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Effective values: implications and uses in nonlinear resistors

Ruvian de Césaro[1], Rodrigo Marques de Figueiredo[1]
[1]*(Department of Electronic Engineering, University do Vale do Rio dos Sinos - UNISINOS, Brazil)*

Abstract: *The effective value (usually calculated through the RMS value) is of great importance in electrical engineering and related areas. This article seeks to review the concepts to know how to proceed in cases of nonlinear resistance. The results are validated with application of theory in conjunction with simulation in LTSpice software, and taking advantage of the numerical convergence of the software for resolution of recursive integrals without algebraic solution.In this article few things are used to demonstrate the importance of correct calculation, which are algebra for deductions, Microsoft Excel for calculations with many values, and LTSpice for simulation of a simple circuit.There is deviation of 8.85% from the RMS voltage to effective voltage even when a small AC voltage (100 mV) is overimposed to the quiescent point of 700 mV on a 1N4148 diode. The power has a more expressive deviation, reaching 38.67% deviation from the correct value.The effective values must be calculated correctly when the influence of body resistance is small, because the apparent resistance of a semiconductor has larger influence than body resistance at this situation. When in doubt whether a circuit element is linear or not, use the general effective value equation for a correct value.*

Key Words*: Effective value; Effective apparent resistance; Effective resistance; RMS value.*

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I. Introduction

After realizing that there is a gap in the definition of effective resistance, it was explored what its definition and implications would be. Mathematical deductions were made about effective values, which are basic for electronics. There were made also other deductions and verified through dimensional analysis, and then applied to a hypothetical case, a specific situation in which there is a semiconductor biased in a quiescent point, superimposed an alternating signal, to demonstrate how what was developed would be used until that point. After that, results were compared and it was concluded that the effective value is not always the RMS value, and an equation for the effective value was also found that is always applicable.

In the initial considerations, some concepts and definitions (necessary to understand this work) are reviewed.In the theoretical developments the appropriate deductions are made and dimensional analysis are performed. In theoretical application in semiconductors, software is used for simulation of electronic circuits and spreadsheet is used to apply the equations already deduced, in order to get to reliable results and reach the conclusions of the work. In sections V, VI, and VII the results, discussion, and conclusion are presented, respectively.

For a long time care has been taken with effective electrical values in order to size correctly and prevent premature burning of electronic components and electrical conductors. It is already known how to calculate the effective values correctly for linear resistors. Therefore, this article seeks to explore what happens to the effective values when there is a nonlinear resistor.

II. Initial Considerations

This section will review some basic concepts and definitions, such as RMS value, effective value, and apparent resistance.

a. RMS value

The RMS value refers to the square root of the quadratic mean of the values which are equally spaced in time. The RMS voltage, for example, is

$$V_{RMS} = \sqrt{\frac{1}{T_f - T_i}\int_{T_i}^{T_f} V^2(t)dt} \quad (1)$$

While RMS value means an effective value, as will be seen, the RMS value is not always correct for the situation, so it is not always that an effective value is an RMS value.

b. Effective value

The effective value is the voltage or current value which produces, over a linear resistor, the same average power that the original signal dissipated (on the nonlinear resistor). If the element is a linear resistor, the effective value will be the RMS value, which may turn out to be just a DC or pure AC voltage. And as will be seen, if it is a nonlinear element, it will be called effective value, since the RMS calculation does not accurately represent the effective value. The equation for calculation is the following for voltage and current, respectively.

$$V_{effective} = \sqrt{\frac{R_{effective}}{T_f - T_i}\int_{T_i}^{T_f} V(t)I(t)dt} \tag{2}$$

$$I_{effective} = \sqrt{\frac{1}{(T_f - T_i)R_{effective}}\int_{T_i}^{T_f} V(t)I(t)dt} \tag{3}$$

Thus, effective resistance is a factor that relates power to voltage and current. The effective resistance that appears here is also important and will be addressed further.

c. Apparent resistance

Apparent resistance is the ratio of the voltage and current applied to a semiconductor junction at a given quiescent point. When there are several values of apparent resistance, distributed in intervals in time, there will be an equation to obtain the effective apparent resistance, which represents the resistance that allows the same total flow of electric charge during the same original period. The effective apparent resistance also establishes the ratio between effective voltage and effective current, much like what the characteristic impedance proposes.

III. Theoretical developments

In this section will be developed the deductions appropriate to this work. In the first subdivision is shown the logical explanation of the need for the use of effective resistance, in which it is shown that in certain cases effective resistance is necessary to be known or calculated. After, the effective resistance equation is developed for constant resistances in discrete periods, and soon after, the relationship between effective resistance and effective conductance is presented. Then the deduction of the effective resistance equation when there is continuous variation of the resistance as a function of timeis presented. Then, the deduction of the effective resistance equation when there is variation of other quantities is performed. Below, it is shown how power, effective signal value and effective resistance relate. Then dimensional analysis is used to prove that the equations are valid in terms of dimensions. Finally, it is departed for the derivation of effective resistance when there is a series or parallel association of linear resistors with nonlinear ones.

a. Logical explanation to the use of effective resistance

The modeling of an impedance across its spectrum (linear and nonlinear) needs further modeling. Thus, the equation

$$\frac{V}{I} = R \tag{4}$$

models only the linear behavior. Thus, equation (4) remains valid, but for slightly different concepts.

Effective resistance aims, and should, be the resistance which obeys the equation

$$V_{effective} = R_{effective}\, I_{effective} \tag{5}$$

Which can be

$$V_{RMS} = R_{effective}\, I_{RMS} \tag{6}$$

as exposed in II.a. and II.b. It is also related to the power dissipated.

$$P_{average} = \frac{{V_{RMS}}^2}{R_{effective}} \tag{7}$$

$$P_{average} = {I_{RMS}}^2 R_{effective} \tag{8}$$

It is necessary when there is a nonlinear resistance involved, for the calculation of unknown quantities (not imposed by the source).If the resistance changes over time, the known RMS value equation is no longer valid, and we have to go back to the average power equation and see how to proceed.

$$P_{average} = \frac{V_{effective}{}^{2}}{R_{effective}} \quad (9)$$

$$\frac{V_{effective}{}^{2}}{R_{effective}} = \frac{1}{T_f - T_i}\int_{T_i}^{T_f} \frac{V(t)^2}{R(V(t))}dt \quad (10)$$

Therefore,

$$V_{effective} = \sqrt{\frac{R_{effective}}{T_f - T_i}\int_{T_i}^{T_f} \frac{V(t)^2}{R(V(t))}dt} \quad (11)$$

And so we see that the effective values cannot be determined, for nonlinear elements, without determining the effective resistance. There is also the fact that R(t) is the relationship between V(t) and I(t), so the effective voltage is actually

$$V_{effective} = \sqrt{\frac{R_{effective}}{T_f - T_i}\int_{T_i}^{T_f} V(t)I(t)dt} \quad (12)$$

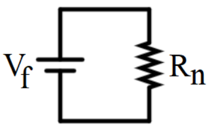
Similarly, the same is true with the current. You can write the relationship between average power and effective current and then the relationship between average power and instantaneous current, andthrough equality, if the effective current is manipulated, we can reach the following equation.

$$I_{effective} = \sqrt{\frac{1}{(T_f - T_i)R_{effective}}\int_{T_i}^{T_f} V(t)I(t)dt} \quad (13)$$

As we can see, effective resistance is important for both correct calculations of effective voltage and current, and therefore must be calculated correctly.

b. The development for discrete time periods of constant resistance

A known voltage source V_f is applied on a resistor R_n, in which "n" is the index relative to the period of time in which R_n is valid, that is, applied to the voltage source. Therefore we can have, for example, 2 periods, one in which R_0 is applied to the source, and another in which R_1 have its turn.



Applying Ohm's Law to find the current for the subscript instants:

$$\frac{V_f}{R_n} = I_n \quad (14)$$

And then, we try to relate the current with the period. Each of the currents delivered an amount of charge due to the time the resistor was applied to the source.

$$Q_n = I_n T_n \quad (15)$$

$$Q_n = \frac{V_R}{R_n}T_n \quad (16)$$

The effective resistance will be the one that will let flow $\sum Q_n$ (charge) for a $\sum T_n$ (total period of time) due, of course, to the source V_f.

$$I_{equiv} = \frac{\sum Q_n}{\sum T_n} = \frac{\sum \frac{V_R}{R_n}T_n}{\sum T_n} \quad (17)$$

$$I_{equiv} = \frac{\sum \frac{V_R}{R_n}T_n}{\sum T_n} \quad (18)$$

$$R_{effective} = \frac{V_f}{I_{equiv}} \quad (19)$$

$$R_{effective} = \frac{V_f \sum T_n}{\sum \frac{V_R}{R_n} T_n} \quad (20)$$

$$R_{effective} = \frac{V_f \sum T_n}{V_R \sum \frac{1}{R_n} T_n} \quad (21)$$

As $V_f = V_R$,

$$R_{effective} = \frac{\sum T_n}{\sum \frac{1}{R_n} T_n} \quad (22)$$

For the case where all $T_n = T_{n-1}$, i.e., in which all periods have the same duration,

$$R_{effective} = \frac{T_n n}{T_n \sum \frac{1}{R_n}} \quad (23)$$

$$R_{effective} = \frac{n}{\sum \frac{1}{R_n}} \quad (24)$$

which is the harmonic mean of the values.

c. Relation between effective resistance and effective conductance

It is known that resistance and conductance are inverse, so

$$G_{effective} = \frac{\sum G_n}{n} \quad (25)$$

which is the arithmetic mean of the conductances involved.

d. Calculation with continuous variation of resistance as a function of time

When the resistance changes continuously as a function of time, the sums become integrals, and the T_n become ∂t. The subscripts "i" and "f" refer to "initial" and "final", respectively.

$$R_{effective} = \frac{\int_{T_i}^{T_f} dt}{\int_{T_i}^{T_f} \frac{1}{R(t)} dt} \quad (26)$$

$$R_{effective} = \frac{T_f - T_i}{\int_{T_i}^{T_f} \frac{1}{R(t)} dt} \quad (27)$$

e. Calculation with continuous variation of resistance as a function of current, voltage, or frequency

The calculation is very similar when the resistance varies depending on the current, or the voltage.

$$R_{effective} = \frac{V_f - V_i}{\int_{V_i}^{V_f} \frac{1}{R(V)} dV} \quad (28)$$

$$R_{effective} = \frac{I_f - I_i}{\int_{I_i}^{I_f} \frac{1}{R(I)} dI} \quad (29)$$

For an impedance (which can be complex), which varies in relation to frequency,

$$Z_{effective} = \frac{\omega_f - \omega_i}{\int_{\omega_i}^{\omega_f} \frac{1}{Z(\omega)} d\omega} \quad (30)$$

This can result in an integral in the complex domain. If the current or voltage varies as a function of time, we will return to a format similar to that of the resistance as a function of time. If the frequency varies (in relation to time), as happens in signal modulation, there is also a composite function.

$$R_{effective} = \frac{T_f - T_i}{\int_{T_i}^{T_f} \frac{1}{R(V(t))} dt} \tag{31}$$

$$R_{effective} = \frac{T_f - T_i}{\int_{T_i}^{T_f} \frac{1}{R(I(t))} dt} \tag{32}$$

$$Z_{effective} = \frac{T_f - T_i}{\int_{T_i}^{T_f} \frac{1}{Z(\omega(t))} dt} \tag{33}$$

Here we say the impedance varies depending on the frequency with the idea that it is a filter (active or passive), so if impedance is a function of time in some way, there will be an effective impedance associated.

f. Average power, signal effective value and effective resistance

The definition of effective resistance is similar to that of the RMS values we are used to. The RMS value is the square root value of the quadratic mean of the instantaneous values, which produces the same amount of power as the variable signal. The effective resistance is the value of the harmonic mean of the resistances (or arithmetic mean of the conductances), which allows the effective signal to produce the predicted average power.

The definition of effective resistance applied to the generic RMS voltage equation obtained earlier is

$$V_{effective} = \sqrt{\frac{R_{effective}}{T_f - T_i} \int_{T_i}^{T_f} V(t)I(t)dt} \tag{34}$$

$$V_{effective} = \sqrt{\frac{\int V(t)I(t)dt}{\int \frac{I(t)}{V(t)} dt}}\Bigg|_{T_i}^{T_f} \tag{35}$$

Or even

$$V_{effective} = \sqrt{\frac{\int P(t)dt}{\int G(t)dt}}\Bigg|_{T_i}^{T_f} \tag{36}$$

It is seen that the RMS value equation takes unexpected and inevitably more complex forms, but they are positive results, as there is a way to calculate and the procedure is now known.

For the current, the general form is

$$I_{effective} = \sqrt{\frac{\int G(t)dt \int P(t)dt}{(T_f - T_i)^2}}\Bigg|_{T_i}^{T_f} \tag{37}$$

Which is a little different from the equation for voltage, but the idea remains the same.

g. Dimensional analysis of the obtained equations

The equation of effective resistance for discrete and arbitrary duration periods has the same dimensions as that for continuous periods in time, and is

$$R_{effective} = \frac{T_f - T_i}{\int_{T_i}^{T_f} \frac{1}{R(t)} dt} \tag{38}$$

Which have dimensions of

$$[\Omega] = \left[\frac{s}{\Omega^{-1} s}\right] \tag{39}$$

and constitute a truth.

The equation of effective resistance for continuous variation of resistance as a function of voltage or current,

$$R_{effective} = \frac{V_f - V_i}{\int_{V_i}^{V_f} \frac{1}{R(V)} dV} \quad (40)$$

$$R_{effective} = \frac{I_f - I_i}{\int_{I_i}^{I_f} \frac{1}{R(I)} dI} \quad (41)$$

have dimensions of

$$[\Omega] = \left[\frac{V}{\Omega^{-1}V}\right] = \left[\frac{A}{\Omega^{-1}A}\right] \quad (42)$$

which are also coherent.

The generic RMS voltage equation for any V(t) and I(t) formats,

$$V_{RMS} = \sqrt{\frac{1}{\int_{T_i}^{T_f} \frac{I(t)}{V(t)} dt} \int_{T_i}^{T_f} V(t)I(t)dt} \quad (43)$$

has dimensions of

$$[V] = \left[\sqrt{\frac{1}{\frac{A}{V}s} VAs}\right] \quad (44)$$

$$[V] = \left[\sqrt{\frac{1}{\frac{A}{V}} VA}\right] \quad (45)$$

$$[V] = \left[\sqrt{\frac{V}{A} VA}\right] \quad (46)$$

$$[V] = \left[\sqrt{V^2}\right] \quad (47)$$

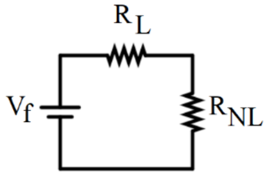
which has the expected coherence.

h. Association of linear resistances with nonlinear resistances

It is necessary to know how to associate linear resistances with nonlinear, because eventually this will happen. Moreover, it is not trivial to understand the reasons for the order of calculations, even after deduced.

i. Series association

The deduction is very similar to the section III.b deduction. Having a source and a series association of a linear resistor, R_L, with a nonlinear resistor, R_{NL}, the deduction begins.



$$\frac{V_f}{R_L + R_{NL}} = I_n \quad (48)$$

$$Q_n = I_n T_n \quad (49)$$

$$Q_n = \frac{V_f}{R_L + R_{NL}} T_n \quad (50)$$

$$I_{equiv} = \frac{\sum Q_n}{\sum T_n} \quad (51)$$

$$I_{equiv} = \frac{\sum \frac{V_f}{R_L + R_{NL}} T_n}{\sum T_n} \tag{52}$$

$$R_{equiv} = \frac{V_f}{I_{equiv}} \tag{53}$$

$$R_{equiv} = \frac{V_f \sum T_n}{\sum \frac{V_f}{R_L + R_{NL}} T_n} \tag{54}$$

$$R_{equiv} = \frac{V_f \sum T_n}{V_f \sum \frac{1}{R_L + R_{NL}} T_n} \tag{55}$$

$$R_{equiv} = \frac{\sum T_n}{\sum \frac{1}{R_L + R_{NL}} T_n} \tag{56}$$

In the limit when T_n tends to zero,

$$R_{equiv} = \frac{\int_{T_i}^{T_f} dt}{\int_{T_i}^{T_f} \frac{1}{R_L + R_{NL}(t)} dt} \tag{57}$$

$$R_{equiv} = \frac{T_f - T_i}{\int_{T_i}^{T_f} \frac{1}{R_L + R_{NL}(t)} dt} \tag{58}$$

This means that the calculation of the effective resistance of the nonlinear resistor depends on the series (linear) resistance. That is, it is not correct to calculate the effective resistance of the nonlinear resistor, and then, add with the series resistance (linear).This may become clearer from the power flow equation for discrete resistance values for the nonlinear resistor, and from the fact that $T_n = T_{n-1}$.

$$P_{dissipated} = \frac{V^2}{R_{equiv}} \tag{59}$$

$$\frac{V^2}{R_{equiv}} = \frac{1}{2}\left(\frac{V^2}{R_L + R_{NL1}} + \frac{V^2}{R_L + R_{NL2}}\right) \tag{60}$$

$$\frac{2}{R_{equiv}} = \frac{1}{R_L + R_{NL1}} + \frac{1}{R_L + R_{NL2}} \tag{61}$$

$$R_{equiv} = \frac{2}{\frac{1}{R_L + R_{NL1}} + \frac{1}{R_L + R_{NL2}}} \tag{62}$$

In the case of "n" nonlinear resistance values, where $T_n = T_{n-1}$ is not necessarily satisfied, there is a weighting for each value.

$$R_{equiv} = \frac{\sum T_n}{\sum \frac{1}{R_L + R_{NL}} T_n} \tag{63}$$

Which is the same result obtained in the equation (56).

ii. Parallel association

Parallel association leads to simpler results, as will be seen. By the dissipated power, with a linear and a nonlinear resistance, we have

$$P_{dissipada} = \frac{V^2}{R_{equiv}} \tag{64}$$

$$\frac{V^2}{R_{equiv}} = \frac{V^2}{R_L} + \frac{V^2}{R_{NL}} \tag{65}$$

For a general case of a linear resistance and "n" nonlinear resistances,

$$\frac{V^2}{R_{equiv}} = \frac{V^2}{R_L} + \frac{V^2}{R_{effective}} \tag{66}$$

$$\frac{1}{R_{equiv}} = \frac{1}{R_L} + \frac{1}{R_{effective}} \tag{67}$$

As

$$R_{effective} = \frac{\sum T_n}{\sum \frac{1}{R_{NL}} T_n} \tag{68}$$

$$\frac{1}{R_{equiv}} = \frac{1}{R_L} + \frac{1}{\frac{\sum T_n}{\sum \frac{1}{R_{NL}} T_n}} \tag{69}$$

$$R_{equiv} = \frac{1}{\frac{1}{R_L} + \frac{\sum \frac{1}{R_{NL}} T_n}{\sum T_n}} \tag{70}$$

That is, first the effective resistance of the nonlinear element is calculated, then the parallel is calculated.

IV. Theoretical application in semiconductors

The semiconductor chosen was the diode, because it is the simplest and can demonstrate the usefulness of equations without other adjacencies being present.

a. Calculation in the diode's exponential region

It begins with the Schockley equation, which describes the current in the diode due to the voltage applied to it.

$$I_d = I_s(e^{kV_D} - 1) \tag{71}$$

In which

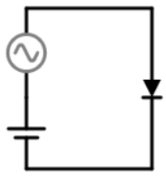
$$k = \frac{q}{nk_B T} \tag{72}$$

Being "n" the coefficient of ideality, and "k_B" Boltzmann's constant. From this, the apparent resistance of the diode is given by the following equation:

$$R_{apparent} = \frac{V_D}{I_s(e^{kV_D} - 1)} \tag{73}$$

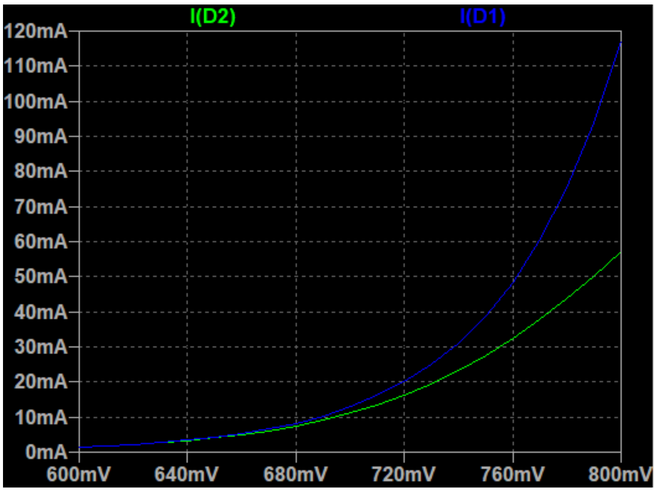
b. Choosing a quiescent point

A possible application is the calculation of the effective apparent resistance given a quiescent point and an alternating voltage on that point. The following figure expresses the example.

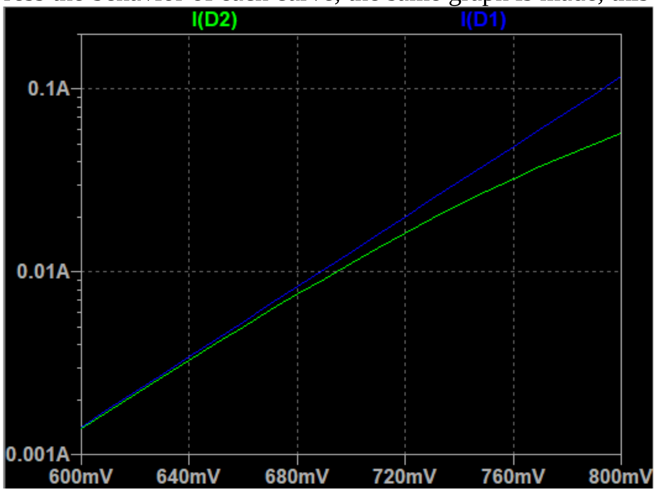


The maximum point (800 mV) is justified by generating a current below the maximum for the chosen diode (200 mA), even in the situation without body resistance, enabling the practical experimentation of this circuit without the burning of the semiconductor. Thus, there are no excessively large currents and voltages, which would make the practice impossible.

The minimum point (600 mV) is practically arbitrary, and could be any non-negative value. This is due to the fact that the diode has different mathematical behavior when in blockage compared to when in forward biased. However, in case of not having too small voltages and currents, 600 mV was chosen to be not below 1 mA. The following figure shows the behavior of diode 1N4148, D2, in green, and the same diode when without body resistance, D1, in blue.



The curve of D2 will always have lower current value (for the same voltage) because there is body resistance. In order to express the behavior of each curve, the same graph is made, this time in log-linear scale.



Therefore, diode D2 exhibits exponential behavior *only* at low voltages or currents, whereas diode D1 (without body resistance) is always exponential. As can be seen in the last figure, up to close to 0.01 A there is a line on a logarithmic scale, characterizing the exponential curve of Shockley. From 0.01 A, there is a transition to a logarithmic curve (in a log-linear graph), characterizing a linear curve, due to the logarithmic scale, because of the already commented diode body resistance.

c. Exponential modeling of the diode

The mathematical modeling of the diode is done by capturing the simulation parameters (of the LTSpice software), and later using these values in the Schockley equation. The parameters that the software uses for the 1N4148 model are

$$I_s = 2{,}52 \cdot 10^{-9}\, A$$

$$R_s = 0{,}568\, \Omega$$

$$n = 1{,}752$$

$$T = 300{,}15\, K\ (27\ °C)$$

And the (physical) constants that are already known, the Boltzmann's constant and the absolute charge of the electron.

$$k_B = 1{,}38064852 \cdot 10^{-23}\, J/K$$

$$e = 1{,}60217662 \cdot 10^{-19} C$$

For the simplification of the later equations, the variable that has already been defined is already calculated here,

$$k = 22{,}067585$$

So the diode equation is

$$I_d = 2{,}52 \cdot 10^{-9}(e^{22{,}067585\ V_D} - 1)\ A \quad (74)$$

when without body resistance. In the other case where there is body resistance, there is no way to include this parameter in the Schockley equation in order to isolate I_d, because the equation becomes recursive, nonlinear, and numerical methods should be used for its resolution.

d. Effective apparent resistance continuous calculation

So the calculation of effective apparent resistance is

$$R_{ap.\ ef.\ D1} = \frac{T_f - T_i}{\int_{T_i}^{T_f} \frac{1}{R(V_D(t))} dt} \quad (75)$$

$$R_{ap.\ ef.\ D1} = \frac{1-0}{\int_0^1 \frac{2{,}52\cdot 10^{-9}(e^{22{,}067585\ V_D(t)}-1)}{V_D(t)} dt} \quad (76)$$

$$R_{ap.\ ef.\ D1} = \frac{1}{\int_0^1 \frac{2{,}52\cdot 10^{-9}(e^{22{,}067585\ (0{,}7+0{,}1\cos(2\pi t))}-1)}{0{,}7+0{,}1\cos(2\pi t)} dt} \quad (77)$$

$$R_{ap.\ ef.\ D1} = 22{,}6439\ \Omega \quad (78)$$

This means that in a 1N4148 diode, a DC voltage of 700 mV added to a sine voltage of 100 mV, there is an effective apparent resistance of 22.6439 ohms.

For the diode with body resistance, reclaiming equation (58)

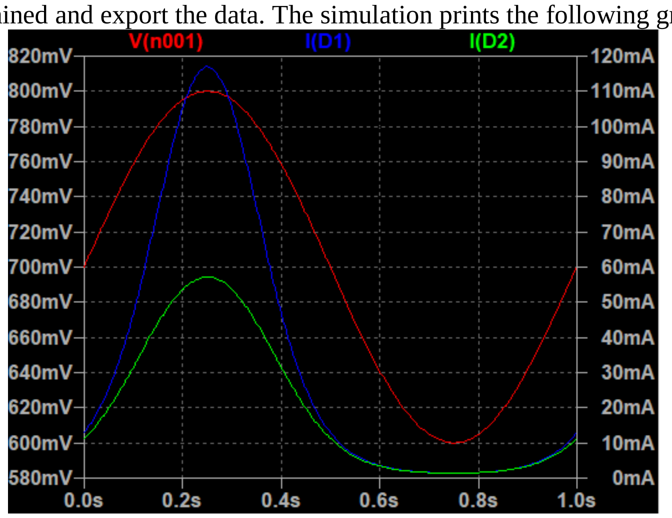
$$R_{ap.\ ef.\ D2+RC} = \frac{T_f - T_i}{\int_{T_i}^{T_f} \frac{1}{R_L + R_{NL}(t)} dt} \quad (79)$$

$$R_{ap.\ ef.\ D2+RC} = \frac{1}{\int_0^1 \frac{1}{0{,}568+\frac{V_D(t)}{I_D(t)}} dt} \quad (80)$$

In order to take advantage of the convergence algorithm already implemented in LTspice for these situations, it is then said that the $R_{ap.\ ef.\ D2+RC}$ will be known from subsequent calculations. Thus, the body resistance (linear) is discounted, obtaining the $R_{ap.\ ef.\ D2}$.

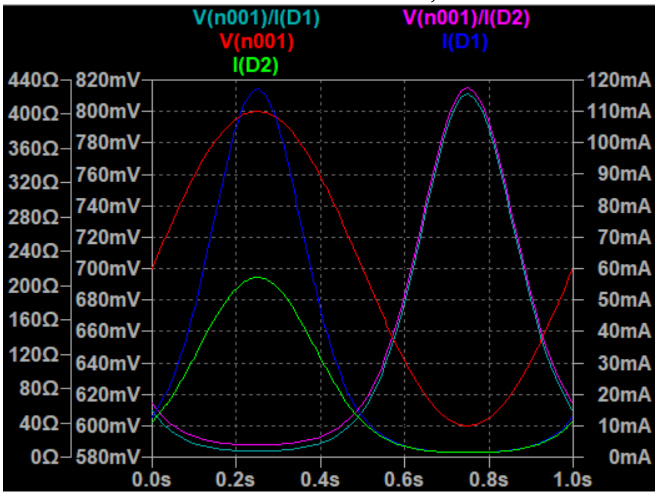
e. Simulation in software

Through the simulation of diode 1N4148 in the LTSpice software, it is possible to compare the simulated results, by a known and reliable software, with the calculations performed.Here we start to simulate the points already determined and export the data. The simulation prints the following graph.



In which one can see the distortion in the current signal, caused by the semiconductor element, although the harmonic distortion itself is not important in this work, even though it is correlated.

A second graph is used to exemplify part of what is intended to calculate. Lines referring to the instantaneous apparent resistance of diode D1 and D2 were added, which are calculated through Ohm's law.



It is also possible to see that the $R_{ap.\ D1} < R_{ap.\ D2}$ is always valid, as expected. The highs and lows are around 431 ohms and 7 ohms, which partially validate the calculations, because an average function, whatever it is, never results in a value outside the source internal range.

The export of 1021 voltage points, current, and time, was made for extension "txt" and then imported and treated in Microsoft Excel according to the following section.

f. Continuous calculation of average power

Before the results, which are the calculations and values made and received with Microsoft Excel, we should know the continuous values first, for comparison purposes.

The calculation of the dissipated average power can be done by integrating the product of the current into the diode (D1) by the source voltage.

$$P_{avg.\ D1} = \frac{1}{1-0}\int_0^1 [0{,}7 + 0{,}1\cos(2\pi t)]\left[2{,}52 \cdot 10^{-9}\left(e^{22{,}067585\,(0{,}7+0{,}1\cos(2\pi t))} - 1\right)\right]dt \quad (81)$$

$$P_{avg.\ D1} = 26{,}3121\ mW \quad (82)$$

For diode D2, as already commented out, current is a recursive function that must be calculated point by point by numerical methods, and then integrated together with the source voltage.

$$P_{avg.\ D1} = \frac{1}{1-0}\int_0^1 [0{,}7 + 0{,}1\cos(2\pi t)][2{,}52 \cdot 10^{-9}(e^{22{,}067585\ V_D} - 1)]dt \quad (83)$$

The equation (83) stays without a possible calculation.

V. Results

First, the values for diode D1, which has no body resistance, are calculated. To obtain effective (discrete) apparent resistance, the following calculations are made in Microsoft Excel.

$$r_{D1}[n] = \frac{V[n]}{I[n]} \quad (84)$$

$$\Delta t[n] = t[n] - t[n-1] \quad (85)$$

$$r_{ap.\ ef.\ D1} = \frac{\sum \Delta t[n]}{\sum \frac{\Delta t[n]}{r_{D1}[n]}} \quad (86)$$

And then it was obtained that

$$r_{ap.\ ef.\ D1} = 22{,}644\ \Omega \quad (87)$$

This characterizes a negligible deviation from the calculated with integral (22,6439 ohms), validating the calculations in the continuous domain. Thus, the discrete calculation is not only very close to continuous calculation, but also shows that the simulation made by LTspice is also very close to that calculated outside the software.

For effective voltage and current calculation, it is necessary to integrate instantaneous power. Because once again there are no fixed time intervals, the average should be weighted.

$$p_{D1}[n] = V[n]I[n] \tag{88}$$

$$p_{avg.D1} = \frac{\sum P[n]\Delta t[n]}{\sum \Delta t[n]} \tag{89}$$

$$p_{avg.D1} = 26{,}311\ mW \tag{90}$$

The difference from the continuum (26,3121 mW), again, is negligible. With the average power (the discrete value) in hands, along with the effective apparent resistance, it is departed for the calculations of the voltage and effective current. By the relationships already established, it is clear that

$$V_{D1\ effective} = 771{,}886\ mV \tag{91}$$

$$I_{D1\ effective} = 34{,}087\ mA \tag{92}$$

In order to show that the traditional RMS calculation would not be correct in this situation, the following procedure is done in table.

$$V_{rms\ D1} = \sqrt{\frac{\sum V[n]^2\Delta t[n]}{\sum \Delta t[n]}} \tag{93}$$

$$V_{rms\ D1} = 703{,}562\ mV \tag{94}$$

Another common calculation is through the independence of the AC and DC source,

$$Vrms = \sqrt{{V_{AC}}^2 + {V_{DC}}^2} \tag{95}$$

$$Vrms = 707{,}107\ mV \tag{96}$$

It is noted that the values are discrepant in relation to what was calculated through discrete power and discrete effective apparent resistance. For the current also happens considerable discrepancy.

$$I_{rms\ D1} = \sqrt{\frac{\sum I[n]^2\Delta t[n]}{\sum \Delta t[n]}} \tag{97}$$

$$I_{rms\ D1} = 51{,}861\ mA \tag{98}$$

Even more, the power is more discrepant.

$$P_{i.\ avg.\ D1} = 36{,}488\ mW \tag{99}$$

Characterizing a deviation of 38.67% in relation to the value calculated discretely, calculated through the equation of the average power, showing that the RMS calculation is *not* the correct way to calculate when there are nonlinear resistors.

The same calculation of the effective apparent resistance performed for D1, through equations (84), (85), and (86) is performed for D2, and it is obtained that

$$r_{ap.\ ef.\ D2+RC} = 37{,}579\ \Omega \tag{100}$$

Which is consistent with what is expected, since D2 should present greater effective apparent resistance due to body resistance. It is also noticeable the increase of almost 15 ohms due only to a body resistance of 0.568 ohms. This is because the body resistance decreases the current by the junction, further increasing the effective apparent resistance. This can be perceived by analyzing the equation (73).

At this point it is possible to estimate the effective apparent resistance only of the semiconductor junction. Since

$$r_{ap.\ ef.\ D2+RC} = r_{ap.\ ef.\ D2} + R_C \tag{101}$$

And then

$$r_{ap.\ ef.\ D2} = 37{,}011\ \Omega \quad (102)$$

In this case it is not of great importance to know the effective apparent resistance only of the junction.

Now the discrete average power is calculated over D2, as calculated for D1, using the equation (89).

$$p_{avg\ .D2} = 15{,}527\ mW \quad (103)$$

Which is according to the expected ($p_{avg\ .D2} < p_{avg\ D1}$), since a higher resistance decreases the average power for the same voltage value. The effective voltage and current, through (7) and (8), are then

$$V_{D2\ effective} = 763{,}876\ mV \quad (104)$$

$$I_{D2\ effective} = 20{,}327\ mA \quad (105)$$

It is seen that $V_{D2\ effective} \neq V_{D1\ effective}$, which was expected, whereas effective resistances are different.

Applying the equations (93) and (97) for D2, we have

$$V_{rms\ D2} = 703{,}562\ mV \quad (106)$$

$$I_{rms\ D2} = 28{,}359\ mA \quad (107)$$

Then

$$P_{i.\ avg\ D2} = 19{,}952\ mW \quad (108)$$

Finally, this section ends with a table to summarize the results obtained by each calculation, remembering that there are calculations that were applied outside its scope purposely, for comparison with the correct calculations.

Table no 1: Shows effective values for diodes D1 and D2.

	$V_{effective}$	$I_{effective}$	$P_{avg.}$	$r_{ap.\ ef.}$
D1	771,886 mV	34,087 mA	26,311 mW	22,644 Ω
D2	763,876 mV	20,327 mA	15,527 mW	37,579 Ω

Table no 2: Shows RMS values for diodes D1 and D2, as if they were linear loads.

	$Vrms$	$Irms$	$P_{i.\ avg}$
D1	703,562 mV	51,861 mA	36,488 mW
D2	703,562 mV	28,359 mA	19,952 mW

VI. Discussion

In the tables 1 and 2 it is possible to see that the effective current is always smaller than the RMS current, and the opposite happens to the voltage. The average power is always lower than the average incorrect power, meaning that every calculation that has made incorrectly overestimated the value. Remembering that these tables are only valid for the situation of 0.7 V DC + 0.1 V AC. Another quiescent point with another sinusoidal amplitude would have other values.

When we have one linear circuit and another nonlinear equivalent to each other, only the power and effective resistance are equal between the circuits, the other effective values (voltage and current) remain different. Therefore, the RMS equation only serves for linear elements.

In short, the calculation of effective apparent resistance for large signals has the following flow:

- Simulate the semiconductor element in the region of interest (AC+DC) or through numerical iterations;
- Export data and apply the calculations made in (84) and (85);
- Equation (86) will give the result;
- Equation (101) can be applied if there is a resistor in series.

VII. Conclusion

Several conclusions can be obtained through this paper. In most applications, the apparent resistance of the semiconductor is not considered, as the limiting (series) resistance is much higher (in high currents). Therefore, there is not much difference between considering or not the apparent effective resistance in most uses of diodes and transistors.

In some applications that require greater accuracy of results, or where there are small currents, such as in integrated circuits, considering effective apparent resistance can become an important factor, whether for calculating input or output impedance, for dissipated power, or for calculating effective voltage or current in the semiconductor. In simulations in which the RMS voltage is calculated by the software itself, it has been shown that the effective voltage differs from the RMS voltage, and it can be seen that this difference is greater the greater the nonlinearity of the element under analysis. In general, we have a semiconductor with its apparent effective resistance much higher than its body resistance (as shown in the calculated values), making it be considered as an ideal voltage drop (due to its variable portion as a function of the applied voltage), along with a linear resistance, which adds a voltage drop.

It is important to note that, in terms of application, any already known calculation of RMS fails for a nonlinear resistor. The solution to this calculation is to use the generic equation (36) for voltage and the generic equation (37) for the current, which can be applied to any situation.

As a note, use the RMS equation for linear resistors, the generic equation for both non-linear or when in doubt whether an element is linear or not. As an example of change of effective voltage and effective current we have the type of load and the quiescent point.

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