

## Channel estimation for multiuser MIMO-OFDM systems based on Kalman algorithms

K.Suresh

Assistant Professor, Sri Venkateswara college of Engineering.

---

**Abstract:** The use of multiple antenna's at both transmitter and receiver to form multiple input and multiple output (MIMO) channels currently hold the potential to drastically improve the spectrum efficiency and capacity of future wireless communication system. For broadband systems, OFDM turns a frequency selective channel into a set of parallel flat channels which reduces the receiver complexity. Designing a wireless receiver is a complicated task, thereby analysis is restricted on three functions of the receiver; Channel estimation, multiuser detectors and data decoding. By using iterative receiver algorithms different combinations of multiuser detectors and channel estimators are compared. The complexity-performance tradeoff and convergence behavior of the iterative receivers with channel estimation for multi user MIMO-OFDM system is analyzed.

**Index Terms:** MIMO, OFDM, Iterative receiver.

---

### I. Introduction

It is now widely accepted that multiple input multiple output (MIMO) systems increase the link reliability and/or spectral efficiency of multiuser wireless communications [1]. Moreover, when channel state information (CSI) is available at the transmitter, linear precoding can be used to further improve system performance by tailoring the transmission to the instantaneous channel conditions [2]–[5] while retaining the benefits of all-linear processing. CSI at the transmitter is mandatory in the multiuser downlink, where a base station attempts to communicate simultaneously with multiple users. On a different front, orthogonal frequency division multiplexing (OFDM) is a simple, and now well-accepted, technique to mitigate the effects of intersymbol interference in frequency selective channels [6]. OFDM converts a broadband frequency selective channel to a series of narrowband channels by transmitting data in parallel over many subcarriers. Combining OFDM with MIMO, producing so called MIMO-OFDM, significantly reduces receiver complexity in wireless multiuser broadband systems [7], thus making it a competitive choice for future broadband wireless communication systems. Since OFDM uses multiple subcarriers, optimal linear precoding for MIMO-OFDM can be implemented by deriving linear precoders for each subcarrier independently. However, due to the generally large number of subcarriers, the computational load is excessive, and this approach is probably impossible to implement in practice. Furthermore, this approach is computationally inefficient since the MIMO channels associated with adjacent subcarriers are highly correlated; the precoder and decoders are correlated as well.

### II. MU-MIMO-OFDM system overview

The MU-MIMO-OFDM system under consideration is shown in Fig. 1. It consists of  $K$  single-antenna users, transmitting to a receiver (the base-station) equipped with  $N$  antennas. The users transmit blocks of  $S$  OFDM symbols, each containing  $M$  sub-carriers. The first  $S_p$  OFDM symbols are reserved for pilot symbols, which are known to the receiver. The following  $S_d = S - S_p$  OFDM symbols contain coded data. The total number of information bearing signal constellation points per block, transmitted from each user, then becomes  $L = S_d M$ . A forward error correcting code (FEC) with rate  $R$  is used to generate codewords, which after interleaving, are mapped onto the  $L$  signal constellation points. We restrict our investigations to the case of QPSK. An extension to other constellations is conceptually straightforward, but in general non-trivial [8]. After OFDM modulation and pilot insertion, the users transmit their signals over a frequency selective block fading channel, where the different multi-antenna links are independent and identically distributed (IID). The block fading assumption holds if the transmitted data blocks are much shorter than the channel coherence time. Thus, a system with short data blocks transmitted over a channel with moderate Doppler spread is considered. Furthermore, to allow for correct OFDM demodulation at the receiver, the users are assumed to be synchronized both in time and frequency. In frequency, the synchronization requirement is strict, but due to the use of a cyclic prefix, the time requirement is somewhat relaxed to the case where the difference in arrival times is less than the duration of the cyclic prefix minus the channel delay spread. At the receiver, the signal is demodulated into the complex baseband, where an iterative receiver is implemented. The complexity-performance trade-off of this receiver is the focal point of this paper. The receiver consists of three blocks; a channel estimator, a MUD, and a bank of soft-input-soft-output (SISO) channel decoders. First, an initial channel estimation is done, based on the transmitted pilot symbols. This estimate is then used in the MUD to

separate the different user streams, which are then fed to the SISO decoders. The output of the decoders are then used in the next iteration to update the channel estimate, and to further improve the user separation in the MUD. Multiple iterations are then performed in the same way.

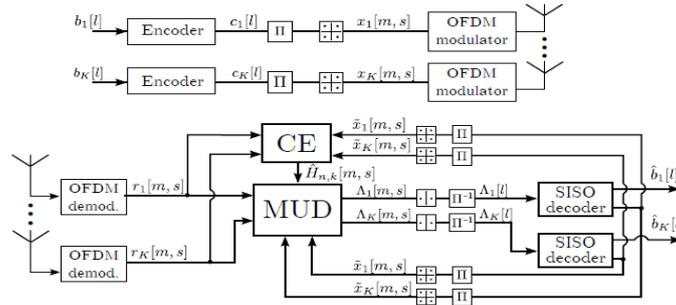


Figure 1: A baseband model of a MIMO-OFDM system with K users includes an iterative multiuser receiver with channel estimation, multiuser detection, and SISO decoders

### III. Channel Estimation Algorithms based on kalman filters

#### 3.1 Pilot-aided channel estimation

Pilot-aided channel estimation inserts known symbols, called pilots, into the time-frequency grid. The specific choice of sub-locations for these pilots is referred to as the pilot pattern. By considering only the pilots in the measurements, disregarding all the payload symbols and inserting zeros into  $\Phi_t$  at payload sub-locations, the pilot matrix  $\Phi_t$  becomes known but time-variant.

It is of interest to keep the pilot overhead low, i.e. the ratio between number of pilots and the total number of sub-symbols. In all the BAs that we study here, the pilot overhead is 1/12. Since persistent scheduling is used in the uplink case, the channel estimator has access to a continuous flow of pilot measurements on which it can base its channel estimates. We are interested in channel estimation after a whole bin has been received, so that measurements all the way to the end of the bin are available. We consider three cases as displayed if Figure 2:

**B-MMSE** filters, which stands for Block-MMSE filters, base their estimates only on measurements within the bin. Historic data are not considered.

**Kalman (KF)** filters continuously produce estimate from the most recent measurement and take all past measurements into account. However, no “future” measurements are used, so that estimates for channel coefficients early in the bin are mostly based on measurements from earlier bins.

**Smoothed Kalman** filters use all available measurements and can be produced only when all the pilots of a bin have been received.

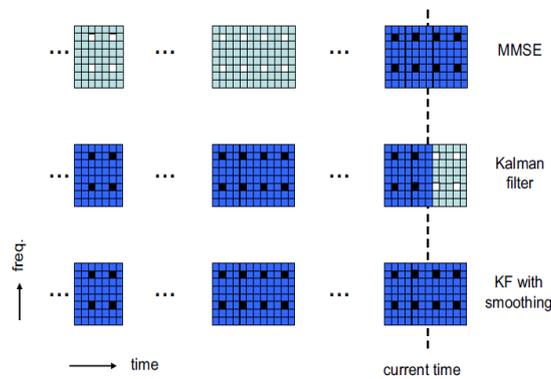


Figure 2: Schematic illustration of the three types of channel estimators examined. Here, the LFDMA block allocation is used. Black squares indicate pilot measurements that are used for forming channel estimates, while white squares are pilots disregarded by the respective estimators.

#### 3.2 Non-smoothed Kalman filter

The state-space model enables us to directly write down the optimal Kalman observer of the state vector  $\mathbf{x}_t$ :

$$\hat{\mathbf{x}}_{t|t} = \underbrace{(\mathbf{I} - \mathbf{K}_{f,t}\mathbf{H})\mathbf{F}}_{\mathbf{A}_t} \hat{\mathbf{x}}_{t-1|t-1} + \mathbf{K}_{f,t}\mathbf{y}_t.$$

The so called filtered state estimate  $\hat{\mathbf{x}}_{t|t}$  is the minimum mean square estimate given all measurements up to time  $t$ . We assume in this chapter that enough pilot data has then been received that the channel estimator has settled to a steady state. The Kalman gain  $\mathbf{K}_{f,t}$ , which is generally time-dependent and needs to be updated by a computationally demanding Riccati equation, will then be periodic-constant since the pilot matrix  $\Phi t$  is periodic. The periodic series of matrices  $\{\mathbf{K}_{f,t}\}$  may therefore be precomputed. For the same reason, the matrix  $\mathbf{A}_t$  is also known. Once the state estimation vector has been computed, channel coefficient estimates are calculated by

$$\hat{\mathbf{h}}_{t|t} = \mathbf{H}\hat{\mathbf{x}}_{t|t}.$$

### 3.3 Smoothed Kalman filter

The KF estimate may be improved upon by smoothing the estimates. Assume that the bin begins at time  $t_1+1$  and ends at time  $t_2$ . Since measurements up to and including time  $t_2$  is available, we may produce the smoothed

estimates  $\hat{\mathbf{h}}_{t_1+1|t_2}, \hat{\mathbf{h}}_{t_1+2|t_2}, \dots, \hat{\mathbf{h}}_{t_2|t_2}$  instead of  $\hat{\mathbf{h}}_{t_1+1|t_1+1}, \hat{\mathbf{h}}_{t_1+2|t_1+2}, \dots, \hat{\mathbf{h}}_{t_2|t_2}$  as the non-smoothed KF would produce. Smoothed Kalman estimates can

be computed from the one-step predictions  $\hat{\mathbf{x}}_{t|t-1}$ , which are computed with the non-smoothed KF:

$$\hat{\mathbf{x}}_{t|t_2} = \hat{\mathbf{x}}_{t|t-1} + \sum_{j=t}^{t_2} \mathbf{B}_j(\mathbf{y}_j - \mathbf{H}\hat{\mathbf{x}}_{j|j-1})$$

### 3.4 B-MMSE

The last channel estimation algorithm that we examine is the most commonly suggested in the literature. It conducts channel estimation bin-wise, basing the estimates only on the measurements taken from the bin. Such algorithms go by many names<sup>2</sup>; we refer to it here as the block minimum mean square error (B-MMSE) estimator. The B-MMSE estimator works as follows: given two Gaussian vectors  $\mathbf{h}$  and  $\mathbf{y}$  with cross-covariance  $\mathbf{R}_{hy}$  and the latter with covariance  $\mathbf{R}_y$ , the optimal least squares estimate of  $\mathbf{h}$  given  $\mathbf{y}$  is

$$\hat{\mathbf{h}} = \boldsymbol{\mu}_h + \mathbf{R}_{hy}\mathbf{R}_y^{-1}(\mathbf{y} - \boldsymbol{\mu}_y),$$

where  $\boldsymbol{\mu}_h$  and  $\boldsymbol{\mu}_y$  are the mean values of  $\mathbf{h}$  and  $\mathbf{y}$ , respectively, which we here set to all-zeros. We here use an alternative method where we use the smoothed Kalman filter to produce the B-MMSE estimate. The vector of channel coefficients  $\mathbf{h}_t$  is associated with a prior distribution that represents the knowledge that we have about the channel before measurements have been received. Accordingly, this prior distribution is set as  $p(\mathbf{h}_t|\mathbf{I}) = \mathcal{CN}(\mathbf{h}; \mathbf{0}, \mathbf{R}_h)$  for all  $t$ , where  $\mathbf{R}_h$  is the (known) covariance matrix for the channel coefficients over the  $w$  subchannels, with the SNR along is diagonal, since the noise has unit variance. By using this prior and running the smoothed KF over only one bin, so that it does not necessarily converge, the B-MMSE estimate is retrieved, because no past data is then utilized. Alternatively, the smoothed KF run to convergence but with the bins extensively separated in time can also be used to produce B-MMSE estimates. Assuming a correct modelling of the actual fading channel conditions, NMSE values can be calculated directly from the model without conducting simulation.

Of the three estimation methods, the smoothed KF will always have the best performance, since it uses all past measurements. The B-MMSE estimator and the non-smoothed KF will outperform one another depending on situation; while the B-MMSE estimator only uses local data and keeps no record of measurements of previous bins, the non-smoothed KF learns from history. However, the B-MMSE filter will use all data that it has available even for estimating the channel coefficients located early in the bin. As a contrast, the non-smoothed KF will generally give poor performance for early channel

coefficients, since it at that point has only historic measurements to base its decision on. Before evaluating the respective channel estimation methods on the six different BAs, we need to define a performance criterion with which the channel estimators can be evaluated.

#### IV. Soft-Input Soft-Output Multi-user detectors

With estimates of the transmission channel having been made available by the channel estimator, the next stage of the iterative receiver structure is to produce likelihood-ratios of the coded data symbols. This operation is performed by the

MUD, which apart from the received signal and channel estimate, uses a-priori information of the transmitted symbols. This information is provided, from the previous iteration, by the channel decoder. The optimal SISO detector is the symbol-wise MAP detector, implemented through the BCJR algorithm

[9]. Unfortunately, the complexity of the MAP detector in the MIMO case is prohibitive in most situations, except for the cases when the number of users  $K$  is small. Therefore, reduced complexity techniques have to be considered for most practical applications. Furthermore, although optimal detection is not generally feasible in practice, it remains important as a benchmark reference, and will therefore be considered in this paper.

##### 4.1 MAP

As stated previously, the optimal MUD is the symbol-wise MAP detector. While the PIC-based algorithms, only make use of the mean values  $\tilde{x}_k[m, s]$ , the symbol-wise MAP detector works with the probability mass function of  $x[m, s]$ , denoted

$P_a(x[m, s])$ . In the case of QPSK transmission, the data vector  $x[m, s]$  contains  $2K$  code bits,  $c_1, \dots, c_{2K}$ . The MAP detector computes the marginal mass functions, represented by LLR values, for these  $2K$  bits:

$$\Lambda(c_q) = \log \left( \frac{\sum_{\mathbf{x}:c_q=1} \exp\left(-\frac{|r[m,s]-H[m]\mathbf{x}|^2}{\sigma_w^2}\right) P_a(\mathbf{x})}{\sum_{\mathbf{x}:c_q=0} \exp\left(-\frac{|r[m,s]-H[m]\mathbf{x}|^2}{\sigma_w^2}\right) P_a(\mathbf{x})} \right), \quad q = 1 \dots 2K.$$

As was discussed above, the complexity of the symbol-wise MAP detector may in many cases be prohibitively large, showing the demand for low complexity schemes.

##### 4.2 PIC based detectors

A popular low-complexity approach is to make use of interference cancellation. Though simple in their implementation, PIC based detectors have shown good performance [9,10,11]. The interference cancellation is operating separately on each subcarrier and OFDM symbol, and makes use of the most recent channel estimate  $\hat{H}[m]$ , and soft symbol estimates  $\tilde{x}_k[m, s]$  from the SISO decoders. Following the notation in (1), the interference cancelled output for user  $k$  is given by

$$\tilde{r}_k[m, s] = r[m, s] - \hat{H}[m] \tilde{\mathbf{x}}_{\neq k}[m, s],$$

where  $\tilde{\mathbf{x}}_{\neq k}[m, s]$  is equal to  $\tilde{\mathbf{x}}[m, s]$ , except for element  $k$ , which is set to zero. A filtering of the signal  $\tilde{r}_k[m, s]$  is then applied to produce an estimate of the transmitted symbol  $\hat{x}_k[m, s]$ . A mapping to LLR values then follows. The first algorithm, which will be referred to as PIC-MF, applies a matched filter (MF) to the interference canceled output, *i.e.*

$$\hat{x}_k[m, s] = \frac{\hat{h}_{k,:}^H[m]}{\left\| \hat{h}_{k,:}[m] \right\|^2} \tilde{r}_k[m, s],$$

where  $\hat{h}_{k,:}[m]$  is an estimate of the channel between user  $k$  and the base-station. In case of QPSK, the complex valued LLRs (with one symbol per complex dimension) are produced as is the variance of the residual interference plus noise for user  $k$ .

The drawback of PIC-MF is that the noise and residual interference is not taken into account when performing user separation. To alleviate this problem,

an MMSE filter can be applied instead of the MF. The resulting algorithm will be referred to as PIC-MMSE. An appropriate MMSE filter can be shown as

$$\Lambda_k[m, s] = \frac{2 \|\hat{h}_{k,:}[m]\|^2}{\sigma_k^2} \hat{x}_k[m, s].$$

where

$$\sigma_k^2 = \sigma_w^2 + \sum_{j \neq k} \left| \hat{h}_{k,:}[m] \hat{h}_{j,:}[m] \right|^2 (1 - |\tilde{x}_j[m, s]|^2).$$

$$\hat{x}_k[m, s] = \frac{i_K^{(k)T} \left( \hat{H}^H[m] \hat{H}[m] + \sigma_w^2 V_{(k)}^{-1}[m, s] \right)^{-1} \hat{H}^H[m]}{i_K^{(k)T} \left( \hat{H}^H[m] \hat{H}[m] + \sigma_w^2 V_{(k)}^{-1}[m, s] \right)^{-1} \hat{H}^H[m] \hat{h}_{k,:}[m]} \tilde{r}_k[m, s],$$

where  $i(k)$  is the  $k$ th column of  $I_K$ , and  $V(k)[m, s] = \text{diag}(d_{k,1}[m, s], \dots, d_{k,K}[m, s])$  is a diagonal matrix with elements

$$d_{k,k'}[m, s] = \begin{cases} 1 - |\tilde{x}_{k'}[m, s]|^2 & , k' \neq k \\ 1 & , k' = k. \end{cases}$$

The output of the MMSE filter can be modeled as

$$\hat{x}_k[m, s] = x_k[m, s] + v_k[m, s], \text{ with } v_k[m, s] \sim \mathcal{CN}(0, \eta_k^2),$$

where

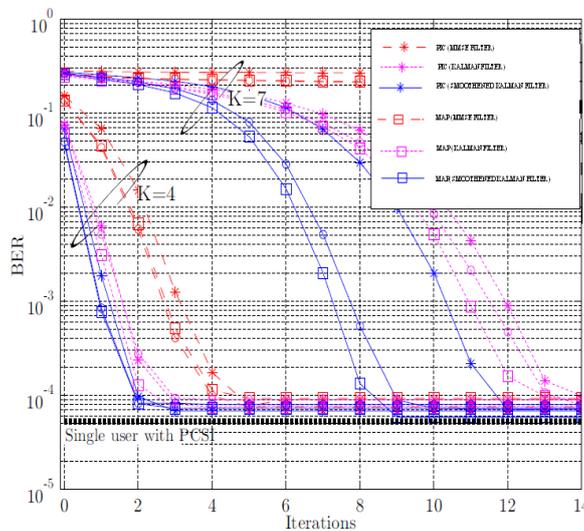
$$\eta_k^2[m, s] = \left( i_K^{(k)T} \left( H^H[m] H[m] + \sigma_w^2 V_{(k)}^{-1}[m, s] \right)^{-1} H^H[m] h_{k,:}[m] \right)^{-1} - 1.$$

The complex LLR output is then produced as

$$\Lambda_k[m, s] = \frac{2}{\eta_k^2[m, s]} \hat{x}_k[m, s]$$

### V. Simulation Results

In order to investigate the receiver performance under the use of the different algorithms, computer simulations were performed. In the simulations, each user transmits  $S = 20$  OFDM symbols, each with  $M = 256$  subcarriers. The channel estimation inside the iterative receiver, simulations were performed for a system with  $N = 4$  receive antennas,  $K = 4$  and  $7$  users at an  $E_b/N_o = 10$ db



	PIC (MMSE FILTER)
	PIC (KALMAN FILTER)
	PIC (SMOOTHENED KALMAN FILTER)
	MAP (MMSE FILTER)
	MAP (KALMAN FILTER)
	MAP(SMOOTHENED KALMAN FILTER)

Figure 3: The BER convergence for  $N = 4$  receive antennas,  $K = 4$  and 7 users, at an  $E_b/N_0 = 10\text{dB}$ .

## VI. Conclusion

In this paper, we have studied the trade-off between BER and no of iterations for uplink receivers in a packet based multi-user MIMO-OFDM system. The considered iterative receivers contained three main components; a MUD, a channel estimator and a convolutional decoder. The fastest convergence is achieved using the MAP based multi user detector with smoothed kalman filter channel estimation while the slowest is obtained if using the PIC based detector with MMSE channel estimation. The results show that low-complexity algorithms provide the best trade-off, even though more receiver iterations are needed to reach a desired performance.

## References

- [1] G. Foschini and M. Gans, "On limits of wireless communications in a fading environment when using multiple antennas," *Wireless Personal Communications*, vol. 6, pp. 311–335, Mar. 1998.
- [2] A. Scaglione, P. Stoica, S. Barbarossa, G. B. Giannakis, and H. Sampath, "Optimal designs for space-time linear precoders and decoders," *IEEE Trans. on Signal Processing*, vol. 50, no. 5, pp. 1051–1064, May 2002.
- [3] A. Bourdoux and N. Khaled, "Joint TX-RX optimisation for MIMOSDM based on a null-space constraint," in *Proc. of IEEE VTC*, Vancouver, Canada, Sept. 2002, pp. 171–174.
- [4] A. M. Khachan, A. J. Tenenbaum, and R. S. Adve, "Linear processing for the downlink in multiuser MIMO systems with multiple data streams," in *Proc. of IEEE ICC - To appear*, June 2006.
- [5] M. Schubert, S. Shi, E. A. Jorswieck, and H. Boche, "Downlink Sum-MSE transceiver optimization for linear multi-user MIMO systems," in *Proc. 39 Asilomar Conf. on Signals, Systems and Computers*, Oct. 2005, pp. 1424–1428.
- [6] A. R. S. Bahai, B. R. Saltzberg, and M. Ergen, *Multi-carrier digital communications theory and applications of OFDM*. New York, Springer, 2004.
- [7] H. Bolcskei, D. Gesbert, and A. Paulraj, "On the capacity of OFDM based spatial multiplexing systems," *IEEE Trans. on Communications*, vol. 50, no. 2, pp. 225–234, Feb. 2002.
- [8] J. Ylioinas, M. Raghavendra, and M. Juntti, "Avoiding matrix inversion in DD SAGE channel estimation in MIMO-OFDM with M-QAM," in *Proc. IEEE Vehicular Technology Conference 2009 fall*, pp. 1–5, Sept. 2009.
- [9] L. Bahl, J. Cocke, F. Jelinek, and J. Raviv, "Optimal decoding of linear codes for minimizing symbol error rate," *IEEE Transactions on Information Theory*, vol. 20, pp. 284–287, Mar. 1974.
- [9] T. Zemen, C. Mecklenbrauker, J. Wehinger, and R. Muller, "Iterative joint time-variant channel estimation and multi-user detection for MC-CDMA," *IEEE Transactions on Wireless Communications*, vol. 5, pp. 1469–1478, Jun. 2006.
- [10] B. Hu, I. Land, L. Rasmussen, R. Piton, and B. Fleury, "A divergence minimization approach to joint multiuser decoding for coded cdma," *IEEE Journal on Selected Areas in Communications*, vol. 26, pp. 432–445, Apr. 2008.
- [11] H. Lee and I. Lee, B.; Lee, "Iterative detection and decoding with an improved V-BLAST for MIMO-OFDM systems," *IEEE Journal on Selected Areas in Communications*, vol. 24, no. 3, pp. 504–513, 2006.