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Global Exponential Stability of Bidirectional Associative Memory Neural Networks with Distributed Delays and Fuzzy Logic

Dianbo Ren
(Associate Professor, School of Automotive Engineering, Harbin Institute of Technology at Weihai, China)
Corresponding Author: Dianbo Ren

Abstract
In this paper, based on the theory of M-matrix, by constructing proper vector Liapunov functions, the globally exponential stability is investigated for a class of bidirectional associative memory neural networks with fuzzy logic and distributed delays. Without assuming the boundedness, monotonicity and differentiability of the activation functions, the new sufficient criterions for ascertaining the exponential stability of the equilibrium point of such neural networks are obtained. Since the criterion is independent of the delays and simplifies the calculation, it is easy to test the conditions of the criterion in practice.
Keywords: *bidirectional associative memory, globally exponential stability; vector Liapunov function; fuzzy logic; distributed delays*

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I. Introduction

Bidirectional associative memory (BAM) neural networks known as an extension of the unidirectional autoassociator of Hopfield was first introduced by Kosto[1,2]. Due to the BAM neural networks has been used in many fields such as pattern recognition, image processing, and automatic control. Therefore, the BAM neural networks have attracted great attention of many researchers. One can refer to the articles [3-17] for detailed discussion on these aspects. When a neural network is employed as an associative memory, the existence of many equilibrium points is a necessary feature. However, in applications to parallel computation and signal processing involving solution optimization problems, it is required that there be a well-defined computable solution for all possible initial states. From a mathematical viewpoint, this means that the network should have a unique equilibrium point that is globally asymptotically or exponential stable. In hardware implementation, time delays occur due to finite switching speeds of the amplifiers, and the existence of time delays frequently causes oscillation or instability in neural networks, thus, the study of globally asymptotical or exponential stability of BAM neural networks with time delays is practically required. In [6-12], some sufficient conditions have been obtained for globally exponential stability of bidirectional associative memory networks with fixed, varying or distributed time delays.

The another fundamental neural networks, fuzzy neural networks (FNN)[18] which combine fuzzy logic with the traditional neural networks structure is an active branch in the field of neural networks. Studies have been shown that FNN can deal with the abstract information and it is good at self-learning and self-tuning. So FNN have their potential in intelligent control, modeling, image processing and pattern recognition [19-21]. Like the traditional neural networks, the stability of the system is very important in the design of the FNN. Some results on stability have been derived for fuzzy neural networks, for example [22-25]. Y. Liu and W. Tang [24] studied the stability of fuzzy cellular neural networks(FCNN)with constant and time varying delays by constructing the suitable Lyapunov functional. T. Huang [25] obtain the exponential stability conditions of FCNN with distributed delays. But, to the best of my knowledge, few results on the stability for BAM neural networks combining fuzzy logic have reported in the literature.

In this paper, we investigate a kind of delayed BAM neural networks with fuzzy logic which integrates fuzzy logic into the structure of traditional BAM neural networks with distributed delays. Our objective is to study the global exponential stability of the kind of BAM neural networks with fuzzy logic and distributed delays. Without assuming the boundedness, monotonicity and differentiability of activation functions, by using M-matrix theory, and vector Liapunov functions method, we present new conditions ensuring the global exponential stability of the equilibrium point for the class of BAM networks with fuzzy logic and distributed delays.

II. Notation and Preliminaries

For convenience, we introduce some notations. $x=(x_1,\cdots,x_n)^{\mathrm{T}}\in R^n$ denotes a column vector. $|x|$ denotes the absolute-value vector given by $|x|=(|x_1|,\cdots,|x_n|)^{\mathrm{T}}$, $\|x\|$ denotes a vector norm defined by $\|x\|=(x_1^2+x_2^2+\cdots+x_n^2)^{1/2}$. For matrix $A=(a_{ij})_{n\times n}$, A^{T} denotes the transpose of A, A^{-1} denotes the inverse of A, $[A]^s$ is defined as $[A]^s=(A^{\mathrm{T}}+A)/2$, and $|A|$ denotes absolute-value matrix given by $|A|=(|a_{ij}|)_{n\times n}$, $\|A\|$ denotes a matrix norm defined by $\|A\|=(\max\{\lambda:\lambda$ is an eigenvalue of $A^{\mathrm{T}}A\})^{1/2}$. $\wedge$ and $\vee$ denote the fuzzy AND and fuzzy OR operation, respectively.

The dynamical behavior of bidirectional associative memory neural networks with fuzzy logic and distributed time delays can be described by the following nonlinear differential equations:

$$\frac{du_i(t)}{dt}=-\alpha_i u_i(t)+\sum_{j=1}^{m}a_{ij}\int_{-\infty}^{t}K_{ij}(t-s)f_j(v_j(s))ds+\bigwedge_{j=1}^{m}b_{ij}\int_{-\infty}^{t}K_{ij}(t-s)f_j(v_j(s))ds$$
$$+\bigvee_{j=1}^{m}c_{ij}\int_{-\infty}^{t}K_{ij}(t-s)f_j(v_j(s))ds+I_i,\quad i=1,2,\cdots,n; \tag{1a}$$

$$\frac{dv_j(t)}{dt}=-\beta_j v_j(t)+\sum_{i=1}^{n}d_{ji}\int_{-\infty}^{t}L_{ji}(t-s)g_i(u_i(s))ds+\bigwedge_{i=1}^{n}w_{ji}\int_{-\infty}^{t}L_{ji}(t-s)g_i(u_i(s))ds$$
$$+\bigvee_{i=1}^{n}z_{ji}\int_{-\infty}^{t}L_{ji}(t-s)g_i(u_i(s))ds+J_j,\quad j=1,2,\cdots,m, \tag{1b}$$

where $\alpha_i>0$, $\beta_j>0$ for $i=1,2,\cdots,n$, $j=1,2,\cdots,m$ denote the passive decay rates; a_{ij}, d_{ji} for $i=1,2,\cdots,n$, $j=1,2,\cdots,m$ are the synaptic connection strengths; b_{ij}, w_{ji} and c_{ij}, z_{ji} for $i=1,2,\cdots,n$, $j=1,2,\cdots,m$ are elements of fuzzy feedforward MIN template and fuzzy feedforward MAX template, respectively; f_j for $j=1,2,\cdots,m$ and g_i for $i=1,2,\cdots,n$ denote the propagational signal functions; I_i for $i=1,2,\cdots,n$, and J_j for $j=1,2,...,m$ are the exogenous inputs. The initial conditions associated with (1) are of the form

$$u_i(s)=\phi_i(s),\quad s\le 0,\quad i=1,2,\cdots,n,$$
$$v_j(s)=\psi_j(s),\quad s\le 0,\quad j=1,2,\cdots,m.$$

where ϕ_i and ψ_j are bounded and continuous on $(-\infty,0]$.

In the following, we let

$$u=(u_1,\cdots,u_n)^{\mathrm{T}},v=(v_1,\cdots,v_m)^{\mathrm{T}},\quad \alpha=\mathrm{diag}(\alpha_1,\cdots,\alpha_n),\quad \beta=\mathrm{diag}(\beta_1,\cdots,\beta_m),$$
$$A=(a_{ij})_{n\times m},B=(b_{ij})_{n\times m},C=(c_{ij})_{n\times m},D=(d_{ji})_{m\times n},W=(w_{ji})_{m\times n},Z=(z_{ji})_{m\times n},$$
$$f(v)=(f_1(v_1),\cdots,f_m(v_m))^{\mathrm{T}},g(u)=(g_1(u_1),\cdots,g_n(u_n))^{\mathrm{T}},\phi=(\phi_1,\cdots,\phi_n)^{\mathrm{T}},\psi=(\psi_1,\cdots,\psi_m)^{\mathrm{T}}.$$

For system (1), we make the following assumptions:

Assumption (A) For each $i\in[1,n]$, $j\in[1,m]$, $f_j:R\to R$ and $g_i:R\to R$ are globally Lipschitz continuous with Lipschitz constant $F_j>0$, $G_j>0$, i.e., $|f_j(y_j)-f_j(v_j)|\le F_j|y_j-v_j|$ for all y_j,v_j; $|g_i(x_i)-g_i(u_i)|\le G_i|x_i-u_i|$ for all x_i,u_i.

In the paper, we let $F=\mathrm{diag}(F_1,F_2,\cdots,F_n)$, $G=\mathrm{diag}(G_1,G_2,\cdots,G_m)$.

Assumption (B) For each $i\in[1,n]$, $j\in[1,m]$, $K_{ij}:[0,\infty)\to[0,\infty)$, $L_{ji}:[0,\infty)\to[0,\infty)$ are piecewise continuous on $[0,\infty)$ and satisfy

$$\int_0^{\infty}\mathrm{e}^{\lambda s}K_{ij}(s)\mathrm{d}s=p_{ij}(\lambda),\int_0^{\infty}\mathrm{e}^{\rho s}L_{ji}(s)\mathrm{d}s=h_{ji}(\rho),i=1,2,\cdots,n,\ j=1,2,\cdots,m.$$

where $p_{ij}(\lambda)$ and $h_{ji}(\rho)$ are continuous functions in $[0,\delta)$ and $[0,\sigma)$, respectively. $\delta>0$, $\sigma>0$ and $p_{ij}(0)=1$, $h_{ji}(0)=1$.

Lemma 1[26]**.** Let Ω be a $(n+m)\times(n+m)$ matrix with non-positive off-diagonal elements. Then the following statements are equivalent:
(i) Ω is an M-matrix,
(ii) The real parts of all eigenvalues of Ω are positive,
(iii) There exists a vector $\xi>0$, such that $\Omega\xi>0$,
(iv) Ω is nonsingular and all elements of Ω^{-1} are nonnegative,
(v) There exists a positive definite $(n+m)\times(n+m)$ diagonal matrix Q such that matrix $\Omega Q+Q\Omega^{\mathrm{T}}$ is positive definite.
From Lemma 3 in Ref**.** [25], we get
Lemma 2. Suppose $(x,y)\in R^{n\times m}$ and $(u,v)\in R^{n\times m}$ are two states of system (1), then

$$|\bigwedge_{i=1}^{n} w_{ji}g_i(x_i)-\bigwedge_{i=1}^{n} w_{ji}g_i(u_i)|\le\sum_{i=1}^{n}|w_{ji}||g_i(x_i)-g_i(u_i)|,$$

$$|\bigvee_{i=1}^{n} z_{ji}g_i(x_i)-\bigvee_{i=1}^{n} z_{ji}g_i(u_i)|\le\sum_{i=1}^{n}|z_{ji}||g_i(x_i)-g_i(u_i)|,\quad i=1,2,\cdots,n;$$

$$|\bigwedge_{j=1}^{m} b_{ij}f_j(y_j)-\bigwedge_{j=1}^{m} b_{ij}f_j(v_j)|\le\sum_{j=1}^{m}|b_{ij}||f_j(y_j)-f_j(v_j)|,$$

$$|\bigvee_{j=1}^{m} c_{ij}f_j(y_j)-\bigvee_{j=1}^{m} c_{ij}f_j(v_j)|\le\sum_{j=1}^{m}|c_{ij}||f_j(x_j)-f_j(v_j)|,\quad j=1,2,\cdots,m.$$

III. Global Exponential Stability of The Equilibrium Point

In the section, we study the global exponential stability of the equilibrium point of system (1).
Theorem 1 Suppose Assumption (A) and Assumption (B) holds, if Ω is an M-matrix, then for any pair of input (I,J), system (1) has a unique equilibrium point (u^*,v^*), which is globally exponential stable, independent of the delays. Ω is defined as

$$\Omega=\begin{bmatrix}\alpha & -[|A|+|B|+|C|]F\\ -[|D|+|W|+|Z|]G & \beta\end{bmatrix}$$

Proof. Since Ω is an M-matrix, similar to the Theorem 1 of Ref.[27], system (1) has a unique equilibrium point (u^*,v^*). By means of coordinate translation $x(t)=u(t)-u^*$, $y(t)=v(t)-v^*$, system (1) can be written as

$$\begin{aligned}\dot{x}_i(t)&=-\alpha_i x_i(t)+\sum_{j=1}^{m}a_{ij}\{\int_{-\infty}^{t}K_{ij}(t-s)f_j(y_j(s)+v_j^*)ds-\int_{-\infty}^{t}K_{ij}(t-s)f_j(v_j^*)ds\}\\&+\bigwedge_{j=1}^{m}b_{ij}\int_{-\infty}^{t}K_{ij}(t-s)f_j(y_j(s)+v_j^*)ds-\bigwedge_{j=1}^{m}b_{ij}\int_{-\infty}^{t}K_{ij}(t-s)f_j(v_j^*)ds\\&+\bigvee_{j=1}^{m}c_{ij}\int_{-\infty}^{t}K_{ij}(t-s)f_j(y_j(s)+v_j^*)ds-\bigvee_{j=1}^{m}c_{ij}\int_{-\infty}^{t}K_{ij}(t-s)f_j(v_j^*)ds,\ i=1,2,\cdots,n;\end{aligned}\tag{2a}$$

$$\begin{aligned}\dot{y}_i(t)&=-\beta_j y_j(t)+\sum_{i=1}^{n}d_{ji}\{\int_{-\infty}^{t}L_{ji}(t-s)g_i(x_i(s)+u_i^*)ds-\int_{-\infty}^{t}L_{ji}(t-s)g_i(u_i^*)ds\}\\&+\bigwedge_{i=1}^{n}w_{ji}\int_{-\infty}^{t}L_{ji}(t-s)g_i(x_i(s)+u_i^*)ds-\bigwedge_{i=1}^{n}w_{ji}\int_{-\infty}^{t}L_{ji}(t-s)g_i(u_i^*)ds\\&+\bigvee_{i=1}^{n}z_{ji}\int_{-\infty}^{t}L_{ji}(t-s)g_i(x_i(s)+u_i^*)ds-\bigvee_{i=1}^{n}z_{ji}\int_{-\infty}^{t}L_{ji}(t-s)g_i(u_i^*)ds,\ j=1,2,\cdots,m.\end{aligned}\tag{2b}$$

System (2) has a unique equilibrium at $(x,y)=(0,0)$. Clearly, (u^*,v^*) is globally exponential stable for (1) if and only if the trivial solution of (2) is global exponential stable.

The initial condition of (2) is $\Phi(s)=\phi(s)-u^*$, $\Psi(s)=\psi(s)-v^*$ $-\infty<s\le0$.

Due to $\Omega = \begin{bmatrix} \alpha & -[|A|+|B|+|C|]F \\ -[|D|+|W|+|Z|]G & \beta \end{bmatrix}$ being an M-matrix, from Lemma 1, there exist $\xi_i > 0$ ($i = 1,2,\cdots,n$), $\eta_j > 0$ ($j = 1,2,\cdots,m$), such that

$$-\xi_i\alpha_i + \sum_{j=1}^{m}\eta_j(|a_{ij}+|b_{ij}|+|c_{ij}|)F_j < 0, i = 1,2,\cdots,n; \tag{3a}$$

$$-\eta_j\beta_j + \sum_{i=1}^{n}\xi_i(|d_{ji}+|w_{ji}|+|z_{ji}|)G_i < 0 \quad j = 1,2,\cdots,m. \tag{3b}$$

Obviously, there exists constants $\lambda_i > 0$, $\rho_j > 0$, such that

$$\xi_i(-\alpha_i+\lambda_i) + \sum_{j=1}^{m}\eta_j(|a_{ij}+|b_{ij}|+|c_{ij}|)F_j < 0, i = 1,2,\cdots,n; \tag{4a}$$

$$\eta_j(-\beta_j+\rho_j) + \sum_{i=1}^{n}\xi_i(|d_{ji}+|w_{ji}|+|z_{ji}|)G_i < 0, \ j = 1,2,\cdots,m. \tag{4b}$$

Let $\lambda = \min_{i=1,2,\cdots,n}\lambda_i$, $\rho = \min_{j=1,2,\cdots,n}\rho_j$. Consider a vector Liapunov function defined by

$$V(t) = (V_1(x_1(t)), V_2(x_2(t)),\cdots,V_n(x_n(t)),V_{n+1}(y_1(t)),V_{n+2}(y_2(t)),\cdots,V_{n+m}(y_m(t)))^{\mathrm{T}},$$

where $V_i(t) = \mathrm{e}^{\lambda t}|x_i(t)|$, $i = 1,2,\cdots,n$; $V_{n+j}(t) = \mathrm{e}^{\rho t}|y_j(t)|$. $j = 1,2,\cdots,m$ .Calculating the upper right derivative of $V_i(t)$ and $V_{n+j}(t)$ along the solutions of (2), by using Assumption (A), and lemma 2, we get

$$\begin{aligned}
&D^+V_i(t) \\
&= \lambda\mathrm{e}^{\lambda t}|x_i| + \mathrm{e}^{\lambda t}\dot{x}_i \operatorname{sgn} x_i \\
&\le \lambda\mathrm{e}^{\lambda t}|x_i| + \mathrm{e}^{\lambda t}\{-\alpha_i|x_i| + \sum_{j=1}^{m}|a_{ij}|\int_{-\infty}^{t}K_{ij}(t-s)|f_j(y_j(s)+v_j^*)-f_j(v_j^*)|ds \\
&+|\bigwedge_{j=1}^{m}b_{ij}\int_{-\infty}^{t}K_{ij}(t-s)f_j(y_j(s)+v_j^*)ds - \bigwedge_{j=1}^{m}b_{ij}\int_{-\infty}^{t}K_{ij}(t-s)f_j(v_j^*)ds| \\
&+|\bigvee_{j=1}^{m}c_{ij}\int_{-\infty}^{t}K_{ij}(t-s)f_j(y_j(s)+v_j^*)ds - \bigvee_{j=1}^{m}c_{ij}\int_{-\infty}^{t}K_{ij}(t-s)f_j(v_j^*)ds|\} \\
&\le \mathrm{e}^{\lambda t}\{(-\alpha_i+\lambda)|x_i| + \sum_{j=1}^{m}(|a_{ij}|+|b_{ij}+|c_{ij}|)\int_{-\infty}^{t}K_{ij}(t-s)|f_j(y_j(s)+v_j^*)-f_j(v_j^*)|ds\} \\
&\le \mathrm{e}^{\lambda t}\{(-\alpha_i+\lambda)|x_i| + \sum_{j=1}^{m}(|a_{ij}|+|b_{ij}+|c_{ij}|)F_j\int_{-\infty}^{t}K_{ij}(t-s)|y_j(s)|ds\} \\
&= (-\alpha_i+\lambda)\mathrm{e}^{\lambda t}|x_i| + \sum_{j=1}^{m}(|a_{ij}|+|b_{ij}+|c_{ij}|)F_j\int_{-\infty}^{t}K_{ij}(t-s)\mathrm{e}^{\lambda(t-s)}|\mathrm{e}^{\lambda s}y_j(s)|ds
\end{aligned}$$

$$= (-\alpha_i+\lambda)V_i + \sum_{j=1}^{m}(|a_{ij}|+|b_{ij}+|c_{ij}|)F_j\int_{-\infty}^{t}K_{ij}(t-s)\mathrm{e}^{\lambda(t-s)}V_{n+j}(s)ds, \quad i = 1,2,\cdots,n; \tag{5a}$$

$$\begin{aligned}
&D^+V_{n+j}(t) \\
&= \lambda\mathrm{e}^{\lambda t}|y_i| + \mathrm{e}^{\rho t}\dot{y}_i \operatorname{sgn} y_i
\end{aligned}$$

$$\le (-\beta_j+\rho)V_{n+j} + \sum_{i=1}^{n}(|d_{ji}|+|w_{ji}+|z_{ji}|)G_i\int_{-\infty}^{t}L_{ji}(t-s)\mathrm{e}^{\rho(t-s)}V_i(s)ds, \ j = 1,2,\cdots,m. \tag{5b}$$

Let

$$\xi_{\max} = \max_{1\le i\le n}\{\xi_i\}, \xi_{\min} = \min_{1\le i\le n}\{\xi_i\}, \quad \eta_{\max} = \max_{1\le j\le m}\{\eta_j\}, \quad \eta_{\min} = \min_{1\le j\le m}\{\eta_j\},$$

$$l_0 = \max\{(1+\theta)\|\Phi\|/\xi_{\min}, (1+\varepsilon)\|\Psi\|/\eta_{\min}\}.$$

where $\theta > 0$, $\varepsilon > 0$ are constants. Then, we get that

$$V_i(s) = e^{\lambda s} |\Phi_i(s)| < \xi_i l_0 \ , \ V_{n+j}(s) = e^{\rho s} |\Psi_j(s)| < \eta_j l_0 \ , -\infty \le s \le 0, \ i = 1,2,\cdots,n, \ j = 1,2,\cdots,m.$$

We claim that

$$V_i(t) < \xi_i l_0, \ V_{n+j}(t) < \eta_j l_0, t \in [0,+\infty), i = 1,2,\cdots,n, \ j = 1,2,\cdots,m.$$

If it is not true, then there exist t_1 and some $l \in [1,n]$ or $k \in [1,m]$, such that

$$V_l(t_1) = \xi_l l_0, \ D^+(V_l(t_1)) \ge 0, \ V_i(t) \le \xi_i l_0, \ -\infty < t \le t_1, i = 1,2,\cdots,n; \tag{6a}$$

or

$$V_{n+k}(t_1) = \eta_k l_0, \ D^+(V_{n+k}(t_1)) \ge 0, \ V_{n+j}(t) \le \eta_j l_0, \ -\infty < t \le t_1, \ j = 1,2,\cdots,m. \tag{6b}$$

However, from (4), (5), (6) and Assumption (B), we get

$$D^+V_l(t_1) \le (-\alpha_l + \lambda)\xi_l l_0 + l_0 \sum_{j=1}^{m} (|a_{lj}| + |b_{lj}| + |c_{lj}|) F_j \eta_j \int_{-\infty}^{t_1} K_{lj}(t_1 - s) e^{\lambda(t_1 - s)} \, ds$$

$$= (-\alpha_l + \lambda)\xi_l l_0 + l_0 \sum_{j=1}^{m} (|a_{lj}| + |b_{lj}| + |c_{lj}|) F_j \eta_j \int_{0}^{\infty} K_{lj}(\tau) e^{\lambda \tau} \, d\tau$$

$$= \{(-\alpha_l + \lambda)\xi_l + \sum_{j=1}^{m} \eta_j (|a_{lj}| + |b_{lj}| + |c_{lj}|) F_j p_{lj}(\lambda)\} l_0 < 0; \tag{7a}$$

or

$$D^+V_{n+k}(t_1) \le \{(-\beta_k + \rho)\eta_k + \sum_{i=1}^{n} \xi_i (|d_{ki}| + |w_{ki}| + |z_{ki}|) G_i h_{ki}(\rho)\} l_0 < 0. \tag{7b}$$

This is contradictory with $D^+(V_l(t_1)) \ge 0$ in (10a) or $D^+(V_{n+k}(t_1)) \ge 0$ in (6b). So $V_i(t) < \xi_i l_0$, $V_{n+j}(t) < \eta_j l_0, i = 1,2,\cdots,n, \ j = 1,2,\cdots,m$, for $t \ge 0$. Furthermore,

$$|x_i(t)| < \xi_i l_0 e^{-\lambda t} \le \xi_{\max} l_0 e^{-\lambda t} = M e^{-\lambda t}, i = 1,2,\cdots,n, t \ge 0;$$

$$|y_j(t)| < \eta_j l_0 e^{-\rho t} \le \eta_{\max} l_0 e^{-\rho t} = N e^{-\rho t}, \ j = 1,2,\cdots,m, t \ge 0.$$

where $M = \xi_{\max} l_0$, $N = \eta_{\max} l_0$, $l_0 = \max\{(1+\theta)\|\Phi\| / \xi_{\min}, (1+\varepsilon)\|\Psi\| / \eta_{\min}\}$. By the standard Liapunov-type theorem in functional differential equations[28], the trivial solution of (2) is global exponential stable, and therefore, (u^*, v^*) is global exponential stable for (1). The proof is completed.

IV. Conclusion

In this paper, by using M-matrix theory and vector Liapunov functions, we present new conditions ensuring the global exponential stability of the equilibrium point of bidirectional associative memory neural networks with fuuzy logic and distributed time delays. The results are applicable to both symmetric and nonsymmetric interconnection matrices, and all continuous non-monotonic neuron activation functions.

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