

2-Dimensional and 3-Dimensional Electromagnetic Fields Using Finite element method

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Abstract: The Finite element method (FEM) is a generalization of the finite difference method (FDM) that was considered in the previous section. Instead of subdividing the area into small squares with a side h , this technique subdivides the area into small triangles. As we will observe, this method is more flexible in its application. For example, the calculation of the potential profile in the region between two concentric rectangular metallic surfaces can be handled with this technique. In this particular subject we have perfected a Graphical User Interface (GUI) in Matlab in which 2-dimensional and 3-dimensional case studies were taken. The technique has a strong foundation using matrix manipulations

KEYWORDS: 2 DIMENSION , 3 DIMENSION , GRAPHICAL USER INTERFACE , FEM

I. INTRODUCTION

There are different methods for the numerical solution of the two-dimensional Laplace's and Poisson's equation. Some of the techniques are based on a differential formulation that was introduced earlier. The Finite difference method is considered here and the Finite element method is discussed in the next section. Other techniques are based on the integral formulation of the boundary value problems such as the Method of moments is described later. The boundary value problems become more complicated in the presence of dielectric interfaces which are also considered in this section.

II. Finite Element Method

Let us consider an example, in Figure (1), one quarter of the cross section of a rectangular coaxial line is shown. Figure (2) depicts the appropriate modeling with the finite triangular elements. The mesh is irregular and the grid is denser in the vicinity of the corners where a more rapid variation of the potential is to be expected. The scalar potential V satisfies Laplace's equation (a) inside the area S

$$\nabla^2 V = 0 \tag{1}$$

while it satisfies two different conditions on the boundaries. 1) On the boundary $L=L_1+L_2$, the voltage is specified to have any particular value. In Figure (2), the voltages are $V = 1$ on one conductor and $V = 0$ on the other conductor. This is called a Dirichlet's boundary condition; 2) Because of the symmetry inherent in this problem, we require that the normal derivative of the voltage be equal to 0 on the plane of symmetry. This is called a Neumann's boundary condition

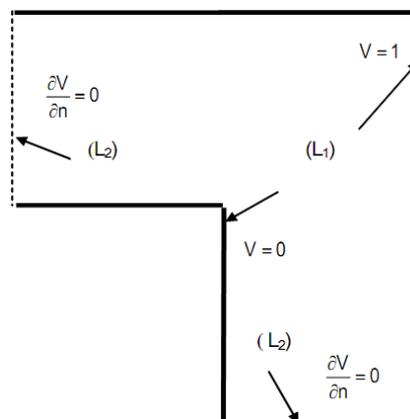


Fig. (1) Cross section of $\frac{1}{4}$ of a square coaxial line is shown.

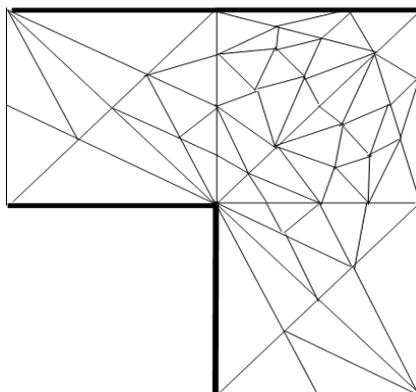


Fig. (2) A finite element mesh is used to subdivide the square coaxial cable

The electric energy that can be stored in a volume with a cross section A is

$$W = \int_{A_2} \frac{\epsilon}{2} |\nabla V|^2 dS \tag{2}$$

This integral or “functional” has a minimum value for the actual solution of this boundary value problem. The necessary condition for this minimum leads to Laplace’s equation for the potential V in the area A. The most widely separated of the elements are the linear node elements for which the potential inside a triangle can be approximated with a linear polynomial

$$V^{(e)}(X,Y) = a + bX + cY \tag{3}$$

where there are three unknown coefficients (a, b, c). These coefficients will have to be determined. In order to accomplish this, it is necessary to apply three additional conditions. The potential at the three nodes of the triangle (1, 2, 3) in Figure (3) are known. This approximation replaces the smooth solution with a piecewise smooth function that is based on a linear interpolation. The coefficients (a, b, c) can be determined from the given node potentials (V₁ , V₂ , V₃) provided that the coordinates of the nodes are known quantities Using the interpolation criteria

$$V^{(e)} = (X_j, Y_j) = V_j \quad (j = 1,2,3) \tag{4}$$

the following matrix equation for the coefficients is obtained

$$\begin{bmatrix} 1 & X_1 & Y_1 \\ 1 & X_2 & Y_2 \\ 1 & X_3 & Y_3 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} \tag{5}$$

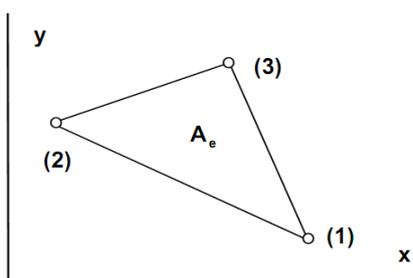


Fig (3) Triangular finite element

Using the explicit expressions, it can be shown that these three linear basis functions satisfy the following interpolation criteria

$$\alpha_j(X_j, Y_j) = f(x) = \begin{cases} 0, & i \neq j \\ 1, & i = j \end{cases} \tag{6}$$

The total energy of the ensemble of all elements in the mesh can be calculated as just being the sum of the energy of each of the individual elements

$$W = \sum_e W^{(e)} \tag{7}$$

There is also a need to know the procedure for adding a new element to the existing elements in the mesh. For simplicity, we consider the case where the mesh consists of only one element as shown in Figure (4). The problem is in obtaining the S-matrix of the new ensemble in terms of the S-matrix of the previous triangular mesh $S^{(1)}$ and the S-matrix of the new triangular mesh $S^{(2)}$. We should expect that the potential to be the same at all of the common nodes for both meshes. This will impose certain boundary conditions in that $V_1=V_4$ and $V_2=V_6$ after coupling. In addition, we also renumber the other node for convenience as $V_4=V_5$ after coupling

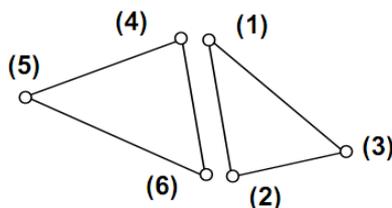


Fig 4(a) Decoupled elements

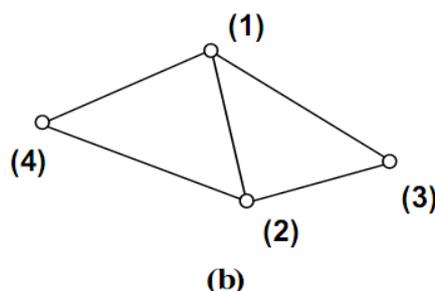


Fig 4(b) Coupled Elements

It is now time to consider the role that sources have on the system and to ascertain their effect on the solution of the boundary value problem. There are two types of sources: 1) surface sources that are prescribed potentials on the boundary L_1 in Laplace's equation and 2) volume sources such as a charge density in Poisson's equation. For simplicity, only the charge free Laplace's equation will be considered in this discussion. Poisson's equation can also be solved using similar techniques but it will not be discussed further. We assume surface sources are known which means that the column matrix of the coupled potentials.

III. Fem Implementation In Matlab

The main advantage of the FEM-method in comparison with the FDM-method is its flexibility. This will be demonstrated by applying it to areas with different shapes. The shapes are covered by triangles in the domain of interest. This may lead to some complication in the mesh generation which is, of course, a disadvantage. This can be avoided to some extent by using a procedure for automatic mesh generation. In this book, the mesh is manually introduced for pedagogical reasons. There is a commercial package FEMLAB that can be used with MATLAB to solve more complicated problems than we will encounter here. The procedure of using this technique has five stages:

- 1) Generation of the mesh;
- 2) Inclusion of the surface and volume sources;
- 3) Construction of the matrices for every element;
- 4) Collection of all the elements of the $[S(e)]$ -matrix;
- 5) Solution of the resulting matrix equation.

For the purpose of finding various electric and magnetic quantities using the FEM method, a simple graphical user interface (GUI) is created whose snapshot is given in figure (5), the graphical user interface has several examples in which various quantities are deduced using graphs etc using the techniques we just saw.



Fig (5) Snapshot of created Graphical User Interface

IV. Results And Discussions

In the GUI that we have created we have incorporated features such as various standard notations and configurations are chosen, whose result we already know, hence the conformality of the result are obvious. We have used various electromagnetic configurations in our GUI, let us see the results of those one by one.

As shown in the figure below, we have created a graphical output showing the magnetic field created by a straight current carrying wire. The computations are performed by finding the magnetic field intensity by finite element methods discussed earlier.

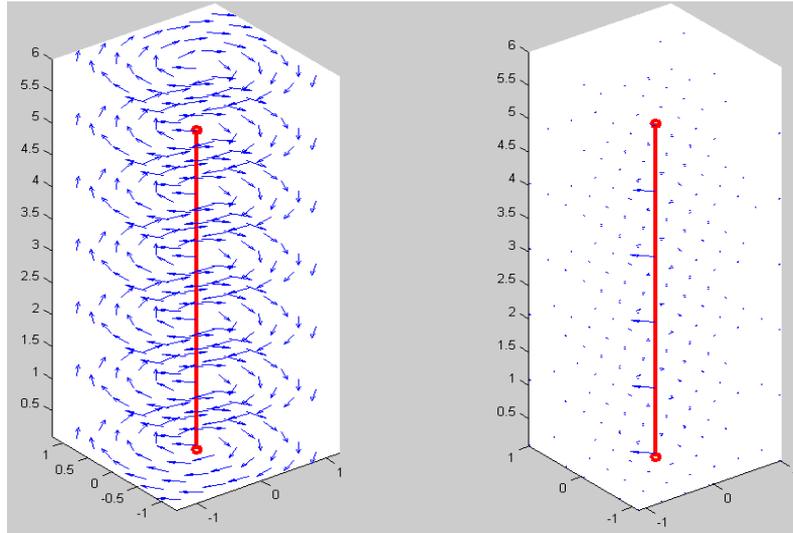


Figure (6) Magnetic Field due to a straight current carrying conductor

On the similar context magnetic field were created due to other configurations namely, magnetic field due to bent wire and also magnetic field due a solenoid. These are depicted in figure (7) and figure (8) respectively

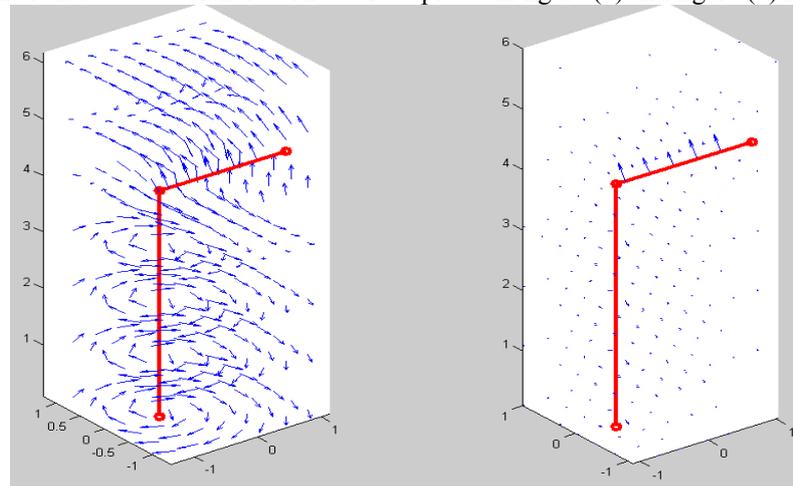


Figure (7) Magnetic Field due to Bent Wire

In figure(8) it is calculated the field and potential distribution to a unit charge dipole, this calculation was based entirely on the techniques of finite element method. This simple program computes the Electric Fields due to dipole in a 2-D plane using the Coulomb's Law on the postulates of finite element method.

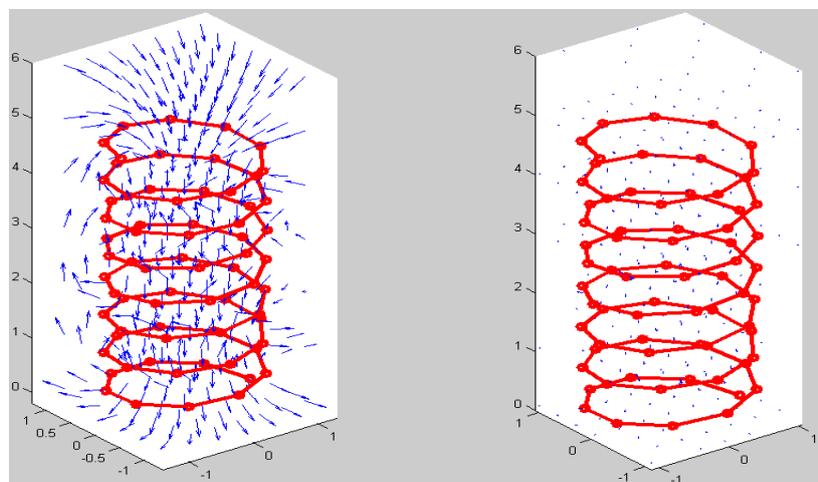


Figure (7) Magnetic Field due to a solenoid

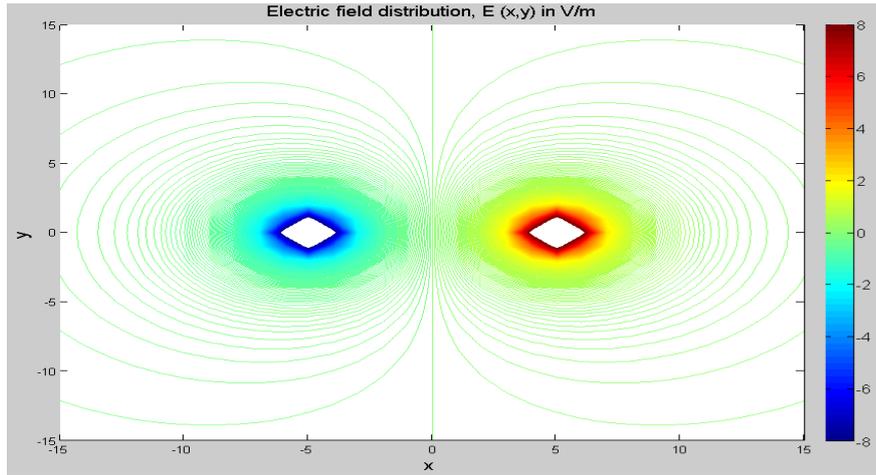


Figure (8) Field and potential due to a dipole

In figure (9) magnetic field calculations were done due a square loop. Square loop is in the X-Y plane and Magnetic Field is Evaluated at every point in the Y-Z plane($X=0$). We calculated R-vector from the loop(X-Y plane) to Y-Z plane where we are interested to find the magnetic field and also the dl-vector along the loop where current is flowing (note that R is the position vector pointing from loop (X-Y plane) to the point where we need the magnetic field (in this case Y-Z plane).) dl is the current element vector which will make up the square loop.

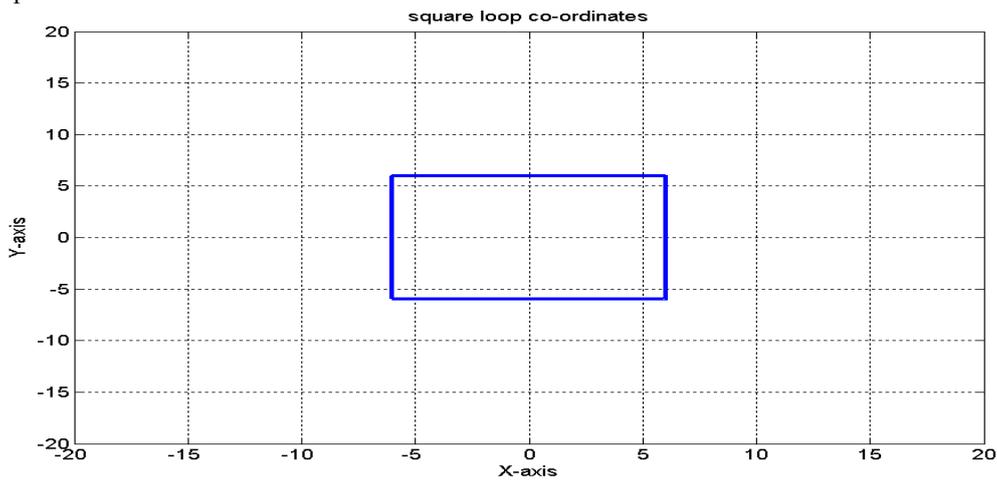


Figure (9) The Square loop Configuration

For the square loop configuration , the y and z components of the magnetic field were evaluated using analytical method formed by FEM techniques.

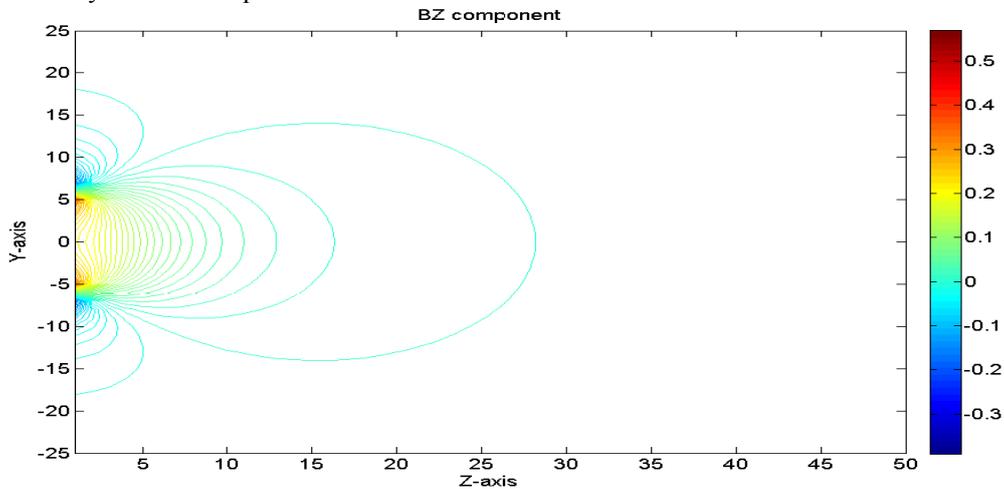


Figure (10) Z component of magnetic field

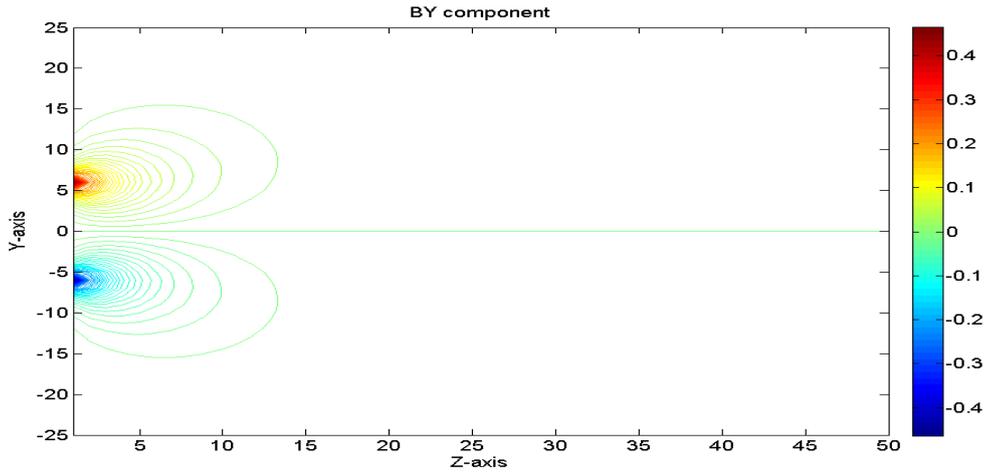


Figure (11) Y component of magnetic field

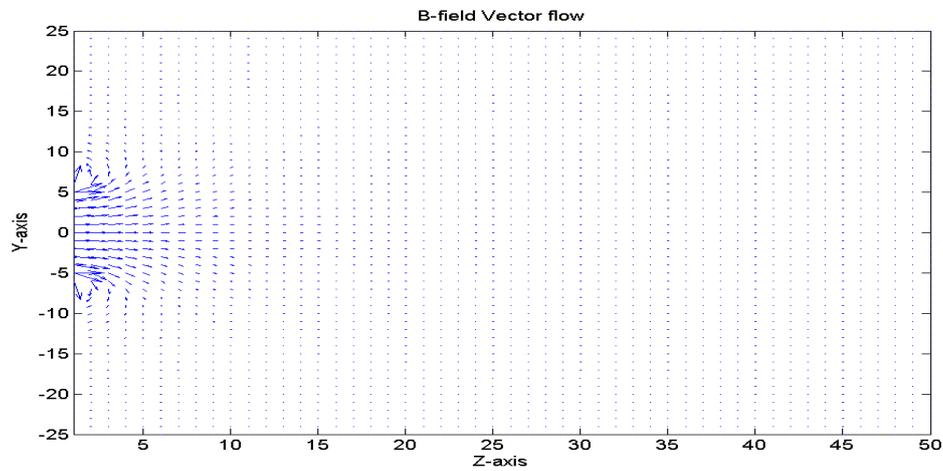


Figure (12) Vector flow for magnetic field of square loop

In figure (13) , a simulation path for a charged material in a magnetic field is designed. The designed program computes the path of the magnetic field as well as allows the user to change all the required parameters such as , mass of the particle , initial position of the particle in rectangular coordinate system , initial velocity of the particle relative to the coordinate system , values of magnetic and electric field surrounding the charged particle. Thus incorporating all the possible intricacies in design , a well thought control layout has been designed as shown in figure (14).

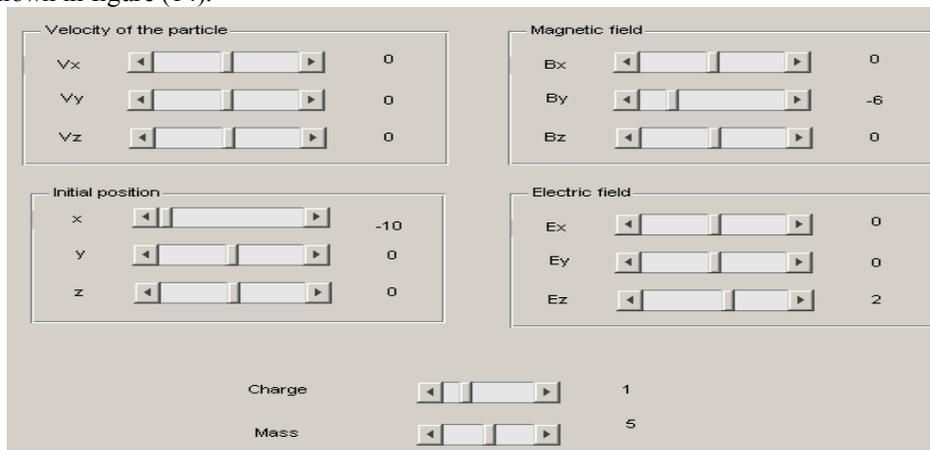


Figure (13) Control layout for simulation of a charged particle

The result for the said animation , depends on the values chosen in the control layout , a sample result has been displayed in figure (14) . Please note here that all the results here confirms mathematically to finite element methods.

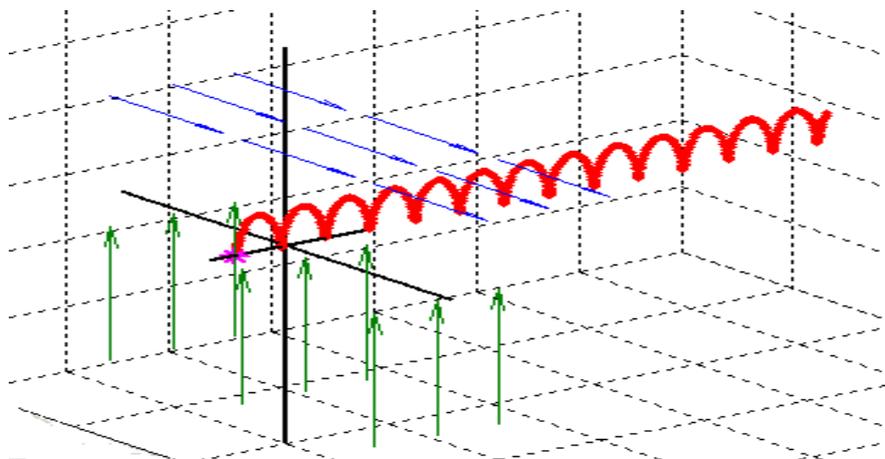


Figure (14) Results for the simulation for a charged particle placed in a magnetic field

V. Conclusions

Solving boundary value problems for potentials has led us to certain general conclusions concerning the methodical procedure. First, nature has given us certain physical phenomena that can be described by partial or ordinary differential equations. In many cases, these equations can be solved analytically. Other cases may require numerical solutions. The analytical solutions contain constants of integration. Nature also provides us with enough information that will allow us to evaluate these constants and thus obtain the solution for the problem. Assuming that neither mathematical nor numerical mistakes have been made, we can rest assured that this is the solution.

The primary idea behind the design was to implement finite element methods in electro magnetic theory for the calculations of various electrical and magnetic quantities . This has been accomplished successfully using several standard results as the foundation for our design . As presented in this paper, all of the methods assume linearity which leads to linear superposition principles. However, you will frequently encounter nonlinearity in nature which will lead to significant alterations in your method of obtaining a solution. Numerical questions concerning other specific programming languages, convergence requirements, numerical errors, aliasing, etc.

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