

Revisions for Aggregation with Fuzzy Diagnosis

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Abstract This paper is a consequence upon the paper of J.Y. Ahn, Y.H. Kim and S.K. Kim that was published in IEICE Transactions on Information & Systems, for fuzzy differential diagnosis under intuitionistic fuzzy set environments. We point out that their aggregation procedure for two intuitionistic fuzzy relations may violate the definition of intuitionistic fuzzy set. Our revisions not only improve the shortcomings of their procedure but also are compatible with the fuzzy set. Our findings provide a solid theoretical foundation for medical diagnosis with the intuitionistic fuzzy set.

Key words: Fuzzy differential diagnosis; Intuitionistic fuzzy set; Quantified data

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I. Introduction

Medical diagnostic investigations are very complex. The doctor is faced with a patient who has his personal experiences, knowledge from books, and mental endowment. The doctor notes the patient's signs and symptoms, combined with the patient's medical history, physical examination, and laboratory findings, and then diagnoses the disease. The fuzzy set framework has been utilized in several different approaches to modeling the diagnostic process. In the approach formulated by Sanchez [18], he adopted the compositional rule of inference by Zadeh [27] as an inference mechanism. It accepts fuzzy descriptions of a patient's symptoms and infers fuzzy descriptions of a patient's diseases employing the fuzzy relationships described before. With P, a set of patients, and a fuzzy relation Q from P to S, and by 'max-min composition', we get the fuzzy relation $T = Q \times R$ with the membership function

$$\mu_T(p, d) = \max_{s \in S} (\min \{ \mu_Q(p, s), \mu_R(s, d) \}), s \in S, d \in D, p \in P.$$

Furthermore, based on the concepts of fuzzy sets theory, several fuzzy approaches to medical diagnosis have been reviewed by Steimann and Adlassnig [20] and shown to be effective in this domain: Yao and Yao [25] based on the fuzzy number and compositional rule of inference to solve medical diagnosis problem; Palma et al. [16] provided a general framework for temporal model-based diagnosis (TMBD) which can deal with the time-varying behavior found in the medical domain; Seising [19] has linked Zadeh's [27] worked on system theory and made reviews for medical diagnosis; Quteishat and Lim [17] provided an application for the fuzzy min-max neural networks in medical diagnosis. In addition, also several investigations in medical diagnosis have addressed these issues based on intuitionistic fuzzy sets, such as De et al. [7], using intuitionistic fuzzy sets to apply in medical diagnosis; Innocent and John [12] presented a new method for computing a diagnostic support index which uses vague symptom and temporal information in a clinical diagnosis context. Szmidt and Kacprzyk [22] provided a measuring method for intuitionistic fuzzy sets and showed its usefulness in medical diagnostic reasoning; Xu [25] proposed a new method for deriving the correlation coefficients to the interval-valued intuitionistic fuzzy set theory, and showed its application in medical diagnosis; Vlachos and Sergiadis [24] presented an information-theoretic approach to discrimination measures for intuitionistic fuzzy sets and their application in medical diagnosis.

Besides, Ahn et al. [1] have also presented a medical diagnostic method by applying intuitionistic fuzzy sets to perform classification of solution sets and linear regression approach. However, their results are questionable. Therefore, this paper aims to point out their errors and to provide a revised method to handle medically diagnosed problems. There are 14 papers: Ahn et al. [2, 3], Chang and Hung [5], Choi et al. [6], Gupta et al. [8], Hung [9], Hung and Tuan [10], Hung et al. [11], Khayamnia et al. [13-15], Symanzik et al. [21], and Tripathy et al. [23] that cited Ahn et al. [1] in their references. However, none of them had pay attention to the questionable results in Ahn et al. [1] such that in this paper, we will provide a detailed discussion and then provide our improvements.

The remainder of this study is organized as follows: Section 2 defines some notations and assumptions for intuitionistic fuzzy sets. Section 3 reviews Ahn et al.'s **Error! Reference source not found.**

method, and provides an example to illustrate their questionable in Section 4; Section 5 provides a revised method for medical diagnosis. Finally, a conclusion is drawn in section 6.

II. Review of Ahn et al. [1]

The intuitionistic fuzzy set (IFS) is introduced in Atanassov [4] as an extension of the fuzzy set. An IFS A in a fixed set E is an objective with the expression

$$A = \left\{ \left\langle x, \mu_A(x), \nu_A(x) \right\rangle \mid x \in E \right\}, \tag{1}$$

where the functions $\mu_A : E \rightarrow [0,1]$ and $\nu_A : E \rightarrow [0,1]$ denote the degree of membership and the degree of nonmembership of the element $x \in E$, respectively. For every $x \in E$,

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1. \tag{2}$$

When $\mu_A(x) + \nu_A(x) = 1$, for every $x \in E$, then the IFS will degenerate to a fuzzy set. Hence, we can consider a fuzzy set with its membership function $\mu_A(x)$, having the IFS expression as

$$A = \left\{ \left\langle x, \mu_A(x), 1 - \mu_A(x) \right\rangle \mid x \in E \right\}. \tag{3}$$

Under the condition of $\nu_A(x) = 1 - \mu_A(x)$, for every $x \in E$.

We recall that $R_s = \left\{ \left\langle (p, s), \mu_{R_s}(p, s), \nu_{R_s}(p, s) \right\rangle \mid (p, s) \in P \times S \right\}$ is the intuitionistic fuzzy relation (IFR) on the set $P \times S$, and $R_o = \left\{ \left\langle (s, d), \mu_{R_o}(s, d), \nu_{R_o}(s, d) \right\rangle \mid (s, d) \in S \times D \right\}$ with $P = \{P_1, \dots, P_q\}$ is the set of patients, $S = \{S_1, \dots, S_m\}$ is the set of symptoms, and $D = \{D_1, \dots, D_n\}$ is the set of diseases. The membership grade $\mu_{R_s}(p, s)$ indicates the degree to which the symptom s appears in the patient p , and $\nu_{R_s}(p, s)$ indicates the degree to which the symptom s does not appear in the patient p . On the other hand, the membership grade $\mu_{R_o}(s, d)$ indicates the frequency of occurrence of a symptom s with a disease d and $\nu_{R_o}(s, d)$ indicates the frequency of occurrence of symptom s not with disease d .

To compute the occurrence between patients and diseases, Ahn et al. [1] assumed that $R_1 = R_s * R_o$ with

$$\mu_{R_1}(p, D_j) = \max_{S_i} \left\{ \min \left\{ \mu_{R_s}(p, S_i), \mu_{R_o}(S_i, D_j) \right\} \right\}, \tag{4}$$

and

$$\nu_{R_1}(p, D_j) = \max_{S_i} \left\{ \min \left\{ \nu_{R_s}(p, S_i), \nu_{R_o}(S_i, D_j) \right\} \right\}. \tag{5}$$

However, Ahn et al. [1] did not check that their new IFR $R_1 = R_s * R_o$ satisfies the condition of Equation (2) as

$$0 \leq \mu_{R_1}(p, D_j) + \nu_{R_1}(p, D_j) \leq 1. \tag{6}$$

Here, we begin to discuss Questionable Results in Ahn et al. [1] for Their Composition of IFRs.

In the following, we will provide a counterexample to demonstrate that according to their approach, Equation (6) is violated.

We assume that the set of patients $P = \{P_1\}$, the set of symptoms $S = \{S_1, S_2, S_3, S_4\}$ and the set of diseases $D = \{D_1\}$ so that the membership function and non-membership function for IFR of patient and symptoms are defined as $\mu_{R_s}(1,1) = 0.1$, $\mu_{R_s}(1,2) = 0.8$, $\mu_{R_s}(1,3) = 0.4$, $\mu_{R_s}(1,4) = 0.5$, $\nu_{R_s}(1,1) = 0.8$, $\nu_{R_s}(1,2) = 0.1$, $\nu_{R_s}(1,3) = 0.4$, $\mu_{R_s}(1,4) = 0.2$. On the other hand, the membership function and non-membership function for IFR of symptoms and disease are defined as $\mu_{R_o}(1,1) = 0.3$, $\mu_{R_o}(2,1) = 0.2$, $\mu_{R_o}(3,1) = 0.7$, $\mu_{R_o}(4,1) = 0.6$, $\nu_{R_o}(1,1) = 0.7$, $\nu_{R_o}(2,1) = 0.5$, $\nu_{R_o}(3,1) = 0.1$, $\mu_{R_o}(4,1) = 0.3$.

It yields that

$$\mu_{R_1}(1,1) = \max_{1 \leq i \leq 4} \min \{ \mu_{R_s}(1,i), \mu_{R_o}(i,1) \} = \max \{ 0.1, 0.2, 0.4, 0.5 \} = 0.5, \tag{7}$$

and

$$\nu_{R_1}(1,1) = \max_{1 \leq i \leq 4} \min \{ \nu_{R_s}(1,i), \nu_{R_o}(i,1) \} = \max \{ 0.7, 0.1, 0.1, 0.2 \} = 0.7. \tag{8}$$

We discover that

$$\mu_{R_1}(1,1) + \nu_{R_1}(1,1) = 1.2 \tag{9}$$

so condition in Equation (2) does not hold. It points out that the construction of Ahn et al. [1] to synthesize two IFRs contained questionable results.

III. Our improvements

The definition of Equation (5) may be revised as

$$\nu_{R_1}(p, D_j) = \min_{S_i} \left\{ \max \{ \nu_{R_s}(p, S_i), \nu_{R_o}(S_i, D_j) \} \right\}. \tag{10}$$

First, we show that our definition is consistent with fuzzy sets. It means that we are under the conditions of

$$\nu_{R_s}(p, S_i) = 1 - \mu_{R_s}(p, S_i), \tag{11}$$

and

$$\nu_{R_o}(S_i, D_j) = 1 - \mu_{R_o}(S_i, D_j). \tag{12}$$

It shows that

$$\begin{aligned} \nu_{R_1}(p, D_j) &= \min_{S_i} \left\{ \max \{ 1 - \mu_{R_s}(p, S_i), 1 - \mu_{R_o}(S_i, D_j) \} \right\} \\ &= \min_{S_i} \left\{ 1 - \min \{ \mu_{R_s}(p, S_i), \mu_{R_o}(S_i, D_j) \} \right\} \\ &= 1 - \max_{S_i} \left\{ \min \{ \mu_{R_s}(p, S_i), \mu_{R_o}(S_i, D_j) \} \right\} \\ &= 1 - \mu_{R_1}(p, D_j). \end{aligned} \tag{13}$$

Hence, our definition of Equation (10) is well-defined for fuzzy sets.

Next, we go back to the IFS environment, under our revision, and then we will prove that $\mu_{R_1}(p, D_j) + \nu_{R_1}(p, D_j) \leq 1$. We assume two auxiliary indexes a and b such that

$$\mu_{R_1}(p, D_j) = \min \{ \mu_{R_s}(p, S_a), \mu_{R_o}(S_a, D_j) \}, \tag{14}$$

and

$$\nu_{R_1}(p, D_j) = \max \{ \nu_{R_s}(p, S_b), \nu_{R_o}(S_b, D_j) \}. \tag{15}$$

We divide into two cases: (1) $a = b$, and (2) $a \neq b$.

For case (1), $a = b$. We will verify that

$$\min \{ \mu_{R_s}(p, S_a), \mu_{R_o}(S_a, D_j) \} + \max \{ \nu_{R_s}(p, S_a), \nu_{R_o}(S_a, D_j) \} \leq 1, \tag{16}$$

Under the conditions $\mu_{R_s}(p, S_a) + \nu_{R_s}(p, S_a) \leq 1$ and $\mu_{R_o}(S_a, D_j) + \nu_{R_o}(S_a, D_j) \leq 1$.

We know that

$$\nu_{R_s}(p, S_a) + \min \{ \mu_{R_s}(p, S_a), \mu_{R_o}(S_a, D_j) \} \leq \mu_{R_s}(p, S_a) + \nu_{R_s}(p, S_a) \leq 1, \tag{17}$$

and

$$\nu_{R_o}(S_a, D_j) + \min \{ \mu_{R_s}(p, S_a), \mu_{R_o}(S_a, D_j) \} \leq \mu_{R_o}(S_a, D_j) + \nu_{R_o}(S_a, D_j) \leq 1, \tag{18}$$

We combine Equations (17) and (18) to obtain that

$$\max \{ \nu_{R_s}(p, S_a), \nu_{R_o}(S_a, D_j) \} + \min \{ \mu_{R_s}(p, S_a), \mu_{R_o}(S_a, D_j) \} \leq 1, \tag{19}$$

so that Equation (16) is verified.

For case (2), $a \neq b$. According to our definition (10), it yields that

$$\nu_{R_1}(p, D_j) \leq \max [\nu_{R_s}(p, S_a), \nu_{R_o}(S_a, D_j)]. \tag{20}$$

We derive that

$$\begin{aligned} &\mu_{R_1}(p, D_j) + \nu_{R_1}(p, D_j) \leq \\ &\min [\mu_{R_s}(p, S_a), \mu_{R_o}(S_a, D_j)] + \max [\nu_{R_s}(p, S_a), \nu_{R_o}(S_a, D_j)] \leq 1, \end{aligned} \tag{21}$$

where the last inequality is based on Equation (19). According to our previous discussion, we summarize our findings in the next theorem.

Theorem 1. After our revision, the sum of the membership and non-membership functions for two IFRs is less than unity to satisfy the definition of an intuitionistic fuzzy set.

IV. Conclusion

We provide a patchwork for the aggregation process of two intuitionistic fuzzy relations so that the resulting intuitionistic fuzzy relation is well-defined. Our improvement will help researchers to apply intuitionistic fuzzy sets to medical diagnosis.

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