$, so the assumption doesn't hold, namely, there is no one moment $t_0 > 0$, such that $\sup_i \| v_i(t_0) \|_\infty = \varepsilon$, thus we get $\sup_i \| v_i(t_0) \|_\infty < \varepsilon$. So for any given $\varepsilon > 0$, there exists a $\delta > 0$, such that

$$\sup_i \| v_i(s) \|_\infty < \delta \Rightarrow \sup_i \| v_i(t) \|_\infty < \varepsilon$$

The proof is completed.

IV. Stability Analysis of Interconnected Systems

Theorem 1. For system （2）, if the following conditions is satisfied , then the zero solution of (2) is exponentially string stable.

（i） $\| f_i(y_1, y_2, y_3) - f_i(z_1, z_2, z_3) \| \le l_1 \| y_1 - z_1 \| + l_2 \| y_2 - z_2 \| + l_3 \| y_3 - z_3 \|$;

（ii） for the system $\dot{x}_i = f_i(x_i, 0, 0)\ (i = 1, 2, \cdots)$, there exists a Liapunov function $v_i(x_i)$ and positive number α_l, α_h, α_1, α_2, such that

$$\alpha_l \| x_i \|^2 \le v_i(x_i) \le \alpha_h \| x_i \|^2, \frac{\partial v_i}{\partial x_i} f_i(x_i, 0, 0) \le -\alpha_1 \| x_i \|^2, \| \frac{\partial v_i}{\partial x_i} \| \le \alpha_2 \| x_i \|;$$

（iii） $-\alpha_1 \alpha_h^{-1/2} + \alpha_2(l_2 + l_3)\alpha_l^{-1/2} < 0$.

Proof. Define a function as

$$w_i(x_i) = e^{\xi t} v_i \tag{6}$$

Where ξ is an arbitrarily small positive number. Take the derivative along the solution of system (2), we have

$$\dot{w}_i = \frac{\partial v_i}{\partial x_i} f_i(x_i, x_{i-1}(t-\tau), x_{i+1}(t-\tau)) e^{\xi t} + \xi e^{\xi t} v_i$$
$$= \{\frac{\partial v_i}{\partial x_i} f_i(x_i, 0, 0) + \frac{\partial v_i}{\partial x_i}[f_i(x_i, x_{i-1}(t-\tau), x_{i+1}(t-\tau)) - f_i(x_i, 0, 0)] + \xi v_i\} e^{\xi t} \tag{7}$$

According to the conditions（i） and （ii）, from（7）, we get

$$\dot{w}_i \le \| x_i \| [(-\alpha_1 + \xi\alpha_h) \| x_i \| + \alpha_2 (l_2 \| x_{i-1}(t-\tau) \| + l_3 \| x_{i+1}(t-\tau) \|)] e^{\xi t}$$

Because ξ is an arbitrarily small positive number, according to the condition （ii）, we can get that

$$\dot{w}_i \le \| x_i \| [(-\alpha_1 + \xi\alpha_h)(\frac{v_i}{\alpha_h})^{1/2} + \alpha_2 l_2 (\frac{v_{i-1}(t-\tau)}{\alpha_l})^{1/2} + \alpha_2 l_3 (\frac{v_{i+1}(t-\tau)}{\alpha_l})^{1/2}] e^{\xi t}$$

Furthermore, wo have

$$\dot{w}_i \le \| x_i \| [(-\alpha_1 + \xi\alpha_h)(\frac{w_i}{\alpha_h})^{1/2} + \alpha_2 l_2 (\frac{w_{i-1}(t-\tau)}{\alpha_l})^{1/2} + \alpha_2 l_3 (\frac{w_{i+1}(t-\tau)}{\alpha_l})^{1/2}] e^{\xi t/2} \tag{8}$$

Take $w_{i0} = 1, w_{i-1,0} = 1, w_{i+1,0} = 1$, then we get

$$\dot{w}_i \le \| x_i \| [(-\alpha_1 + \xi\alpha_h)(\frac{1}{\alpha_h})^{1/2} + \alpha_2 l_2 (\frac{1}{\alpha_l})^{1/2} + \alpha_2 l_3 (\frac{1}{\alpha_l})^{1/2}] e^{\xi t/2}$$

In addition, from the conditions（iii）, due to

$$-\alpha_1 \alpha_h^{-1/2} + \alpha_2(l_2 + l_3)\alpha_l^{-1/2} < 0$$

So there exists a positive number ξ, such that

$$(-\alpha_1 + \xi\alpha_h)\alpha_h^{-1/2} + \alpha_2(l_2 + l_3)\alpha_l^{-1/2} < 0 \tag{9}$$

Therefore, there exists w_{i0}, $w_{i-1,0}$, and $w_{i+1,0}$, such that

$$(-\alpha_1 + \xi\alpha_h)(\frac{w_{i0}}{\alpha_h})^{1/2} + \alpha_2 l_2 (\frac{w_{i-1,0}}{\alpha_l})^{1/2} + \alpha_2 l_3 (\frac{w_{i+1,0}}{\alpha_l})^{1/2} < 0, \quad i = 2,3,\cdots \tag{10}$$

According to the Lemma 1, from (8) and (10), we get that for given any $\varepsilon_0 > 0$, there exists $\delta_0 > 0$, such that

$$\sup_i \| w_i(s) \|_\infty < \delta_0 \Rightarrow \sup_i \| w_i(t) \|_\infty < \varepsilon_0$$

Since $\alpha_h > 0$, so there exists $\varepsilon > 0$ and $\delta > 0$, such that $\varepsilon_0 = \alpha_h \varepsilon^2$, and $\delta_0 = \alpha_h \delta^2$, therefore, we get that

$$\sup_i \| x_i(s) \|_\infty < \delta \Rightarrow \sup_i \| x_i(t) \|_\infty < \varepsilon$$

So according to Definition 1, the system (2) is string stable.

Due to $\sup_i \| w_i(t) \|_\infty < \varepsilon_0$, according to the conditions (ii), we have

$$\sup_i \| x_i(t) \| \le \frac{1}{\alpha_l}(v_i(x_i))^{1/2} = \frac{1}{\alpha_l} w_i^{1/2} e^{-\xi t/2} \le \frac{1}{\alpha_l} \varepsilon_0^{1/2} e^{-\xi t/2}, \; i = 2,3,\cdots$$

According to the Definition 2 of exponential string stability, the system (2) is exponential string stable. The proof is completed.

V. Conclusion

In order to study the stability of longitudinal following control system of vehicle in automated highway system, In this paper, the stability of the infinite-dimensional nonlinear interconnected large-scale system with delays was studied by applying vector Liapunov function. Under the assumption that the system satisfied Lipschitz condition, based on the stability of isolated subsystems decomposed from the large-scale system, a sufficient criterion of exponential stability on the large-scale system was obtained. The obtained sufficient criterion is delay-independent, thus it is easy to test the conditions of the criterion in practice.

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