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A Systematic Current and Voltage Transfer Function Realizations with a Single Active Element

T. S. RATHORE
Independent Researcher,
G-803, Country Park, Dattapada Road, Borivali (E), Mumbai 400066

Abstract: *A systematic synthesis procedure is given for realizing current transfer functions using only one active device: four terminal floating nullor (FTFN) or current differencing transconductance amplifier (CDTA), and voltage transfer functions using only one active device: operational transresistance amplifier (OTRA), Current conveyor (CC) II, or operational amplifier (OA). Realizations of second order current transfer functions using CDTA require one less passive element for LP and BP filters, and equal number of passive elements for HP and BR filters, and 1 more passive element for AP filter, compared to corresponding FTFN realizations. Realizations of voltage transfer functions using OTRA, CCII or OA use the same number of total passive elements. Illustrative examples are given.*

Keywords: *Current transfer function, voltage transfer function, all pass filters, FTFN, CDTA, CC, OA*

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I. Introduction

Circuit realizations with ***several*** FTFNs [1][2] have been proposed for filters with ***specific orders***. Cicekoglu Biquad [1] requires 3 FTFNs. It realizes second order low-pass/high-pass and band-pass filters simultaneously. The filter proposed by Liu and Lee [2] realizes second order low-pass and band-pass filters with 2 FTFNs.

There are first order all-pass realizations [15][16] where all the passive components are grounded but require ***two active devices***.

Higashimura [3], Abuelma'atti [4], Liu and Hwang [5] proposed filters circuits using a ***single FTFN*** for specific orders; while Rathore and Khot [6] proposed a systematic method for realizing a wide class of functions.

Salama and Soliman [7], Clinc and Cam [8], Cakir *et al* [9], and Rathore and Khot [10] have proposed filters using ***single OTRA***.

Toker *et al* [11] and Rathore [12] have proposed filters using a ***single current differencing amplifier***. Rathore [13] has given a systematic synthesis procedure for realizing filters using a ***single current conveyor II*** which can be extended to the circuit that has the same voltage transfer function using *OA* [14].

Here, we introduce a ***systematic procedure*** for realizing ***current and voltage transfer functions of any order with distinct negative real poles using minimum number of passive elements and only one active device.***

II. Realizations

2.1 Current *transfer functions*

(a) using FTFN

Consider the circuit shown in Fig. 1, where Y_1, Y_2, Y_3 and Y_4 are the 2-terminal RC driving point admittances (DPAs) and FTFN has the following terminal characteristics.

$$\begin{bmatrix} V_x \\ I_x \\ I_y \\ I_z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \pm 1 \end{bmatrix} \begin{bmatrix} V_y \\ V_z \\ V_w \\ I_w \end{bmatrix} \tag{1}$$

The + and - signs of the I_z denote plus- and minus-type FTFNs.
Analysis of the circuit leads to

$$T(s) = \frac{I_o}{I_i} = K\frac{N(s)}{D(s)} = \frac{Y_1 - \mu Y_2}{Y_1 + Y_2} \tag{2}$$

where K is a gain constant, $N^o \leq D^o$ and

$$\mu = Y_3/Y_4. \tag{3}$$

It is interesting to note that scaling of Y_1 and Y_2 by one factor and Y_3 and Y_4 by another factor do not change the $T(s)$.

Let

$$\frac{Y_1 - \mu Y_2}{Y_1 + Y_2} = K\frac{\prod_{j=1}^{m}(s + zj)}{\prod_{i=1}^{n}(s + yi)} = K\frac{\frac{N(s)}{Q(s)}}{\frac{D(s)}{Q(s)}} \tag{4}$$

where $Q(s)$ is defined in (6) and $m \leq n$. From (4), let us identify

$$Y_1 + Y_2 = \frac{D(s)}{Q(s)} \quad \text{and} \quad Y_1 - \mu Y_2 = K\frac{N(s)}{Q(s)}. \tag{5}$$

Since Y_1 + Y_2, being the sum of two RC DPAs, is also an RC DPA, it must have poles and zeros on the negative

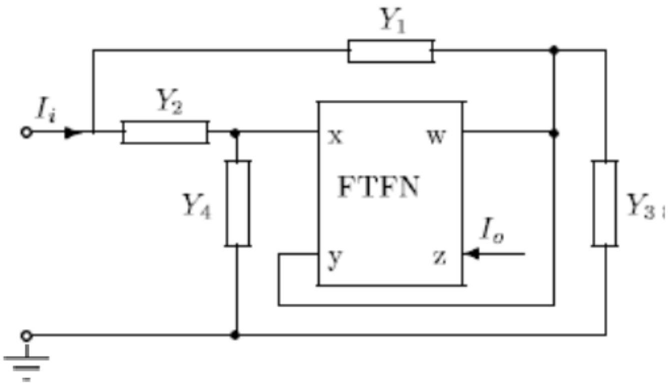


Fig. 1 Circuit for realizing current transfer function using FTFN

real axis, interlaced and the lowest (highest) critical frequency a zero (pole). With these restrictions,

$$Q(s) = \prod_{k=1}^{n-1}(s + p_k) \tag{6}$$

A factor (s + p_n) such that $p_n > y_n$ could be added in $Q(s)$, but the above choice is made to have less number of elements in Y_1 and Y_2. Equation (5) can be expressed as

$$Y_1 + Y_2 = \left[A_\infty s + A_o + \sum_{k=1}^{n-1}\frac{A_k s}{(s + p_k)}\right] \tag{7}$$

and

$$Y_1 - \mu Y_2 = \left[KB_\infty s + KB_o + K\sum_{k=1}^{n-1}\frac{B_k s}{(s + p_k)}\right] \tag{8}$$

where

$$A_\infty = \begin{cases} 0, & m < n \\ \left.\frac{D(s)}{sQ(s)}\right|_{s\to\infty,} & m = n, \end{cases}$$

$$B_\infty = \begin{cases} 0, & m < n \\ \left.\frac{N(s)}{sQ(s)}\right|_{s\to\infty,} & m = n, \end{cases}$$

$$A_o = \frac{D(s)}{Q(s)}\Big|_{s=0}$$

$$B_o = \frac{KN(s)}{Q(s)}\Big|_{s=0}$$

$$A_k = \frac{(s+p_k)D(s)}{sQ(s)}\Big|_{s=-p_k}$$

$$B_k = \frac{(s+p_k)N(s)}{sQ(S)}\Big|_{s=-p_k}$$

A_k, being the residues at the poles of an RC DPA, will be positive real. Thus,

$$A_k > 0. \tag{9}$$

From (7) and (8),

$$Y_1 = \begin{bmatrix} s(\mu A_\infty + KB_\infty) + (\mu A_o + KB_o) \\ + \sum_{k=1}^{n-1} \frac{(\mu A_k + KB_k)s}{s+p_k} \end{bmatrix} \times \left(\frac{1}{1+\mu}\right) \tag{10}$$

and

$$Y_2 = \begin{bmatrix} s(A_\infty - KB_\infty) + (A_o - KB_o) \\ + \sum_{k=1}^{n-1} \frac{(A_k - KB_k)s}{s+p_k} \end{bmatrix} \times \left(\frac{1}{1+\mu}\right) \tag{11}$$

For Y_1 and Y_2 to be RC DPAs, the residues at the poles must be positive real, i.e.,

$$\mu A_k + KB_k \geq 0, \quad k = 0, 1, 2, \ldots n-1, \infty \tag{12}$$

and

$$A_k - KB_k \geq 0, \quad k = 0, 1, 2, \ldots n-1, \infty \tag{13}$$

Thus, K must be chosen such that (10) and (11) satisfy for both Y_1 and Y_2 to be RC realizable, that is,

$$K \leq \min\left[\frac{\mu A_k}{B_k^-}, \frac{A_k}{B_k^+}\right], k = 0, 1, 2, \ldots n-1, \infty \tag{14}$$

Where B_k^- is B_k with negative sign and B_k^+ is B_k with positive sign. Thus, ***there are many possible realizations depending upon the choice of K and μ.*** It may be noted from (2) that the poles of $T(s)$ are the zeros of the RC DPA (Y_1 + Y_2). ***Hence, the method can realize the current transfer functions with distinct negative real poles only.***

***Total number of elements*:** If K and μ are chosen such that the equality condition in (14) holds then the number of elements can be reduced. For keeping the number of circuit elements low, admittances Y_3 and Y_4 can be selected as single conductances. Therefore, the maximum number of elements required is

$$\begin{aligned} N_E &= \text{ total number of elements in} \\ &\quad (Y_1, Y_2, Y_3, Y_4) \\ &= [2 + 2(N-1)] + [2 + 2(N-1)] + 1 + 1 \\ &= (4N + 2). \end{aligned} \tag{15}$$

The minimum number of elements will lie $4N + 2$ – the number of elements reduced due to the choice of K and μ.

(b) Using CDTA

Consider the circuit shown in Fig. 2 which uses current differencing transconductance amplifier (CDTA). The symbol of CDTA is shown in Fig. 3, and its terminal characteristics are

$$\begin{bmatrix} V_p \\ V_n \\ I_z \\ I_{x\pm} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \pm 1 \end{bmatrix} \begin{bmatrix} I_p \\ I_n \\ V_x \\ V_\pm \end{bmatrix}. \quad (16)$$

The current transfer function of the circuit is

$$T(s) = \frac{I_o}{I_i} = K\frac{N(s)}{D(s)} = \frac{g_m}{Y_3}\,\frac{(Y_1 - Y_2)}{(Y_1 + Y_2)}. \quad (17)$$

Ignoring the scaling factor g_m, and letting

$$Y_3 = 1 \quad (18)$$

the current transfer function reduces to the same as given by (1) when μ = 1. Hence the same method can be

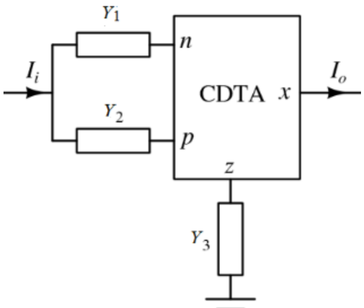
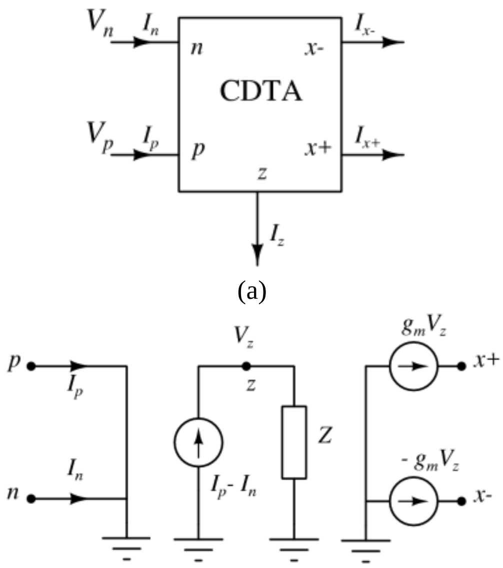


Fig. 2: Circuit for realizing current transfer function using CDTA



(b)

Fig. 3 Current differential transconductance amplifier
(a) Symbol, (b) Ideal model of CDTA

applied here. However, *the flexibility of choosing μ, in order to get many possible realizations with minimum elements* is missing here. Nonetheless, it uses *only 3 two- terminal admittances instead of 4.*

2.2 Realization of *voltage transfer* functions

It is interesting to note that the circuits in Fig. 4(a) [12], 4(b) [13] and 4(c) [14] have the *voltage* transfer function

$$\frac{V_o}{V_i} = \frac{Y_1 - \mu Y_2}{Y_1 + Y_2}, \tag{19}$$

the same as the *current* transfer function given by (1). OTRA in Fig. 4(a) has the following terminal characteristics.

$$\begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -R_m & R_m & 0 \end{bmatrix} \begin{bmatrix} I_x \\ I_y \\ I_z \end{bmatrix}, \tag{20}$$

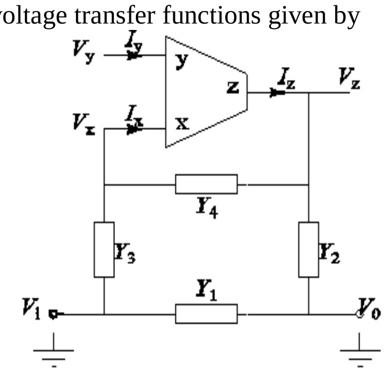
The current conveyor CCII in Fig. 4(b) has the following characteristic.

$$\begin{bmatrix} V_x \\ I_x \\ I_z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_y \\ I_z \\ V_z \end{bmatrix}.$$

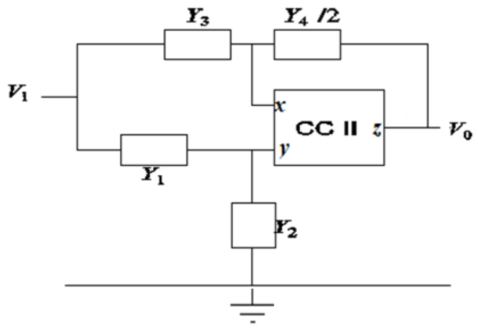
Therefore, the above method, can be used for realizing voltage transfer function by these circuits.

Note that Cakir *et al.* [9] used the same configuration shown in Fig. 4(a) for realizing *only all-pass filters of first and second orders*.

Example 1: Realize the current and voltage transfer functions given by



(a)



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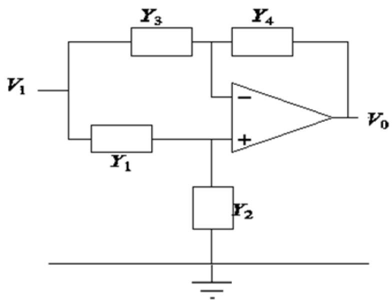


(c)

Fig. 4: Realization of voltage transfer functions using
(a) OTRA, (b) CCII, (c) OA

$$T(s)=K\frac{(s-z)}{(s+p)}. \tag{21}$$

(a) Current transfer function Using FTFN
Choosing $Q(s)$ = 1, and using (10) and (11), we get

$$\begin{aligned} Y_1 &= \left(\frac{1}{1+\mu}\right)\left[(\mu+K)s+\left(\mu p-Kz\right)\right], \\ Y_2 &= \left(\frac{1}{1+\mu}\right)\left[(1-K)s+\left(p+Kz\right)\right]. \end{aligned} \tag{22}$$

Obviously, for $Y_{1,2}$ to be RCDPAs,

$$K \le \begin{cases} 1, & \mu p \ge z \\ \mu\dfrac{p}{z}, & \mu p \le z. \end{cases} \tag{23}$$

Thus there are *many possible realizations depending upon the values of K and μ.* However, larger μ means larger spread in the values of Y_3 and Y_4. Choosing μ = 1, $p \ge z$ and choosing K =1,

$$Y_1 = s+\frac{p-z}{2}, \quad Y_2=\frac{p+z}{2}. \tag{24}$$

The complete realization of $T(s)$ of equation (21) is shown in Fig. 5(a) when $p = z$, i.e., when $T(s)$ is a first order all-pass function. Thus we require 1C and 3R passive elements, and 1 active element for a first order all–pass filter. These numbers, obtained systematically, are the same as obtained intuitively by Cikar *et al* [9].

(b) Current transfer function using CDTA
In this case μ = 1. Choosing Y_3 = 1 and $Q(s)$ = 1, from (10) and (11)

$$\left(Y_1-Y_2\right)=K(s-z), \quad \text{and} \quad \left(Y_1+Y_2\right)=(s+p). \tag{25}$$

By solving

$$\begin{aligned} Y_1 &= \frac{1}{2}\left[(1+K)s+\left(p-Kz\right)\right], \\ Y_2 &= \frac{1}{2}\left[(1-K)s+\left(p+Kz\right)\right]. \end{aligned} \tag{26}$$

Now, the restriction is

$$K \le \begin{cases} 1, & p \ge z \\ \dfrac{p}{z}, & p \le z. \end{cases}$$

Assuming $p = z$ and choosing K =1, the realization is as shown in Fig, 5(b). Here we require 1C and 2R and one active element. *Unlike FTFN realization, in this case there is only one possible realization, but requires less number of elements.*

(c) voltage trtanfer function realization
Following the above procedure, the realizations are shown in Fig. 6 using OTRA, CCII and OA.

Example 2: Realize the all-pass current and voltage functions given by

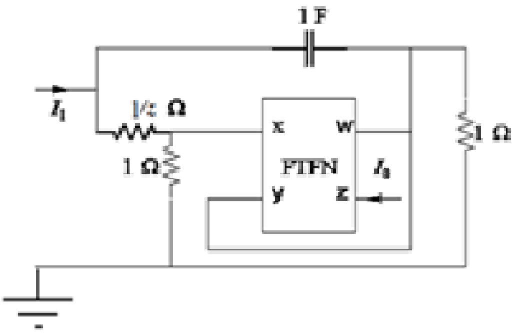
$$T(s)=K\frac{(s-1)(s-3)}{(s+1)(s+3)} \tag{27}$$

(a) Current transfer function using FTFN
Let $Q(s) = (s + 2)$. Then

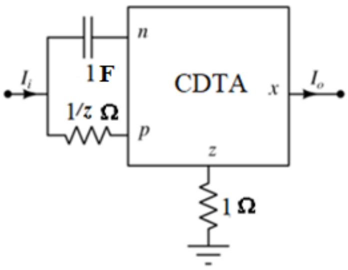
$$T(s)=\frac{\dfrac{K(s^2-4s+3)}{s+2}}{\dfrac{(s^2+4s+3)}{s+2}}=\frac{\left(s+\dfrac{3}{2}\right)K-\dfrac{15s}{2(s+2)}K}{\left(s+\dfrac{3}{2}\right)+\dfrac{s}{2(s+2)}}$$

Here, $A_\infty = B_\infty = 1$, $A_o = B_o = (3/2)$, $A_1 = 1/2$, $B_1 = -(15/2)$. From (10) and (11)

$$Y_1=\left[s(\mu+K)+\frac{3}{2}(\mu+K)+\frac{\frac{1}{2}(\mu-15K)s}{s+2}\right]\times\left(\frac{1}{1+\mu}\right) \tag{28}$$

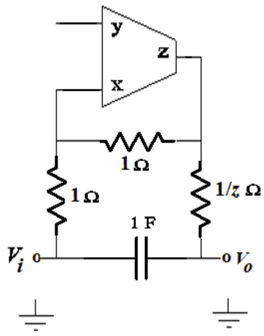


(a)



(b)

Fig. 5: Realization of first order current transfer function of (21) using when z = p using
(a) FTFN, (b) CDTA



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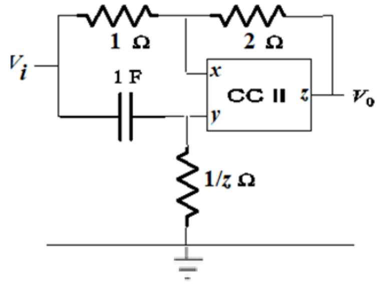

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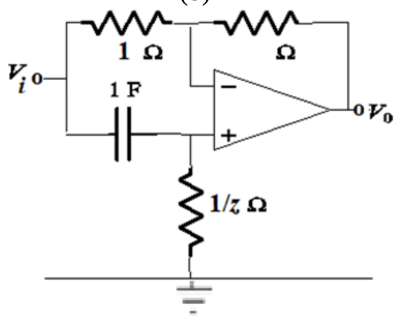


(c)

Fig. 6 Realization of voltage transfer function of (21) using
(a) OTRA, (b) CCII, (c) OA

$$Y_2 = \left[s(1-K) + \frac{3}{2}(1-K) + \frac{\frac{1}{2}(1+15K)s}{s+p_k} \right] \times \left(\frac{1}{1+\mu} \right) \tag{29}$$

From (14),

$$K \leq \min\left[1, \frac{\mu}{15}\right]$$

$$\leq \begin{cases} 1, & \mu \geq 15 \\ \frac{\mu}{15}, & \mu \leq 15. \end{cases}$$

Choosing μ = 15 and K = 1, we get from (28) and (29),

$$Y_1 = s + \frac{3}{2} \tag{30}$$

and

$$Y_2 = \frac{s}{2(s+2)} \tag{31}$$

The complete realization of $T(s)$ is given in Fig. 7(a). Here we require $2C + 4R$ realizations derived *systematically*, while in [9], the same number of elements was obtained *intuitively*.

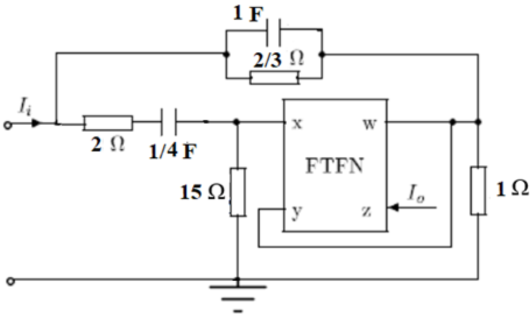
(b) Current transfer function using CDTA

In this case, substituting μ = 1, (28) and (29) reduce to

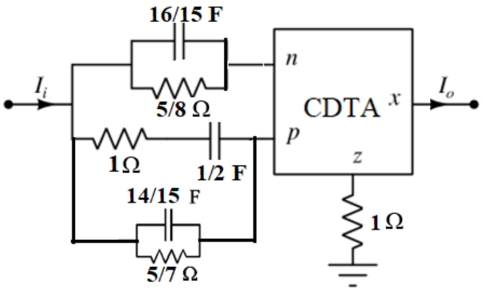
$$Y_1 = \left[s(1+K) + \frac{3}{2}(1+K) + \frac{\frac{1}{2}(1-15K)s}{s+2} \right] \times \left(\frac{1}{2} \right) \tag{32}$$

$$Y_2 = \left[s(1-K) + \frac{3}{2}(1-K) + \frac{\frac{1}{2}(1+15K)s}{s+2} \right] \times \left(\frac{1}{2} \right) \tag{33}$$

Now the restriction becomes

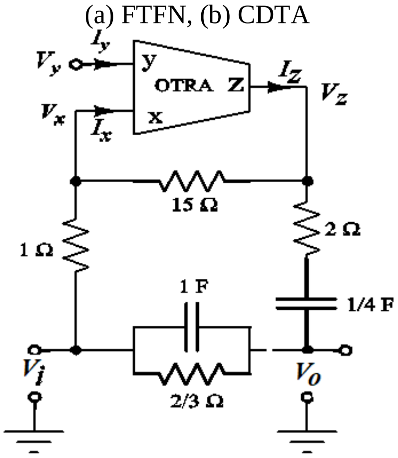


(a)

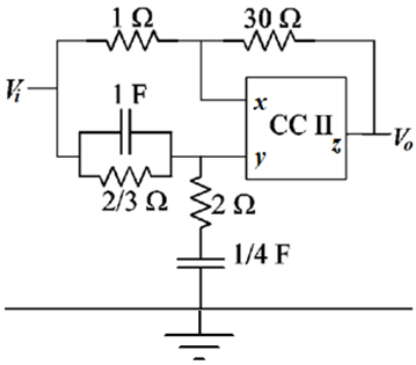


(b)

Fig. 7: Realizations of current transfer function of (28) using (a) FTFN, (b) CDTA



(a)



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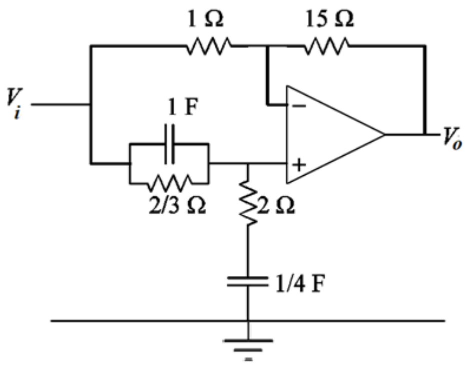

(c)

Fig. 8. Realizations of voltage transfer function of (28) using
(a) OTRA, (b) CCII, (c) OA

Table 1: Number of elements required for second order filters with a single active device

Filter Type	FTFN (current) OTRA/CCII/OA (voltage)			CDTA (current)	
	μ	K	N	K	N
LP	1	1	3C+4R=7	6	3C+3R=6
BP	Arb, 1	1/2	3C+5R=8	1/2	3C+4R=7
HP	4	1	2C+5R=7	¼	3C+4R=7
BR	5	1	2C+5R=7	1/5	3C+4R=7
AP	15	1	2C+4R=6	1/15	3C+4R=7

$$K \le \min\left[1,\ \frac{1}{15}\right].$$

Now choosing $K = 1/15$, the realization obtained is as shown in Fig. 7(b).

(c) Voltage transfer function using OTRA, CCII and OA

Following the similar procedure, realizations of the voltage transfer function given by (27) and obtained using OTRA, CCII and OA are shown in Fig. 8(a), (b) and (c), respectively.
Following the above method, the realization of current/voltage transfer function of (27) with the same denominator, but different numerators (1 for LP, s for BP, s^2 for HP and $s^2 + 1$ for BR) and $Q(s) = s+2$ were realized and the results are given in Table 1. The following remarks are in order from the table.

1. Realizations of second order current transfer functions using CDTA require one less passive element for LP and BP filters, and equal number of passive elements for HP and BR filters, and 1 more passive element for AP filter, compared to corresponding FTFN realizations.
2. FTFN (current) realizations require one less capacitor for HP, BR and AP filter realizations, and equal number of capacitors for LP and BP filters compared to corresponding CDTA (current) realizations.
3. For all types of voltage filters realizations, OTRA, CCII and OA require the same number of total passive elements.

III. Conclusion

A systematic realization procedure for both the current and voltage transfer functions using a single active device have been derived. It is found for current transfer functions that CDTA realizations require equal number of passive elements for HP and BR filters, one less for LP and HP filters, and 1 more for AP filter as compared to corresponding FTFN realizations. Further, CDTA realizations require equal number of capacitors for LP and BP filters, and one more capacitor for HP, BR and AP filters compared to those of FTFN realizations. The total number of passive elements, for all types of voltage transfer functions using OTRA, CCII or OA, is the same. The method has been illustrated with two examples. No attempt is made to produce simulation and practical results as they are available in various references given.

Conflict of interest

Authors have no conflict of interest relevant to this article.

REFERENCES

[1]. Cicekoglu, O.: ‘Current-mode biquad with a minimum number of passive elements’, *IEEE Trans. Circuits Syst. II, Analog and Digital Signal Processing,* **48**, 221-222 (2001).

[2]. Liu S. I., and Lee, J. L., ‘Insensitive current/voltage mode filters using FTFNs’, *Electron Lett.,* **32**, 1079-1080, (1996)

[3]. Higashimura, M.: ‘Current mode all-pass filter using FTFN with grounded capacitor’, *Electron. Lett.*, **27**, 1182-1183 (1991)

[4]. Abuelmaátti, M. T.: ‘Cascadable current mode filters using single FTFN’, *Electron. Lett.*, **32**, pp. 1457-1458 (1996).

[5]. Liu S. I., and Hwang, C. S.: ‘Realization of current mode filters using single FTFN’, *Int. J. Electron,* **82**, 499-502 (1997)

[6]. Rathore T. S., and Khot U. P.: ‘Single FTFN realization of current transfer functions’, *IETE J of Research,* **51**, 193-199 (2005)

[7]. Salama K. N., and Soliman, A. M.: ‘CMOS operational transconductance amplifier for analog signal processing applications’, *Microelectronic. J.,* **30**, pp. 235-245 (1999)

[8]. Kilinc S., and Cam, U.: ‘Operational trans-conductance amplifier based first order all pass filter with an application’, *Proc IEEE Int Midwest Symp, Circuits Syst., Hiroshima, Japan,* 65-68 (2004)

[9]. Cakir, C., Cam U., and Cicekoglu, O.: ‘Novel all pass filter configuration employing single OTRA’, *IEEE Trans Circuits Syst. II, Analog and Digital Signal processing,* **52**, pp. 122-125 (2005)

[10]. Rathore, T. S., and Khot, U. P.: ‘Single OTRA realization of transfer functions’, *IE(I) Journal-ET Section*, **89**, 33-38 (2008)

[11]. Toker, A., Ozoguz, S., Cicekoglu, O., and Acar, C.: ‘Current mode all pass filters using current differencing buffered amplifier and a new high-*Q* band pass filter configuration’, *IEEE Trans Circuits, Syst. II, Analog and digital Signal Processing,* **47**, 949-954 (2000)

[12]. Rathore, T. S., ‘Realizations of current transfer functions using current differencing trans-conductance amplifier’, *Circuits, Systems, and Signal processing*, DOI 10.1007/s00034-019-01036-x

[13]. Rathore, T. S., and Dasgupta, S. M.: ‘Current conveyor realization of transfer functions’, *Proc IEE,* **122**, 1119-1120 (1975)

[14]. Mitra, S. K.: ‘Analysis and synthesis of linear active networks’, New York: Wiley, 1969.

[15]. Maheshwari, S.: ‘High input impedance VM-APSs with grounded passive elements’, *IET Proc. Circuits, Devices and Syst.,* **1,** pp. 72-78 (2007)

[16]. Horng, Jiun-Wel, Hou, Chun-Li, Hsu, Chih-Hou, Yang, Dun-Yih, Hp, Min-Jie.: ‘Low input and high output impedance current-mode first order all pass filter employing grounded passive components’, *Int. J. Circuits and Syst.,* **3**, pp. 176-179 (2012)

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