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Trajectory Planning for Vehicle Lane Changing on Circular Road in Automated Highway System

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Abstract :*In this paper, we study the virtual trajectory planning method for lane changing of vehicle in automated highway system, derivate the trajectory model for lane changing on circular road by using odd-order polynomial constraints. Based on the trajectory model for lane changing on straight road, the motion for lane changing of vehicle on circular road can be decomposed into the linear centripetal motion and the circular motion around the instantaneous centre of the circle road. Assuming that the centripetal motion displacement and rotational angular displacement meet the odd-order polynomial constraints, from the restrictive conditions for lane changing, such as time requirements, position limitation and desired state of vehicle at start time and end time of the lane changing behavior, the mathematical model of virtual trajectory for lane changing is designed. Compared with the existing trajectory planning method for lane changing on circular road, the road section length constraint is increased in this paper, consequently the trajectory model for lane changing is more general. The simulation results verify the feasibility of trajectory planning method for lane changing proposed in this paper on circular road.*

Keywords -*automated highway system, lane changing, trajectory planning, odd-order polynomial constraints*

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I. Introduction

Automated Highway System (AHS) is an important branch of Intelligent Transportation System, in which the car and the road was comprehensively considered. The research objective of AHS is to make the highway systems automatic, safe and efficient based on the research and development of driverless technology [1]. Automatic control of vehicles is the key to achieve the unmanned driving, generally, it is divided into two areas to study, the longitudinal control and the lateral control. Automatic vehicle lane changing is a research content belongs to lateral control, which means a behavior to control vehicle to change its lane along the changing lane trajectory from one lane into another lane automatically [2, 3]. There are a variety of methods to plan the lane changing trajectory, such as assuming the lateral and longitudinal position meet the boundary constraints, or the expected lateral acceleration of vehicle is assumed to be two positive and negative trapezoids that have the same size [4]. By using the former method, the curvature of lane changing trajectory changes suddenly, this would cause running instability of vehicle, the latter method don’t have such flaws, but the desired lateral acceleration should be chosen based on the power performance of the vehicle, and the application is not flexible enough to meet the requirement of the complex changing lane scene. Montes et al. [5] planed the lane changing trajectory by using Bezier curve, but it is difficult to select the control point when there are some obstacles on road. Recently, Li et al. [6] studied a fast trajectory planning algorithm for lane changing based on polynomial constraints, which can improve the timeliness of trajectory planning. The above strategies are based on geometric method, and the trajectory model for lane changing is acquired by the formula derivation, different from the above method, we can also use the test method, such as established the trajectory empirical model for lane changing by extracting data information of vehicle position and states during the realistic lane changing process [7]. Yang [8] corrected the key parameters of the empirical model through the test data for lane changing. Ranjeet and Shekhar [9] studied the effectiveness of lane change trajectory prediction on the basis of past positions, and modeled the lane change trajectory of vehicle as back propagation neural network. Hou et al. [10] developed a fuzzy logic-based lane changing model for mandatory lane changes at lane drops, and used genetic algorithm to optimize the widths of membership functions. Liu et al. [11] investigated drivers’ lane-changing behavior under different information feedback strategies, made a microscopic traffic simulation based on the cellular automaton model on the typical freeway with a regular lane and a high-occupancy one. Rahman et al. [12] conducted a detailed review and systematic comparison of existing microscopic lane-changing models, discussed also the possible measures to improve the accuracy and reliability of lane-changing models.

At present, the research result for lane changing on curve section of road is very little. Hatipoglu et al [13] researched the control method for lane changing based on the yaw rate tracking, by using the curvature of the circular road, designed the mathematical model of desired yaw angle and yaw rate for lane changing, but it ignored the curvature difference between the inside lane and the outside lane of the road, so the vehicle position can't be guaranteed at the centerline of the target lane at the end of the lane changing process, namely lane changing trajectory exists a deviation. In reference [14], authors considered the impact of lateral movement of vehicle on curvature of lane changing trajectory, without assuming that the starting lane and the target lane have the same curvature, the trajectory model planed for lane changing did not exist deviation. In above references [13] and [14], the length constraint of the road section corresponds to the lane changing process had not been considered, but in fact lane changing behavior usually occurs in the case when there is an obstacle in front of the current vehicle position or the spacing between vehicles is limited, it is needed to consider the driving distance during the process of lane changing, therefore, the road section length constraints for lane changing behavior should be considered when planning the lane changing trajectory, such as in reference [15], authors established a minimum safety distance model for lane changing by applying elliptic model, on which the changing lane model considering spatial constraints was designed based, but it did not consider the curvature of road.

Based on references [13] and [14], assuming that the road is circular, this paper continues to research trajectory planning method for lane changing, not only consider the lateral distance between lanes of circular road, but the longitudinal length constraints for road section is also included, make a further promotion for existing research results.

II. Lane Changing Trajectory For Straight Road

2.1 Constraint conditions

Assume that the vehicle running on a straight road changes lanes from the lane 1 into the lane 2, as shown in Figure 1. The road shown in the figure contains two lanes that have the same width, l_0 is the boundary of the two lanes and is also the centerline of the road; l_1 and l_2 are the centerlines of lane 1 and lane 2 respectively; S and F denote the vehicle position at the start time and end time respectively, M is the centroid of the vehicle. Suppose the lane pacing, namely the distance between two lane centerlines is d, the length of road section is L.

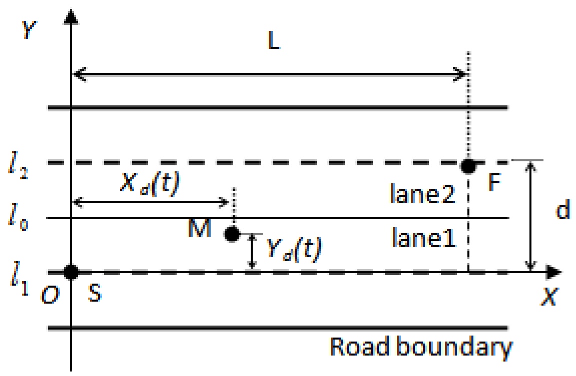

Figure 1. Schematic diagram of trajectory planning for lane changing on straight road

Take S as origin point to establish a Cartesian coordinate system XOY, in which X-axis is along the l_1, and take the vehicle traveling direction as positive; Y-axis is perpendicular to X-axis, take the direction that lane 1 point to lane 2 as positive. The displacement of the vehicle centroid M moved along the X-axis direction is $X_d(t)$, the displacement moved along the Y-axis direction is $Y_d(t)$, where $t \in [t_{on}, t_{off}]$, and t_{on}, t_{off} represent the start time and the end time of the lane changing process, respectively. According to Figure 1, the position coordinates of the vehicle at start time and the end time should meet the following form.

$$\begin{cases} X_d(t_{on}) = 0 \\ Y_d(t_{on}) = 0 \\ X_d(t_{off}) = \mathrm{L} \\ Y_d(t_{off}) = \mathrm{d} \end{cases} \tag{1}$$

2.2 Trajectory model

According to the reference [6], based on a quintic polynomial, the expected lane changing trajectory model can be expressed as:

$$\begin{cases} X_d(t) = a_5t^5 + a_4t^4 + a_3t^3 + a_2t^2 + a_1t + a_0 \\ Y_d(t) = b_5t^5 + b_4t^4 + b_3t^3 + b_2t^2 + b_1t + b_0 \end{cases} \tag{2}$$

where a_i and b_i are undetermined coefficient, $i = 0,1,\cdots,5$. For convenience, they can be written in vector forms as $\mathrm{A} = [a_0, a_1, a_2, a_3, a_4, a_5]^{\mathrm{T}}$ and $\mathrm{B} = [b_0, b_1, b_2, b_3, b_4, b_5]^{\mathrm{T}}$. If a vehicle travel along this trajectory, its velocity and acceleration would be expected as following:

$$\begin{cases} \dot{X}_d(t) = 5a_5t^4 + 4a_4t^3 + 3a_3t^2 + 2a_2t + a_1 \\ \ddot{X}_d(t) = 20a_5t^3 + 12a_4t^2 + 6a_3t^2 + 2a_2 \\ \dot{Y}_d(t) = 5b_5t^4 + 4b_4t^3 + 3b_3t^2 + 2b_2t + b_1 \\ \ddot{Y}_d(t) = 20b_5t^3 + 12b_4t^2 + 6b_3t + 2b_2 \end{cases} \tag{3}$$

Assume that the speed and acceleration at the start time and the end time are all known as constraint condition, take their values into equation (2) and (3), by solving the following linear equations, the undetermined coefficient of trajectory model (2) can be acquired.

$$\begin{cases} [0, \dot{X}_d(t_{on}), \ddot{X}_d(t_{on}), \mathrm{L}, \dot{X}_d(t_{off}), \ddot{X}_d(t_{off})]^{\mathrm{T}} = \mathrm{TA} \\ [0, \dot{Y}_d(t_{on}), \ddot{Y}_d(t_{on}), \mathrm{d}, \dot{Y}_d(t_{off}), \ddot{Y}_d(t_{off})]^{\mathrm{T}} = \mathrm{TB} \end{cases} \tag{4}$$

where

$$\mathrm{T} = \begin{bmatrix} \mathrm{t_{on}^5} & \mathrm{t_{on}^4} & \mathrm{t_{on}^3} & \mathrm{t_{on}^2} & \mathrm{t_{on}} & 1 \\ 5\mathrm{t_{on}^4} & 4\mathrm{t_{on}^3} & 3\mathrm{t_{on}^2} & 2\mathrm{t_{on}} & 1 & 0 \\ 20\mathrm{t_{on}^3} & 12\mathrm{t_{on}^2} & 6\mathrm{t_{on}} & 2 & 0 & 0 \\ \mathrm{t_{off}^5} & \mathrm{t_{off}^4} & \mathrm{t_{off}^3} & \mathrm{t_{off}^2} & \mathrm{t_{off}} & 1 \\ 5\mathrm{t_{off}^4} & 4\mathrm{t_{off}^3} & 3\mathrm{t_{off}^2} & 2\mathrm{t_{off}} & 1 & 0 \\ 20\mathrm{t_{off}^3} & 12\mathrm{t_{off}^2} & 6\mathrm{t_{off}} & 2 & 0 & 0 \end{bmatrix}$$

When change lanes, the direction of longitudinal velocity should be consistent with the tangent direction of lane changing trajectory, so the expected yaw angle ψ_d of vehicle should be equal to the angle between tangent direction of trajectory and the direction of lane 1 centerline, namely

$$\psi_d(t) = \mathrm{atan}\frac{\dot{Y}_d(t)}{\dot{X}_d(t)} \tag{5}$$

Desired vehicle speed v_d should be equal to the change rate of trajectory displacement along its tangent direction, namely

$$v_d(t) = \sqrt{\dot{X}_d^2(t) + \dot{Y}_d^2(t)} \tag{6}$$

III. Lane Changing Trajectory For Circular Road

3.1 Constraint conditions

In fact, road sections are usually not linear, it is necessary to study how to change lanes on the curved road. In the following, we should extend the trajectory model for lane changing on straight road to circular road based on polynomial constraint. It is different from Figure1, the centerlines l_0, l_1 and l_2 of road and its two lanes are changed from straight line into a arc line, as shown in Figure 2. Assuming l_0, l_1 and l_2 have the same center of curvature O_r, namely the instantaneous center of the road, let the radius of curvature of l_0 as R, the difference between the radius of curvature of l_1 and l_2 is lane spacing d, so the radius of curvature of l_1 and l_2 are R+d/2 and R-d/2, respectively. Let S′ and S″ as the intersection points of l_0, l_2 with the connect line between start point S and the instantaneous center of road O_r, F′ and F″ as the points of intersection between l_1, l_0 and the extension of line connecting O_r and the end point F. Assuming the arc length corresponding a lane changing process, namely the length of road centerline between the point S′ and point F′ is L, then the corresponding circumference angle $\alpha = \mathrm{L/R}$, and arc length between points S″ and F is $(\mathrm{R} - \mathrm{d}/2)\alpha$, arc length between points S and F″ is $(\mathrm{R} + \mathrm{d}/2)\alpha$. When a vehicle change lanes on a circular road, the motion of vehicle centroid M can be decomposed into the linear centripetal motion from point S to instantaneous center O_r and the circular motion around O_r.

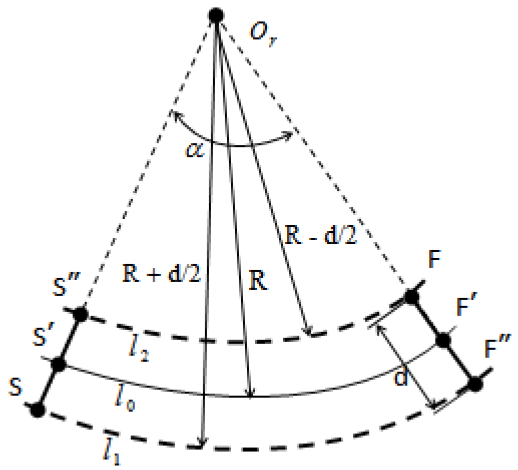

Figure 2. Model of a segment of circular road

Take the start point S as origin point to establish coordinate system, X-axis is along the tangential direction of arc l_1 at point S, and Y-axis towards the instantaneous centre of road, as shown in Figure 3, where $X_c(t)$ and $Y_c(t)$ denote the position coordinates of vehicle centroid M at time t; $r(t)$ and $\theta(t)$ denote the instantaneous radius of changing lane trajectory and rotate angular displacement of vehicles around the instantaneous center, respectively.

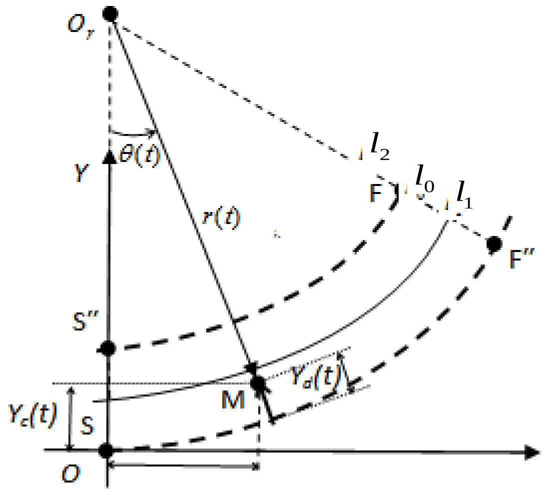

Figure 3. Schematic diagram of trajectory planning for lane changing on circular road

Assuming that the lane spacing, time requirement, length of road section traveled during the lane changing process, and the constraints on vehicle states at the start point and end point are the same as above changing lane scene on a straight road, based on the coordinate system shown in Figure 3, the constraint conditions to be satisfied of the vehicle displacement, velocity and acceleration at the start time and end time for lane changing on circular road can be determined as

$$\begin{cases} X_c(t_{on}) = 0 \\ Y_c(t_{on}) = 0 \\ X_c(t_{off}) = (\mathrm{R} - \mathrm{d}/2)\sin\alpha \\ Y_c(t_{off}) = \mathrm{R} + \mathrm{d}/2 - (\mathrm{R} - \mathrm{d}/2)\cos\alpha \end{cases} \tag{7}$$

$$\begin{cases} \dot{X}_c(t_{on}) = \dot{X}_d(t_{on}) \\ \dot{Y}_c(t_{on}) = \dot{Y}_d(t_{on}) \\ \dot{X}_c(t_{off}) = \dot{X}_d(t_{off})\cos\alpha - \dot{Y}_d(t_{off})\sin\alpha \\ \dot{Y}_c(t_{off}) = \dot{X}_d(t_{off})\sin\alpha + \dot{Y}_d(t_{off})\cos\alpha \end{cases} \tag{8}$$

$$\begin{cases} \ddot{X}_c(t_{on}) = \ddot{X}_d(t_{on}) \\ \ddot{Y}_c(t_{on}) = \ddot{Y}_d(t_{on}) + \dot{X}_d^2(t_{on})/(\mathrm{R} + \mathrm{d}/2) \\ \ddot{X}_c(t_{off}) = \ddot{X}_d(t_{off})\cos\alpha - [\ddot{Y}_d(t_{off}) + \dot{X}_d^2(t_{off})/(\mathrm{R} - \mathrm{d}/2)]\sin\alpha \\ \ddot{Y}_c(t_{off}) = \ddot{X}_d(t_{off})\sin\alpha + [\ddot{Y}_d(t_{off}) + \dot{X}_d^2(t_{off})/(\mathrm{R} - \mathrm{d}/2)]\cos\alpha \end{cases} \tag{9}$$

In above equation (9), $\dot{X}_d^2(t_{on})/(\mathrm{R} + \mathrm{d}/2)$ and $\dot{X}_d^2(t_{off})/(\mathrm{R} - \mathrm{d}/2)$ denote the centripetal acceleration caused by desired velocity along the tangential direction of the lane changing trajectory at the start time and end time.

3.2 Trajectory model

Instantaneous radius of lane changing trajectory $r(t)$ should meet the boundary conditions $r(t_{on}) = \mathrm{R} + \mathrm{d}/2$ and $r(t_{off}) = \mathrm{R} - \mathrm{d}/2$. Using the trajectory planning results for lane changing of vehicle on straight road, during a lane changing process, the linear motion displacement of the vehicle centroid M to O_r is just as the same as $Y_d(t)$ at the time t , so it is reasonable to design $r(t)$ as the fowling form.

$$r(t) = \mathrm{R} + \mathrm{d}/2 - Y_d(t) \tag{10}$$

According to Figure 3, along the directions of coordinate axes, the vehicle displacement, velocity and acceleration model for the lane changing behavior on circular road are deduced as follows:

$$\begin{cases} X_c(t) = [\mathrm{R} + \mathrm{d}/2 - Y_d(t)]\sin\theta \\ Y_c(t) = \mathrm{R} + \mathrm{d}/2 - [\mathrm{R} + \mathrm{d}/2 - Y_d(t)]\cos\theta \end{cases} \tag{11}$$

$$\begin{cases} \dot{X}_c(t) = [\mathrm{R} + \mathrm{d}/2 - Y_d(t)]\dot{\theta}(t)\cos\theta - \dot{Y}_d(t)\sin\theta \\ \dot{Y}_c(t) = [\mathrm{R} + \mathrm{d}/2 - Y_d(t)]\dot{\theta}(t)\sin\theta + \dot{Y}_d(t)\cos\theta \end{cases} \tag{12}$$

$$\begin{cases} \ddot{X}_c(t) = [\mathrm{R} + \mathrm{d}/2 - Y_d(t)][\ddot{\theta}(t)\cos\theta - \dot{\theta}^2(t)\sin\theta] \\ \qquad - \ddot{Y}_d(t)\sin\theta - 2\dot{Y}_d(t)\dot{\theta}(t)\cos\theta \\ \ddot{Y}_c(t) = [\mathrm{R} + \mathrm{d}/2 - Y_d(t)][\ddot{\theta}(t)\sin\theta + \dot{\theta}^2(t)\cos\theta] \\ \qquad + \ddot{Y}_d(t)\cos\theta - 2\dot{Y}_d(t)\dot{\theta}(t)\sin\theta \end{cases} \tag{13}$$

Accordingly, the desired velocity along the tangential directions of lane changing trajectory can be calculated as the following form.

$$v_c(t) = \sqrt{\dot{X}_c^2(t) + \dot{Y}_c^2(t)} \tag{14}$$

The desired yaw angle and yaw rate are deduced as

$$\psi_c(t) = \arctan\frac{\dot{Y}_c(t)}{\dot{X}_c(t)} \tag{15}$$

$$\dot{\psi}_c(t) = \frac{\ddot{Y}_c(t)\dot{X}_c(t) - \dot{Y}_c(t)\ddot{X}_c(t)}{\dot{X}_c^2(t) + \dot{Y}_c^2(t)} \tag{16}$$

Due to the rotate angular displacement $\theta(t)$ desired of vehicles around the instantaneous center O_r should meet the conditions such as continuity, smoothness and monotonic, in this paper, we assume it meets the quintic polynomial constraint, namely:

$$\theta(t) = c_5t^5 + c_4t^4 + c_3t^3 + c_2t^2 + c_1t + c_0 \tag{17}$$

Boundary conditions of $\theta(t)$ and its rate of change met can be determined bases on the vehicle's states at the start time and the end time of the lane changing process, based on the constraints conditions for lane changing behavior, such as lane spacing and location requirement of road section, the multinomial coefficient of $\theta(t)$ can be determined from the boundary conditions.

According to formula (7) and (11), the angular displacement constraints that $\theta(t)$ met at the start and end time of lane changing can be determined as

$$\begin{cases} \theta(t_{on}) = 0 \\ \theta(t_{off}) = \alpha \end{cases} \tag{18}$$

According to formula (8) and (12), the angular velocity constraints that $\dot{\theta}(t)$ met at the start and end time of lane changing can be determined as

$$\begin{cases} \dot{\theta}(t_{on}) = \dot{X}_d(t_{on})/(\mathrm{R}+\mathrm{d}/2) \\ \dot{\theta}(t_{off}) = \dot{X}_d(t_{off})/(\mathrm{R}-\mathrm{d}/2) \end{cases} \tag{19}$$

Furthermore, according to formula (9) and (13), the angular acceleration constraints that $\ddot{\theta}(t)$ met at the start and end time of lane changing can be determined as

$$\begin{cases} \ddot{\theta}(t_{on}) = [(\mathrm{R}+\mathrm{d}/2)\ddot{X}_d(t_{on}) + 2\dot{X}_d(t_{on})\dot{Y}_d(t_{on})]/(\mathrm{R}+\mathrm{d}/2)^2 \\ \ddot{\theta}(t_{off}) = [(\mathrm{R}-\mathrm{d}/2)\ddot{X}_d(t_{off}) + 2\dot{X}_d(t_{off})\dot{Y}_d(t_{off})]/(\mathrm{R}-\mathrm{d}/2)^2 \end{cases} \tag{20}$$

Let vector $\mathrm{C} = [c_0, c_1, c_2, c_3, c_4, c_5]^{\mathrm{T}}$, from the boundary constraints conditions that $\theta(t)$, $\dot{\theta}(t)$ and $\ddot{\theta}(t)$ met, i.e., (18), (19) and (20), by solving the following equation

$$[\theta(t_{on}), \dot{\theta}(t_{on}), \ddot{\theta}(t_{on}), \theta(t_{off}), \dot{\theta}(t_{off}), \ddot{\theta}(t_{off})]^{\mathrm{T}} = \mathrm{TC}$$

the undetermined coefficient involved in the multinomial (17) can be acquired.

3.3 Comparative analysis

According to the method in reference [14], assume that the angular velocity that $r(t)$ rotates around the O_r is $\dot{\theta}(t) = \dfrac{\dot{X}_d(t)}{\mathrm{R}+\mathrm{d}/2-Y_d(t)}$, from formula (12) , we have

$$\begin{cases} \dot{X}_c(t) = \dot{X}_d(t)\cos\theta - \dot{Y}_d(t)\sin\theta \\ \dot{Y}_c(t) = \dot{X}_d(t)\sin\theta + \dot{Y}_d(t)\cos\theta \end{cases} \tag{21}$$

From (21), the velocity at the start time and the end time of lane changing meet the requirement of formula (8), and the velocity at time t can be get according to formula (14) and (21) as

$$v_c(t) = \sqrt{\dot{X}_d^2(t) + \dot{Y}_d^2(t)} \tag{22}$$

It is the same as the velocity $v_d(t)$ when changing lanes on straight road, so the lengths of lane changing trajectory are also the same with each other. By the formula (15) and (21), the vehicle yaw angle can be obtained as

$$\psi_c(t) = \arctan\frac{\dot{X}_d(t)\sin\theta + \dot{Y}_d(t)\cos\theta}{\dot{X}_d(t)\cos\theta - \dot{Y}_d(t)\sin\theta} = \arctan\frac{\dot{Y}_d(t)}{\dot{X}_d(t)} + \theta(t) \tag{23}$$

From (23), It is obviously, that $\psi_c(t)$ can be decomposed into the sum of yaw angle $\psi_d(t)$ expected to change lanes on straight road and the angular displacement $\theta(t)$ which rotate around the instantaneous center of the road. In this case angular displacement can be calculated as

$$\theta(t) = \int_{t_{on}}^{t} \frac{\dot{X}_d(\tau)}{\mathrm{R}+\mathrm{d}/2-Y_d(\tau)} d\tau = \int_{t_{on}}^{t} \frac{\dot{X}_d(\tau)}{\mathrm{R}} d\tau + \int_{t_{on}}^{t} \frac{[Y_d(\tau)-\mathrm{d}/2]\dot{X}_d(\tau)}{\mathrm{R}[\mathrm{R}+\mathrm{d}/2-Y_d(\tau)]} d\tau$$

The Angular displacement at the end time of lane changing is easily get as follows

$$\theta(t_{off}) = \alpha + \int_{t_{on}}^{t_{off}} \frac{[Y_d(\tau)-\mathrm{d}/2]\dot{X}_d(\tau)}{\mathrm{R}[\mathrm{R}+\mathrm{d}/2-Y_d(\tau)]} d\tau \tag{24}$$

From (24), it can't be guaranteed that $\theta(t_{off}) = \alpha$, according to the formula (11), there is a deviation between vehicle position and its expected value required by formula (7) at the end time of lane changing. The deviation has something to do with curvature radius of the road, distance between lanes, and the longitudinal velocity at the start time and the end time.

According to the research result of reference [14], centroid position of the vehicle is on the centerline of the target lane at the end time of lane changing, but from the above analysis, it can’t guarantee that the circular length of the road centerline that corresponds with lane changing trajectory is L.

Reference [13] studied the lane changing on a circular road, and the expected yaw velocity model for lane changing was designed, where assume that $\dot{\theta}(t) = \dot{X}_d(t)/\mathrm{R}$, this is independent of the lateral displacement. In this case, angular displacement is calculated as

$$\theta(t) = \int_{t_{on}}^{t} \frac{\dot{X}_d(\tau)}{\mathrm{R}} d\tau = X_d(t)/\mathrm{R}$$

If the lane changing trajectory is planned by applying formula (11), then at the end time, $\theta(t_{off}) = X_d(t_{off})/\mathrm{R} = \mathrm{L/R} = \alpha$, this shows that vehicle position satisfies the requirement of formula (7). According to formula (12), the vehicle velocity should meet the following form.

$$\begin{cases} \dot{X}_c(t) = \dfrac{[\mathrm{R} + \mathrm{d}/2 - Y_d(t)]\dot{X}_d(t)\cos\theta}{\mathrm{R}} - \dot{Y}_d(t)\sin\theta \\ \dot{Y}_c(t) = \dfrac{[\mathrm{R} + \mathrm{d}/2 - Y_d(t)]\dot{X}_d(t)\sin\theta}{\mathrm{R}} + \dot{Y}_d(t)\cos\theta \end{cases} \tag{25}$$

So at the start time and end time of lane changing, the values of vehicle velocity is obtained as

$$\begin{cases} \dot{X}_c(t_{on}) = \dot{X}_d(t_{on})[\mathrm{R} + \mathrm{d}/2]/\mathrm{R} \\ \dot{Y}_c(t_{on}) = \dot{Y}_d(t_{on}) \end{cases}$$

$$\begin{cases} \dot{X}_c(t_{off}) = \dfrac{(\mathrm{R} - \mathrm{d}/2)\dot{X}_d(t_{off})\cos\alpha}{\mathrm{R}} - \dot{Y}_d(t_{off})\sin\alpha \\ \dot{Y}_c(t_{off}) = \dfrac{(\mathrm{R} - \mathrm{d}/2)\dot{X}_d(t_{off})\sin\alpha}{\mathrm{R}} + \dot{Y}_d(t_{off})\cos\alpha \end{cases} \tag{26}$$

From (26), it is easily to verify that the values of vehicle velocity have deviations from the requirement of formula (8).

In reference [13], automated control for Lane changing is achieved bases on yaw angle tracking, and the expected yaw angle is not determined by the formula (25) but using formula (23) directly, so the trajectory deviation for lane changing exist inevitably.

Considering the time requirement and lane spacing, references [13] and [14] planed the lateral movement behavior from the start lane to the target lane based on positive and negative trapezoid acceleration constraint. Because the length of road section during lane changing process was not considered in above references, so the longitudinal movement constraints are not necessary, and the longitudinal velocity can be assumed as a constant, therefore, the trajectory planning results on the straight road can be used directly to calculate the trajectory coordinates and the expected vehicle state when vehicle changes lanes on circular road, not needed to consider whether the length of the road section corresponded with the lane changing trajectory is the same as changing lanes on the straight road. If the length constraint of road section is considered, according to the approach in references [13] and [14], based solely on the desired state of vehicle changing lanes on straight road, it is difficult to find the suitable $\theta(t)$ to make the values of position deviation and velocity deviation are zero simultaneously, differently from references [13] and [14], in this paper, $\theta(t)$ is obtained by inversing the expected model for lane changing based on constraints conditions, the lane changing model deduced here doesn’t exist deviation.

IV. Simulation Results

Assume that curvature radius of the road $\mathrm{R} = 200\mathrm{m}$, the distance between the centerlines of lanes $\mathrm{d} = 3.5\mathrm{m}$, the length of the centerline of the road corresponded with the lane changing process $\mathrm{L} = 80\mathrm{m}$; the start time for lane changing $t_{on} = 0$, the end time $t_{off} = 4s$; the states of vehicle at the start time are $\dot{X}_d(0) = 15\mathrm{m/s}$, $\ddot{X}_d(0) = 5\mathrm{m/s}^2$, $\dot{Y}_d(0) = 0.5\mathrm{m/s}$, $\ddot{Y}_d(0) = 0.2\mathrm{m/s}$, the states of vehicle at the end time are $\dot{X}_d(4) = 25\mathrm{m/s}$, $\ddot{X}_d(4) = 0$, $\dot{Y}_d(4) = 0$, $\ddot{Y}_d(4) = 0$.

4.1 Model for lane changing

Figure 4 shows the trajectory model and the desired states of vehicle for lane changing. Figure 4 (a), (c) and Figure 4(b), (d) denote vehicle velocity and acceleration along the X-axis and Y-axis, respectively; Figure 4 (e) shows the changes of arc’s length along the centerline of the road during the changing lanes process. As shown in the figure, the arc length meets the requirement of the length of the road; Figure 4 (f) shows the trajectory of the position during changing lanes, in which the dotted lines represent centerlines of two lanes; Figure 4 (g) and (h) denote the desired yaw angle and yaw rate of vehicle changing lanes.

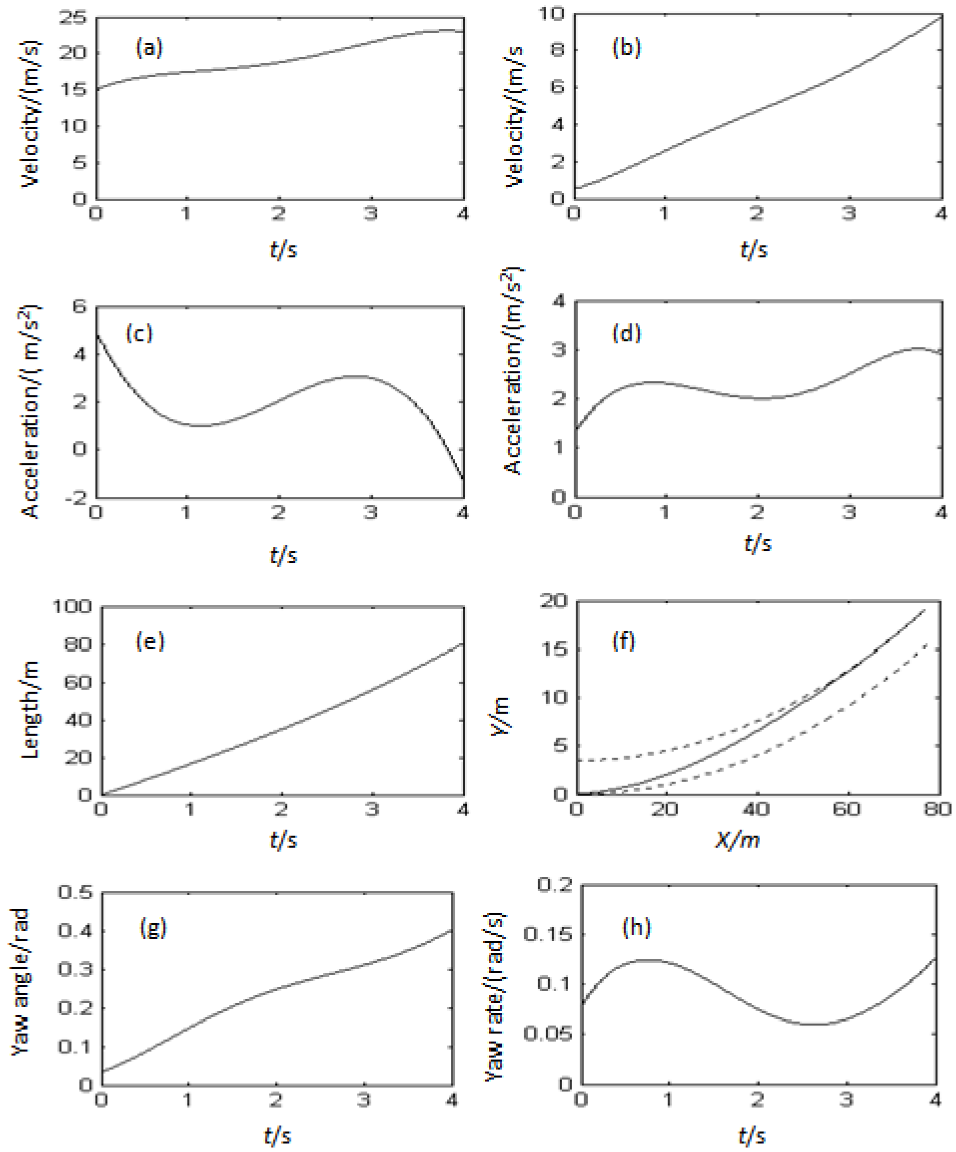

Figure 4. Desired trajectory model and vehicle states for lane changing: (a)Velocity along X-axis; (b)Velocity along Y-axis; (c) Acceleration along X-axis; (d)Acceleration along Y-axis; (e) Length of the road section; (f) Trajectory model; (g)Expected yaw angle; (h)Expected yaw rate

4.2. Model comparison

According to the above analysis, the existing design methods for $\theta(t)$ are based on the lane changing model on straight road, and the changing lane model obtained has a deviation when there is a length constraint of road section for the lane changing behavior. In the following, we examine the deviations of the lanes changing model from the expected value at the end time by the simulation.

Assume that θ_{off} , v_{off} , X_{off} and Y_{off} denote the expected angular displacement, velocity, displacement along X-axis and displacement along Y-axis at the end time of the lane changing process, respectively, from formula (18), (8), (7), we can get

$$\theta_{off} = \alpha = \mathrm{L/R} \tag{27}$$

$$v_{off} = \sqrt{\dot{X}_c^2(t_{off}) + \dot{Y}_c^2(t_{off})} = \sqrt{\dot{X}_d^2(t_{off}) + \dot{Y}_d^2(t_{off})} \tag{28}$$

$$\begin{cases} X_{off} = (\mathrm{R} - \mathrm{d}/2)\sin(\mathrm{L}/\mathrm{R}) \\ Y_{off} = \mathrm{R} + \mathrm{d}/2 - (\mathrm{R} - \mathrm{d}/2)\cos(\mathrm{L}/\mathrm{R}) \end{cases} \tag{29}$$

According to the above constraints conditions on lane changing behavior, by formula (27), (28) and (29), we can obtain that

$$\theta_{off} = 0.4\,,\ v_{off} = 25\ ,\ X_{off} \approx 77.202\,,\ Y_{off} \approx 19.150$$

Figure 5 shows the deviations of the trajectory models from the expected value, where the trajectory models are planned by using three different design methods, respectively, ① refers to use the form of the angular velocity as $\dot{\theta}(t) = \dfrac{\dot{X}_d(t)}{\mathrm{R} + \mathrm{d}/2 - Y_d(t)}$; ② refers to use the form of the angular velocity as $\dot{\theta}(t) = \dfrac{\dot{X}_d(t)}{\mathrm{R}}$; ③ refers to use the polynomial angular displacement model designed in this paper. As is shown in the detail figures (a_2), (c_2) and (d_2), if the method ① is used, at the end time of lane changing behavior, there exist deviations between the angular displacement, position of vehicle and their expected value; as is shown in the detail figure (b_2), if the method ② is used, at the end time of lane changing behavior, there exist deviation between the velocity of vehicle and its expected value; as we can seen from the all enlarged detail figures, the angular displacement, velocity and position of vehicle do not exist deviations at the end time of lane changing behavior if method ③ is used.

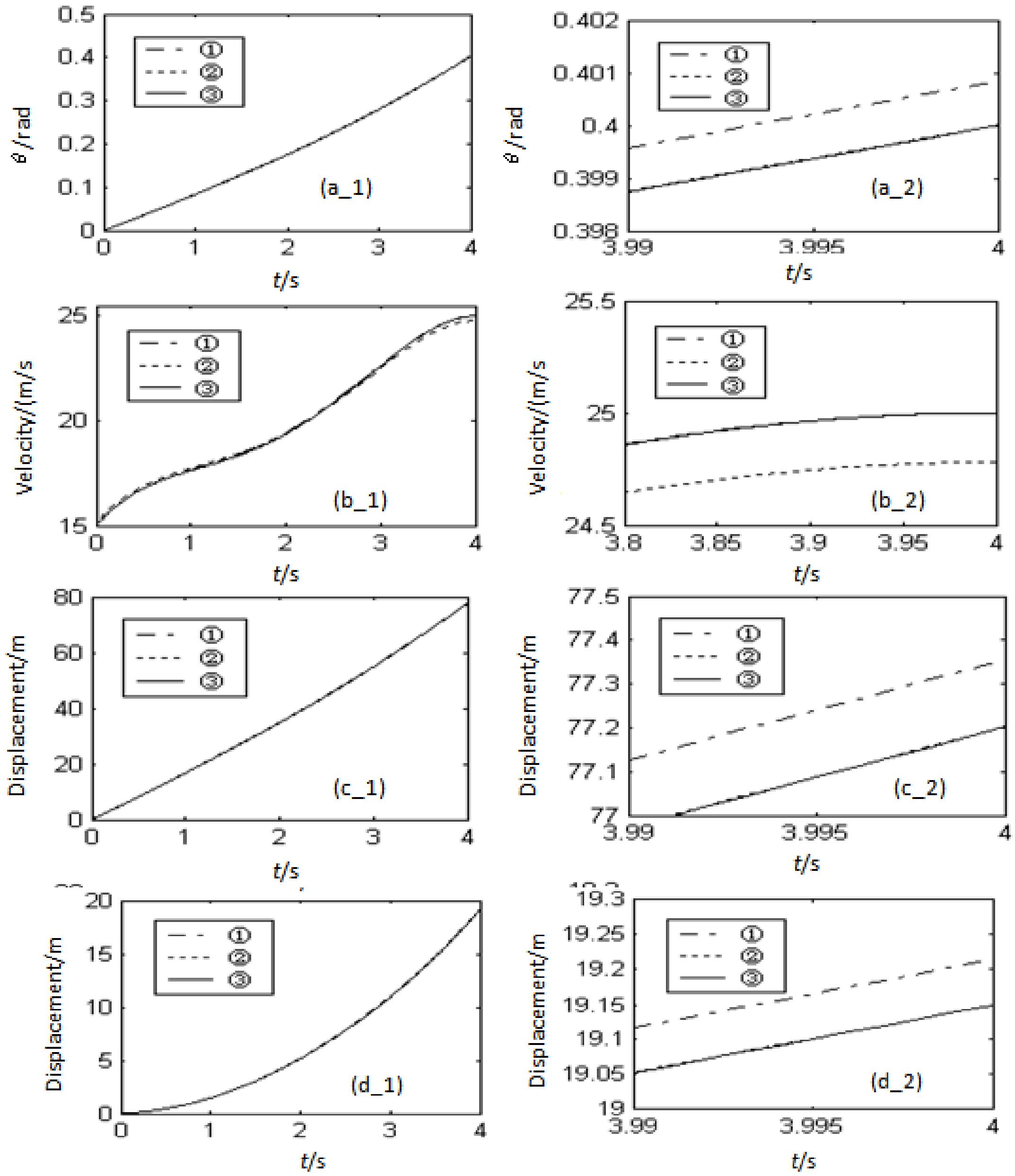

Figure 5. Model comparison for lane changing: (a_1) Angular displacement; (a_2) Partial enlargement of (a_1); (b_1) Velocity of vehicle; (b_2) Partial enlargement of (b_1); (c_1) Displacement along X-axis; (c_2) Partial enlargement of (c_1); (d_1) Displacement along Y-axis; (d_2) Partial enlargement of (d_1)

4.3. Model deviation

Let $e(t)=\sqrt{[X_{off}-X_c(t)]^2+[Y_{off}-Y_c(t)]^2}$ as the position deviation function, which reflecting the case that the vehicle position approach to the expected value at the end stage of lane changing process. So, we have $e(t_{off})=\sqrt{[X_{off}-X_c(t_{off})]^2+[Y_{off}-Y_c(t_{off})]^2}$, it denotes the position deviation at the end time of the lane changing process.

Let $\varepsilon(t)=v_{off}-v_c(t)$ as the velocity deviation function, which reflecting the case that the vehicle velocity approach to the expected value at the end stage of lane changing process. So, we get $\varepsilon(t_{off})=v_{off}-v_c(t_{off})$, it represents the deviation of velocity at the end time of lane changing process.

Change the constraints conditions on lane changing behavior, then the values of X_{off} , Y_{off} and v_{off} is changed correspondingly in the simulation. The impacts on the deviation of the model for lane changing can be reflected from the values of $e(t_{off})$ and $\varepsilon(t_{off})$ changed with the constraints conditions.

Figure 6 represents the simulation results when the curvature radius of road takes different values, shows how the changes of the curvature radius of road influence the position deviation and the velocity deviation. As is shown in Figure 6 (a), if method ① is used, the position deviation at the end time will increase as the decrease of curvature radius, when the curvature radius is 300 m, the deviation is about 0.1m; when the curvature radius is 100 m, and the deviation exceeds 0.3m. Deviation $e(t)$ appears to decrease at first and then to increase, which illustrates that at the end time the angular displacement $\theta(t_{off})>\alpha$. At the time when $\theta(t)=\alpha$, $e(t)$ get the value of minimum, correspondingly, this shows that the vehicle centroid position is right in the normal direction of the end point of, and is the nearest point to the end point.

As shown in Figure 6 (b), if method ② is used, the velocity deviation at the end time will increase as the decrease of curvature radius, when the curvature radius is 300 m, the deviation is less than 0.2m/s; when the curvature radius is 100 m, and the deviation exceeds 0.4m/s. According to the Figure 6 (a) and (b), if method ③ is used, the position deviation and the velocity deviation are all zero at the end time and have nothing to do with the change of curvature radius.

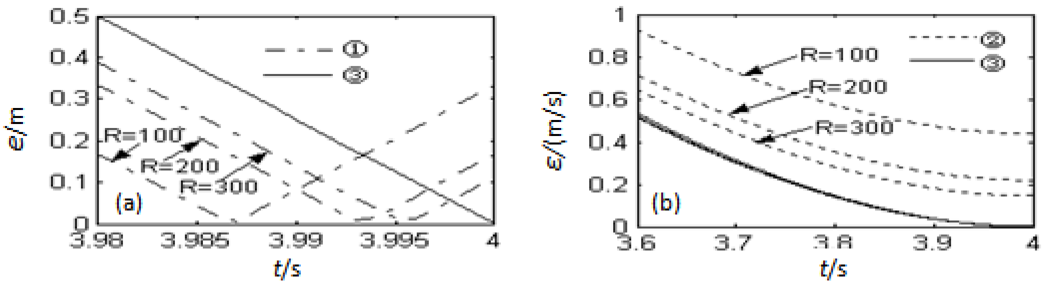

Figure 6. Simulation result with different radii of road: (a) Position deviation; (b) Velocity deviation

Figure 7 represents the simulation results that the spacing between lanes take different values, shows how the change of the distance between lanes influence the position deviation and the velocity deviation.

According to the given constraints to simulation, as is shown in detail Figure 7 (a), if method ① is used, the position deviation at the end time will increase as the increase of the distance between lanes, when the spacing is 3.5m, the deviation is below 0.2m; when the spacing is 10.5m, the deviation exceeds 0.2m. As is shown in detail Figure 7 (b), if method ② is used, the velocity deviation at the end time will increase as the increase of the lane spacing, when the spacing is 3.5 m, the deviation is below 0.3m; when the spacing is 10.5m, the deviation exceeds 0.6m. Figure 7 (a) and (b) show that if the method ③ is used, the position deviation and the velocity deviation are all zero at the end time and have nothing to do with the change of spacing between lanes.

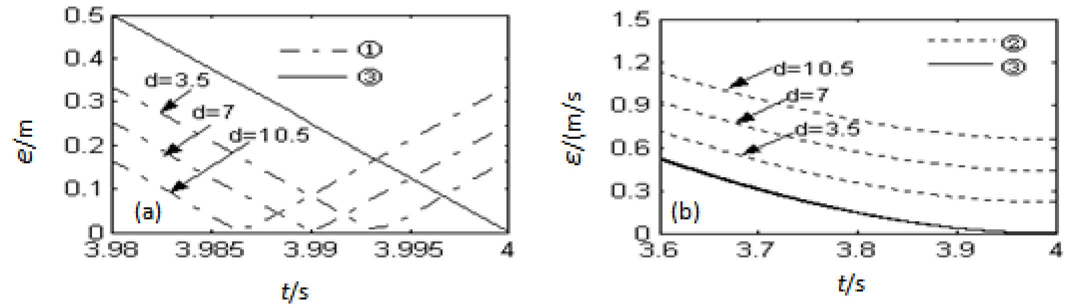

Figure 7. Simulation results with different lane spacing: (a) Position deviation; (b) Velocity deviation

Figure 8 represents the simulation results that the longitudinal velocity takes different values at the start time; it shows how the change of the values of longitudinal velocity at the start time influence the position deviation and the velocity deviation, where $\dot{X}_d(t_{on})$ takes different values from v_1 to v_6, respectively, v_1=28, v_2=29, v_3=30, v_4=32, v_5=33, v_6=34, and the unit is m/s. Assume that at the end time the vehicle velocity $\dot{X}_d(t_{off})$ = 25m/s.

As the Figure 8 (a) shows, using the method ①, the deviation $e(t)$ decreases at first and then increases when $\dot{X}_d(t_{on})$ successively takes values of v_1, v_2, and v_3, this means $\theta(t_{off}) > \alpha$, and the position deviation at the end time decreases as $\dot{X}_d(t_{on})$ increases; the deviation $e(t)$ decreases all the time, when $\dot{X}_d(t_{on})$ successively taking values of v_4, v_5 and v_6, this means $\theta(t_{off}) < \alpha$, and the position deviation increases as $\dot{X}_d(t_{on})$ increases. From above analysis, there exists a critical value between v_3 and v_4, and it can make the value of $\theta(t_{off})$ is α, namely, the deviation is zero at the end time. According to the detail Figure 8 (b), using the method ②, the velocity deviation $\varepsilon(t)$ is a fixed value at the end time of the lane changing process, the reason for this is as follows. Bases on formula (14) and (26), the value of velocity at the end time can be obtained as

$$v_c(t_{off}) = \sqrt{(\frac{\mathrm{R - d/2}}{\mathrm{R}})^2 \dot{X}_d^2(t_{off}) + \dot{Y}_d^2(t_{off})},$$

then by formula (27), its desired value is get as $v_{off} = \sqrt{\dot{X}_d^2(t_{off}) + \dot{Y}_d^2(t_{off})}$, according to the simulation conditions, $\dot{Y}_d^2(t_{off})$ is zero, so we get that the velocity deviation

$$\varepsilon(t_{off}) = v_{off} - v_c(t_{off}) = \frac{\mathrm{d}}{2\mathrm{R}} \dot{X}_d(t_{off}) \approx 0.2188.$$

As is shown in Figure 8 (a) and (b), using the method ③, the position deviation and the velocity deviation are all zero and have nothing to do with the change of constraints to the longitudinal velocity at the start time.

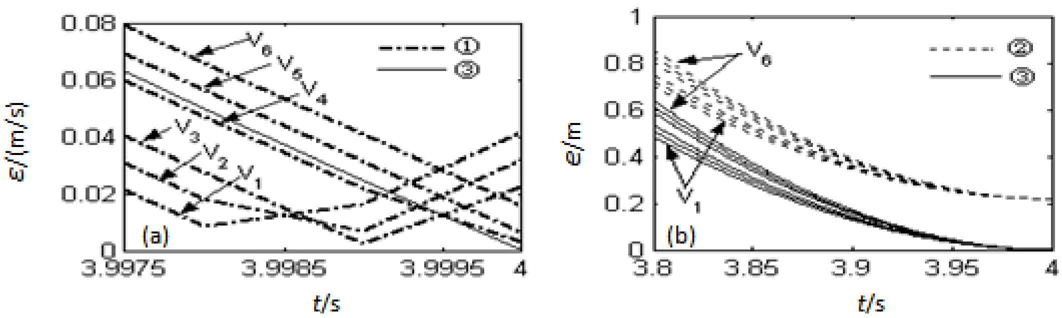

Figure 8. Simulation result with different values of velocity at start time: (a) Position deviation; (b) Velocity deviation

Figure 9 represents the simulation results that the longitudinal velocity takes different values at the end time; it shows how the change of the longitudinal velocity at the end time influence the position deviation and the velocity deviation, where $\dot{X}_d(t_{off})$ takes different values from u_1 to u_6, respectively, u_1 =5, u_2 =10, u_3=15, u_4 =20, u_5 =25, u_6 =30, the unit is m/s. Assume that at the end time, the vehicle velocity $\dot{X}_d(t_{on})$ = 25m/s.

As the Figure 9 (a) shows, using the method ①, the deviation $e(t)$ decreases all the time when $\dot{X}_d(t_{off})$ successively takes values of u_1, u_2 and u_3, this means $\theta(t_{off}) < \alpha$, and the position deviation decreases as $\dot{X}_d(t_{off})$ increases; when $\dot{X}_d(t_{off})$ successively takes values of u_4, u_5 and u_6, the deviation $e(t)$ decreases at first and then increases, this means $\theta(t_{off}) > \alpha$, and the position deviation increases as $\dot{X}_d(t_{off})$ increases. So there exists a critical value between u_3 and u_4, it can make the value of $\theta(t_{off})$ is α, namely, the deviation is zero at the end time. According to the Figure 9 (b), using method ②, the velocity deviation $\varepsilon(t)$ increases as $\dot{X}_d(t_{off})$ increases at the end time of the lane changing process.

As is shown in Figure 9 (c) and (d), using the method ③, the position deviation and the velocity deviation are all zero and have nothing to do with the change of constraints to the longitudinal velocity at the end time.

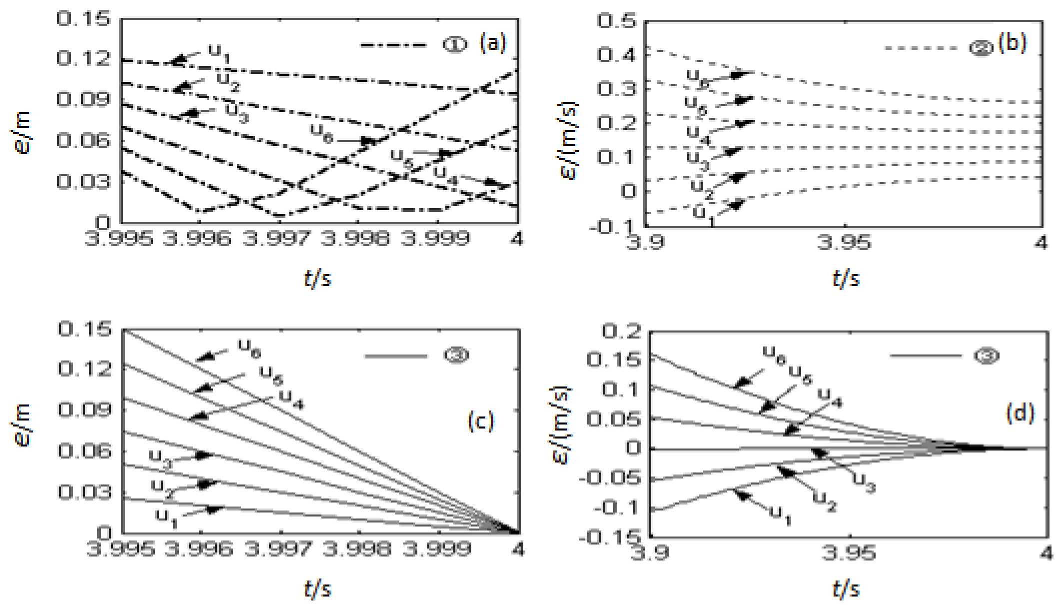

Figure 9. Simulation result with different values of velocity at end time: (a) Position deviation using method①; (b) Position deviation using method②; (c) Position deviation using method③; (d) Velocity deviation using method③

V. Conclusion

(1) If the road section length constraint is considered, by the lane changing model existing on circular road, it can not be guaranteed that the position deviation and velocity deviation are all zero, the deviation has something to do with constraints such as curvature radius of the road, the distance between lanes, longitudinal velocity at the start and end time, etc.

(2) Assuming that the angular displacement rotating around the instantaneous center of the road meet the constraints of the odd-order polynomial, lane changing model designed in this paper can ensure that both position deviation and velocity deviation are all zero at the end time of lane changing process.

(3) Both the lane spacing and the length constraint of the road section corresponds to the lane changing process are considered in this paper, the lane changing trajectory model designed here is more general than the existed research results of lane changing on circular road.

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