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Optimal Limit Order Placement Depth With Consideration Of Volume Size On Limit Order Book

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Abstract:
Determining the optimal depth at which to place a limit order in a Limit Order Book (LOB) presents a significant challenge for traders. If a trader places an order too shallow in the LOB, they may incur lower profits if their order is matched. Conversely, placing an order too deep can increase potential profits but decrease the probability of the order being matched. This project aims to find the optimal placement depth for limit orders based on the trader's utility function by solving the Hamilton-Jacobi-Bellman (HJB) equations, which incorporate several factors, including the order matching probability. We explore different methods for defining order matching probability, utilizing the order volume information posted on the LOB at that moment.

Keywords: *Limit Order Placement, Limit Order Book (LOB), Hamilton-Jacobi-Bellman (HJB) Equations, Dynamic Programming*

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I. Introduction

When a trader posts a limit order in a Limit Order Book (LOB), the depth at which the order is placed has a direct impact on the potential profit. A deeper placement increases the profit if the order is eventually matched, but it also reduces the likelihood of a match. Consequently, there exists an optimal depth for placing a limit order, which balances the trade-off between higher potential profits and the probability of execution. The order should not be placed too close to the best bid or ask, as this would result in lower profits. Conversely, placing the order too deep within the book reduces the likelihood of the order being matched.

There are many researches that examine the optimization of a trader's utility function to identify the optimal trading strategy. Each study typically focuses on different objectives and market models. For instance, Cartea and Jaimungal (2015) proposed an optimal execution policy aimed at maximizing a trader's utility function for the liquidation of large orders, specifically by selecting the optimal limit order placement depth or choosing to execute with a market order. Their model assumes that the probability of a trader's limit order being matched follows an exponential distribution. However, as the focus of their study is solely on the liquidation of an asset, and as such, they consider only one side of the Limit Order Book (LOB), without mentioning the imbalance between the bid and ask sides. Additionally, they assume that the volume size distribution on the LOB is uniform across the price levels.

In Chapter 10 of "Algorithmic and High-Frequency Trading (2015)", Cartea, Jaimungal, and Penalva proposed a methodology for determining the optimal depth for a market maker's limit order based on his utility function. They model the matching probability using a variable representing the LOB under the assumption that the volume in the LOB is balanced between the bid and ask sides, and that the volume is uniformly distributed across price levels. However, this assumption does not necessarily reflect real-world conditions, where the volume on the bid and ask sides of the LOB is often unbalanced and the distribution across price levels may not be uniform. Given that the volume in the LOB is publicly observable, it may be advantageous for a trader to incorporate this information when determining the optimal depth for posting a limit order.

In a more recent contribution, Cartea, Jaimungal, and Ricci (2018) developed an optimal market-making strategy based on a more complex market model, wherein market orders are modeled as a mutually exciting process. In this model, the arrival of a market order can trigger the arrival of subsequent market orders, creating short-lived price trends. While this approach provides a more dynamic view of market behavior, it requires intricate mathematical derivations, numerous assumptions, and the estimation of several parameters.

Aydoğan et al. (2023) introduced another approach to optimal market-making strategies, assuming that the volatility of asset prices follows the Heston Stochastic Volatility model. This approach is notable for its ability to suggest deeper limit order placements in the LOB when volatility is high. However, like previous studies, it is highly parameterized and does not account for the shape of the LOB as it is presented to the market maker in real time.

These studies reflect the diversity of approaches to optimizing trading strategies, each offering valuable insights but also presenting certain limitations, particularly in terms of simplifying assumptions or the lack of consideration for the full complexity of the LOB.

The motivation behind this project is to utilize the information contained in the Limit Order Book (LOB) to determine the optimal placement depth for limit orders. The structure of the LOB can provide valuable insights into the likely direction of asset price movements. For example, if the volume on the bid side of the LOB is significantly larger than the ask side, it suggests that demand exceeds supply, and the asset price is likely to rise. Conversely, if the bid side exhibits considerably lower volume than the ask side, this indicates that supply outstrips demand, and the asset price is likely to fall.

This suggests that if a trader assumes a balanced volume between the bid and ask sides when determining the optimal order placement depth, they may forgo potential profits in situations where the LOB clearly exhibits an imbalance. Thus, accounting for such imbalances is crucial for the trader's strategy.

The goal of this project is to identify the optimal limit order placement depth for various shapes of the LOB, including both balanced and imbalanced volume distributions between the bid and ask sides. Furthermore, recognizing that real-world LOB volumes are often not uniformly distributed across price levels, we will also identify the optimal limit order placement depth under non-uniform volume distributions in each side of the LOB to better align with actual market.

II. Methodology

In this project, we will use and compare three methods for determining the optimal placement depth of limit orders.

1. A method that does not fully utilize current data from the LOB, but instead assumes a constant volume size that is balanced and uniformly distributed across the LOB. This method is from Chapter 10 of "Algorithmic and High-Frequency Trading (2015)", Cartea, Jaimungal, and Penalva, serves as the reference method for this study. It is the simplest to implement, as it can be solved analytically.
2. A method that calculates the weighted average of the volume on each side of the LOB. This method accounts for imbalances between the bid and ask sides, but requires numerical methods to solve.
3. A method that utilizes the volume posted at each price level in the LOB. This method addresses both volume imbalances and the non-uniform distribution of volume across price levels. However, it is the most computationally complex, as it requires numerical optimization at each time step.

Define common functions and variables for all three methods

First, let's define price dynamics.

$$dS_t = \sigma dW_t$$

Where, we define S_t as an asset mid-price, σ as a volatility of an asset mid-price, and W_t as a standard Brownian motion.

Then, we define M_t^+ as a Poisson Process (with parameter λ^+) denotes Market Order arrival on ask side, M_t^- as a Poisson Process (with parameter λ^-) denotes Market Order arrival on bid side, N_t^+ as a counting process denotes trader's Limit Order being matched on ask side, N_t^- as a counting process denotes trader's Limit Order being matched on bid side. We can see that, $N_t^{\pm}$ can jump only when $M_t^{\pm}$ jumps. Let Q_t be the position of asset that the trader is holding, which follows

$$dQ_t = dN_t^- - dN_t^+$$

We define two more variables related to asset position, $\overline{q}$ as an upper limit of position that the trader allowed to hold, and $\underline{q}$ as a lower limit of position that the trader allowed to hold.

Define X_t as a wealth of the trader.

$$dX_t = (S_t + \frac{\varepsilon}{2} + \delta_t^+ \varepsilon)dN_t^+ - (S_t - \frac{\varepsilon}{2} - \delta_t^- \varepsilon)dN_t^-$$

Where, δ_t^+ as a price level under best ask that the trader posted his ask Limit Order, δ_t^- as a price level under best bid that the trader posted his bid Limit Order, ε as a price tick.

Next, we define the utility function of the trader.

$$H^{\delta_t^{\pm}}(t, x, S, q) = \mathbb{E}_{t,x,S,q}[X_T + Q_T(S_T - \alpha Q_T) - \phi \int_t^T Q_u^2 \, du]$$

Where, T - time at the end of the trading period (terminal time), α - constant of a penalty for walking a limit order book at the end of trading period, and ϕ - a constant of a running penalty for holding positions during the trading period

The utility function is composed of three parts. The first part, $\boldsymbol{X_T}$ represents the trader's wealth at the end of the trading period. The second part, $\boldsymbol{Q_T(S_T - \alpha Q_T)}$ represents the value of the assets the trader holds at the terminal time, adjusted for the liquidation penalty. Given the trader's risk aversion, he prefers not to hold positions, as the future asset price movement is uncertain. The third part, $\boldsymbol{-\phi \int_t^T Q_u^2 \, du}$ accounts for the running penalty associated with holding positions throughout the trading period.

The objective is to maximize this utility function by selecting the optimal limit order placement depth ($\boldsymbol{\delta_t^{\pm}}$) at each time step during the trading period.

Define matching probability (each method is different)

The key distinction among the three methods lies in their approach to the matching probability of limit orders, which will be defined in the following section.

Define matching probability: Method 1

For the first method, when a market order arrives, the matching probability is defined as $\boldsymbol{e^{-\kappa(\delta_t^{\pm}+1)}}$, where $\boldsymbol{\kappa}$ is a variable that reflects the volume of limit orders. It is important to note that $\boldsymbol{\kappa}$ is assumed to be the same for both the bid and ask sides in this method, which means that it assumes that the LOB is balanced and its volume are uniformly distributed across all price levels. The below figure shows that as the placement depth increases, the matching probability decreases exponentially.

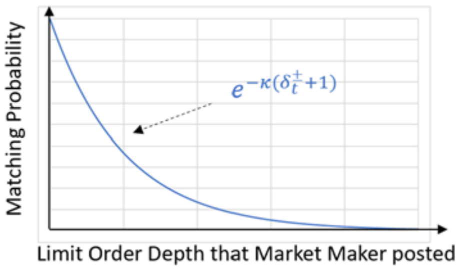


Figure 1: Graph of matching probability of the 1st method

Define matching probability: Method 2

For the second and third methods, we define $\boldsymbol{Z^{\pm}}$ as an exponentially distributed random variable with rate parameter $\boldsymbol{\kappa^{\pm}}$, which represents the volume size associated with each market order. The expected volume size for each market order is given by $\frac{1}{\kappa^{\pm}}$. It is important to note that $\boldsymbol{\kappa^{\pm}}$ in the second and third methods is different from $\boldsymbol{\kappa}$ in the first method.

Next, let $\boldsymbol{V_{i,t}^{+}}$ ($\boldsymbol{V_{i,t}^{-}}$) denote the volume at the i^{th} level under the best ask (bid) in the Limit Order Book on the ask (bid) side. The volume and price at each price level in the Limit Order Book are presented in the table below.

Table 1: Notation of volume size and price in Limit Order Book

Bid Volume	Bid Price	Ask Price	Ask Volume
$V_{0,t}^{-}$	$S - \frac{\varepsilon}{2}$	$S + \frac{\varepsilon}{2}$	$V_{0,t}^{+}$
$V_{1,t}^{-}$	$S - \frac{\varepsilon}{2} - \varepsilon$	$S + \frac{\varepsilon}{2} + \varepsilon$	$V_{1,t}^{+}$
$V_{2,t}^{-}$	$S - \frac{\varepsilon}{2} - 2\epsilon$	$S + \frac{\varepsilon}{2} + 2\epsilon$	$V_{2,t}^{+}$
$V_{3,t}^{-}$	$S - \frac{\varepsilon}{2} - 3\varepsilon$	$S + \frac{\varepsilon}{2} + 3\varepsilon$	$V_{3,t}^{+}$
$V_{4,t}^{-}$	$S - \frac{\varepsilon}{2} - 4\varepsilon$	$S + \frac{\varepsilon}{2} + 4\varepsilon$	$V_{4,t}^{+}$

We define the condition for a trader's limit order to be matched as $Z^{\pm} \geq \sum_{i=0}^{\delta_t^{\pm}} V_{i,t}^{\pm}$. In other words, the volume size of the market order must be greater than or equal to the total volume of limit orders posted at and above the relevant price level.

In the second method, we use a weighted average to represent the overall volume size on the ask and bid sides, denoted as $\boldsymbol{\overline{V}_t^{+}}$ and $\boldsymbol{\overline{V}_t^{-}}$, respectively. The weights used in the calculation of the weighted average correspond to the matching probability at each price level. This approach assigns greater weight to the volume sizes closer to the best bid and ask, and less weight to the volumes deeper in the Limit Order Book. The probability that a trader's limit order will be matched upon the arrival of a market order is simplified from $\boldsymbol{e^{-\kappa^{\pm}\sum_{i=0}^{\delta_t^{\pm}} V_{i,t}^{\pm}}}$ to $\boldsymbol{e^{-\kappa^{\pm}\bar{V}_t^{\pm}(\delta_t^{\pm}+1)}}$.

Define matching probability: Method 3

In the third method, we model the volume size at each price level as a function of the depth in the Limit Order Book. Specifically, we fit the volume size at each price level to an affine function $\boldsymbol{V_{i,t}^{\pm} = \alpha^{\pm} + \beta^{\pm} i}$. It is important to note that the values of $\boldsymbol{\alpha^{\pm}}$ and $\boldsymbol{\beta^{\pm}}$ must be constrained such that $\boldsymbol{V_{i,t}^{\pm}}$ remains non-negative for all selectable values of $\boldsymbol{\delta_t^{\pm} = i}$.

The probability that a trader's limit order will be matched upon the arrival of a market order is represented as $\boldsymbol{e^{-\kappa^{\pm}\int_0^{\delta_t^{\pm}+1}(\alpha^{\pm}+\beta^{\pm}i)di}}$, which equals to $\boldsymbol{e^{-\kappa^{\pm}[\alpha^{\pm}(\delta_t^{\pm}+1)+\frac{\beta^{\pm}}{2}(\delta_t^{\pm}+1)^2]}}$.

The table below summarizes matching probability of each method.

Table 2: Summary of matching probability for each method

Method	Limit Order Matching Chance
Method 1	$e^{-\kappa(\delta_t^{\pm}+1)}$
Method 2	$e^{-\kappa^{\pm}\bar{V}_t^{\pm}(\delta_t^{\pm}+1)}$
Method 3	$e^{-\kappa^{\pm}[\alpha^{\pm}(\delta_t^{\pm}+1)+\frac{\beta^{\pm}}{2}(\delta_t^{\pm}+1)^2]}$

Characterize the Hamilton-Jacobi-Bellman (HJB) equations

In this section, the characterization will be presented for the first method. The second and third methods follow a similar characterization, with the only difference being in the matching probability.

From the trader's utility function

$$H^{\delta_t^{\pm}}(t,x,S,q) = \mathbb{E}_{t,x,S,q}[X_T + Q_T(S_T - \alpha Q_T) - \phi \int_t^T Q_u^2\, du]$$

We define

$$H(t,x,S,q) = \sup_{\delta_t^{\pm}}\{H^{\delta_t^{\pm}}(t,x,S,q)\}$$

It can be shown that the Hamilton-Jacobi-Bellman equation for the Method 1 is as follows.

$$\phi q^2 = \frac{dh(t,q)}{dt} + \lambda^{+}\sup_{\delta_t^{+}}\left\{e^{-\kappa(\delta_t^{+}+1)}\left(\delta_t^{+}\varepsilon + \frac{\varepsilon}{2} + h(t,q-1) - h(t,q)\right)\right\}\mathbb{I}_{q>\underline{q}} + \lambda^{-}\sup_{\delta_t^{-}}\left\{e^{-\kappa(\delta_t^{-}+1)}\left(\delta_t^{-}\varepsilon + \frac{\varepsilon}{2} + h(t,q+1) - h(t,q)\right)\right\}\mathbb{I}_{q<\overline{q}}$$

For the Method 2, the matching probability will change as below

$$\phi q^2 = \frac{dh(t,q)}{dt} + \lambda^{+}\sup_{\delta_t^{+}}\left\{e^{-\kappa^{+}\bar{V}_t^{+}(\delta_t^{+}+1)}\left(\delta_t^{+}\varepsilon + \frac{\varepsilon}{2} + h(t,q-1) - h(t,q)\right)\right\}\mathbb{I}_{q>\underline{q}} + \lambda^{-}\sup_{\delta_t^{-}}\left\{e^{-\kappa^{-}\bar{V}_t^{-}(\delta_t^{-}+1)}\left(\delta_t^{-}\varepsilon + \frac{\varepsilon}{2} + h(t,q+1) - h(t,q)\right)\right\}\mathbb{I}_{q<\overline{q}}$$

For the Method 3, the matching probability will change as below

$$\phi q^2 = \frac{dh(t,q)}{dt} + \lambda^{+}\sup_{\delta_t^{+}}\left\{e^{-\kappa^{+}\left[\alpha^{+}(\delta_t^{+}+1)+\frac{\beta^{+}}{2}(\delta_t^{+}+1)^2\right]}\left(\delta_t^{+}\varepsilon + \frac{\varepsilon}{2} + h(t,q-1) - h(t,q)\right)\right\}\mathbb{I}_{q>\underline{q}} + \lambda^{-}\sup_{\delta_t^{-}}\left\{e^{-\kappa^{-}[\alpha^{-}(\delta_t^{-}+1)+\frac{\beta^{-}}{2}(\delta_t^{-}+1)^2]}\left(\delta_t^{-}\varepsilon + \frac{\varepsilon}{2} + h(t,q+1) - h(t,q)\right)\right\}\mathbb{I}_{q<\overline{q}}$$

Solve for optimal Limit Order placement depth

Consider the supremum function inside the Method 1

$$\sup_{\delta_t^{\pm}}\left\{e^{-\kappa(\delta_t^{\pm}+1)}\left(\delta_t^{\pm}\varepsilon + \frac{\varepsilon}{2} + h(t,q-1) - h(t,q)\right)\right\}$$

To determine the optimal value of $\boldsymbol{\delta_t^{\pm}}$ that maximizes the function, one can proceed as follows: First, compute the derivative of the function within the supremum operator. Then, set the derivative equal to zero and

rearrange the resulting equation such that $\boldsymbol{\delta_t^{\pm}}$ appears on the left-hand side, with the other variables on the right-hand side.
We get the optimal limit order placement depth of the first method in semi-closed form as

$$\boldsymbol{\delta_t^{\pm,*}} = \frac{\mathbf{1}}{\boldsymbol{\kappa}} - \frac{\mathbf{1}}{\mathbf{2}} - \frac{\boldsymbol{h(t, q \mp 1)}}{\boldsymbol{\varepsilon}} + \frac{\boldsymbol{h(t, q)}}{\boldsymbol{\varepsilon}}$$

However, the value of $\boldsymbol{h(t, q \mp 1)}$ and $\boldsymbol{h(t, q)}$ remain unknown, so we must substitute the optimal limit order placement depth back into the Hamilton-Jacobi-Bellman equation.
We get,

$$\mathbf{0} = \frac{\boldsymbol{dh(t, q)}}{\boldsymbol{dt}} + \left[\frac{\boldsymbol{\lambda^+ \varepsilon}}{\boldsymbol{\kappa}} \boldsymbol{e}^{-\kappa\left(\frac{1}{\kappa} + \frac{1}{2} - \frac{h(t,q-1)}{\varepsilon} + \frac{h(t,q)}{\varepsilon}\right)}\right] \mathbb{I}_{q > \underline{q}} + \left[\frac{\boldsymbol{\lambda^- \varepsilon}}{\boldsymbol{\kappa}} \boldsymbol{e}^{-\kappa\left(\frac{1}{\kappa} + \frac{1}{2} - \frac{h(t,q+1)}{\varepsilon} + \frac{h(t,q)}{\varepsilon}\right)}\right] \mathbb{I}_{q < \overline{q}} - \boldsymbol{\phi q^2}$$

which is a system of Ordinary Differential Equations, which can be solved analytically.

For the second method, we repeat the same steps as the Method 1, we get the optimal limit order placement depth of the Method 2 in semi-closed form as

$$\boldsymbol{\delta_t^{+,*}} = \frac{\mathbf{1}}{\boldsymbol{\kappa^+ \overline{V}_t^+}} - \frac{\mathbf{1}}{\mathbf{2}} - \frac{\boldsymbol{h(t, q \mp 1)}}{\boldsymbol{\varepsilon}} + \frac{\boldsymbol{h(t, q)}}{\boldsymbol{\varepsilon}}$$

After, substitute the optimal limit order placement depth back into the Hamilton-Jacobi-Bellman equation.
We get,

$$\mathbf{0} = \frac{\boldsymbol{dh(t, q)}}{\boldsymbol{dt}} + \left[\frac{\boldsymbol{\lambda^+ \varepsilon}}{\boldsymbol{\kappa^+ \overline{V}_t^+}} \boldsymbol{e}^{-\kappa^+ \overline{V}_t^+\left(\frac{1}{\kappa^+ \overline{V}_t^+} + \frac{1}{2} - \frac{h(t,q-1)}{\varepsilon} + \frac{h(t,q)}{\varepsilon}\right)}\right] \mathbb{I}_{q > \underline{q}} + \left[\frac{\boldsymbol{\lambda^- \varepsilon}}{\boldsymbol{\kappa^- \overline{V}_t^-}} \boldsymbol{e}^{-\kappa^- \overline{V}_t^-\left(\frac{1}{\kappa^- \overline{V}_t^-} + \frac{1}{2} - \frac{h(t,q+1)}{\varepsilon} + \frac{h(t,q)}{\varepsilon}\right)}\right] \mathbb{I}_{q < \overline{q}} - \boldsymbol{\phi q^2}$$

which is a system of Ordinary Differential Equations. In this case, it should be solved numerically. We will start from the terminal condition of $\boldsymbol{h(T, q)}$, which is a known value. Then, we will solve the system of ODE backward, until we reach the current time. We will repeat the process like this every time the information on the Limit Order Book change.

For the Method 3, we cannot write the semi-closed form for optimal Limit Order placement depth ($\delta_t^{\pm,*}$), so we have to directly solve

$$\begin{aligned}\mathbf{0} &= \frac{\boldsymbol{dh(t, q)}}{\boldsymbol{dt}} \\ &+ \boldsymbol{\lambda^+} \sup_{\delta_t^+} \left\{ \boldsymbol{e}^{-\kappa^+\left[\alpha^+(\delta_t^+ + 1) + \frac{\beta^+}{2}(\delta_t^+ + 1)^2\right]} \left(\boldsymbol{\delta_t^+ \varepsilon} + \frac{\boldsymbol{\varepsilon}}{\mathbf{2}} + \boldsymbol{h(t, q-1)} - \boldsymbol{h(t, q)} \right) \right\} \mathbb{I}_{q > \underline{q}} \\ &+ \boldsymbol{\lambda^-} \sup_{\delta_t^-} \left\{ \boldsymbol{e}^{-\kappa^-\left[\alpha^-(\delta_t^- + 1) + \frac{\beta^-}{2}(\delta_t^- + 1)^2\right]} \left(\boldsymbol{\delta_t^- \varepsilon} + \frac{\boldsymbol{\varepsilon}}{\mathbf{2}} + \boldsymbol{h(t, q+1)} - \boldsymbol{h(t, q)} \right) \right\} \mathbb{I}_{q < \overline{q}} - \boldsymbol{\phi q^2}\end{aligned}$$

This system can be solved numerically backward like the Method 2, but we have to find optimal $\delta_t^{+,*}$ that maximize the function inside the supremum operators at every time step backward.

Method 1 is the most computationally efficient, as it can be solved analytically (please refer to appendix); however, it captures limited information about the Limit Order Book. Method 2, while more computationally expensive due to it has to be solved numerically, it provides a more comprehensive understanding of the Limit Order Book, particularly in terms of its imbalance. Method 3 is the most computationally expensive, as it requires numerical solution along with optimization at each time step when solving the backward ODE. Nevertheless, it offers the most detailed representation of the Limit Order Book.

III. Results

This section is divided into two parts. The first part outlines the parameter configurations used for solving the system of ordinary differential equations (ODEs). The second part presents the results of solving the system of ODEs to determine the optimal Limit Order placement depth for each method.

Parameters Configuration

The table below presents the parameter values for each variable used in this project.

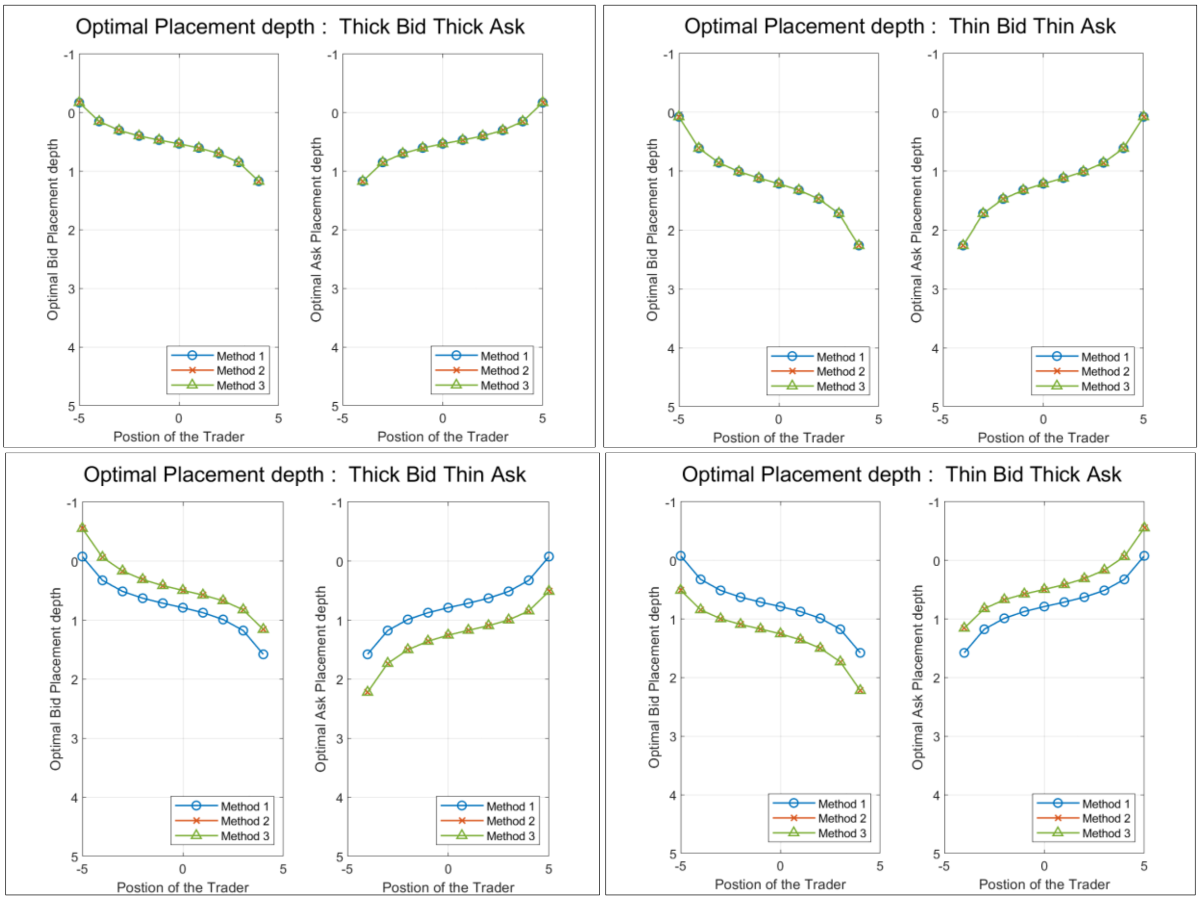
Table 4: Parameters Configuration

Symbol	Description	Value	Unit
$\boldsymbol{S_0}$	Mid-price at time $t = 0$	50.125	\$
$\boldsymbol{\varepsilon}$	Price tick	0.25	\$
$\boldsymbol{S_0 + \frac{\varepsilon}{2}}$	Best ask price at time $t = 0$	50.25	\$
$\boldsymbol{S_0 - \frac{\varepsilon}{2}}$	Best bid price at time $t = 0$	50.00	\$

T	Trading period	100	seconds
-	Recalculating frequency	0.1	second
λ^{+}	Market Order arrival rate on ask side	0.25	$second^{-1}$
λ^{-}	Market Order arrival rate on bid side	0.25	$second^{-1}$
$\frac{1}{\kappa^{+}}$	Mean of Market Order volume size on ask side	10000	-
$\frac{1}{\kappa^{-}}$	Mean of Market Order volume size on bid side	10000	-
-	Volume size that defines thick volume on LOB	10000	-
-	Volume size that defines thin volume on LOB	6000	-
$\overline{q}$	Maximum position allowed for the trader	5	-
$\underline{q}$	Minimum position allowed for the trader	-5	-
ϕ	Constant which reflects risk averseness of the trader	10^{-4}	-
α	Constant of a penalty of remaining position at the end of trading period	0.001	-
-	Rate at which other traders fill or cancel Limit Orders for each price levels	2	$second^{-1}$
$\begin{bmatrix} \gamma_{S_0+\frac{\varepsilon}{2}} \\ \gamma_{S_0+\frac{\varepsilon}{2}+\varepsilon} \\ \gamma_{S_0+\frac{\varepsilon}{2}+2\varepsilon} \\ \vdots \end{bmatrix}$	Mean-reverting constants to let the Limit Order Book correct itself back to the desired test shape. (First row is for best ask, second row is for second-best ask, and so on)	$\begin{bmatrix} 1 \\ 1/2 \\ 1/3 \\ \vdots \end{bmatrix}$	-
$\begin{bmatrix} \gamma_{S_0-\frac{\varepsilon}{2}} \\ \gamma_{S_0-\frac{\varepsilon}{2}-\varepsilon} \\ \gamma_{S_0-\frac{\varepsilon}{2}-2\varepsilon} \\ \vdots \end{bmatrix}$	Mean-reverting constants to let the Limit Order Book correct itself back to the desired test shape. (First row is for best bid, second row is for second-best bid, and so on)	$\begin{bmatrix} 1 \\ 1/2 \\ 1/3 \\ \vdots \end{bmatrix}$	-

Optimal Placement Depth based on solving ODEs

In this part, we present the results of solving the systems of Ordinary Differential Equations (ODEs) associated with each of the three methods outlined above. The figures below present the optimal placement depth for each asset position held by the trader, for both the bid and ask sides, for each snapshot of LOB shape.



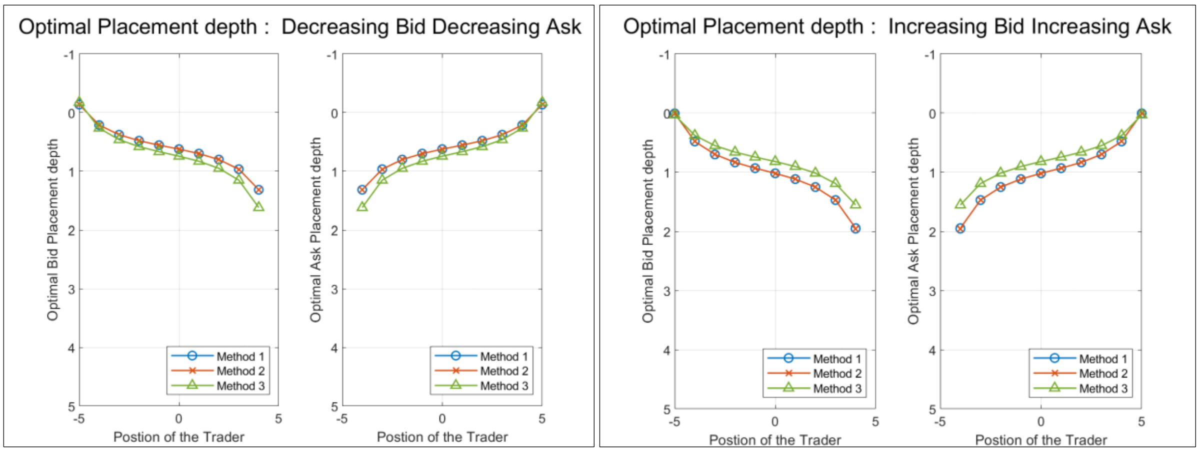


Figure 7: Optimal Placement Depth of each of Limit Order Book (LOB) shape

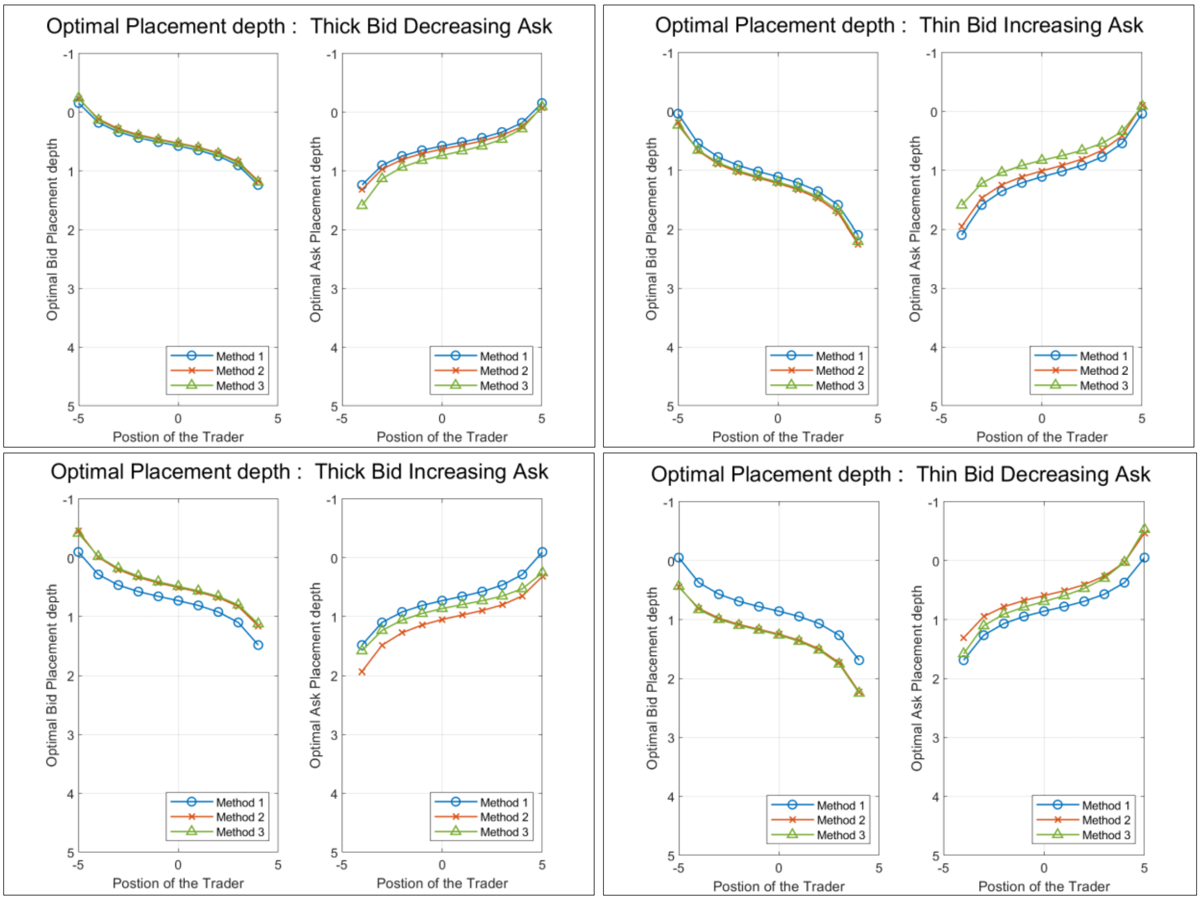


Figure 7 (continued): Optimal Placement Depth of each of Limit Order Book (LOB) shape

Note that for the trader's position, a positive value indicates a long position, a negative value indicates a short position, and a value of zero indicates no position. Furthermore, if the optimal placement depth is denoted as n, this indicates that the method recommends posting a Limit Order at the nth price level below the best bid or ask. It is important to note that since the price level must be an integer, the calculated value of n should be rounded to the nearest integer during implementation. If the optimal placement depth is 0, it signifies that the method suggests posting a Limit Order at the best bid or ask. Conversely, if the optimal placement depth is negative, the method implies the execution of a Market Order instead of a Limit Order.

Balanced Limit Order Books with uniformly distributed volume

This group of Limit Order Books (LOBs) includes two cases: Thick Bid, Thick Ask, and Thin Bid, Thin Ask. Both cases are balanced, with volume uniformly distributed across all price levels.

For this type of Limit Order Book (LOB), since the lines representing all three methods overlap entirely, it demonstrates that all three methods suggest the same limit order placement depth. This is because the balanced LOB and the uniform distribution of volume lead to mathematically equivalent across system of ODEs of all three methods.

Furthermore, the figure shows that as the trader holds more positive positions, bid (ask) orders are placed deeper (shallower), while more negative positions lead to bid (ask) orders being placed shallower (deeper). This pattern is driven by the penalty associated with holding a position, which reflects the trader's risk aversion. If the trader is more risk-averse, holding the same positive position will result in placing bid (ask) order even deeper (shallower) in the Limit Order Book. Conversely, if the trader is less risk-averse, holding the same positive position will lead to placing bid (ask) order shallower (deeper) in the LOB. For example, to encourage the trader to hold a larger position, we can reduce his risk aversion factor.

Lastly, in the case of thinner volume, the optimal limit order placement occurs deeper in the Limit Order Book (LOB) compared to the thicker volume case. This is because the lower volume in the order book increases the likelihood of matching at deeper levels, making it more advantageous to place orders further away from the current market price. To clarify, the difference in optimal placement depth between a thick and thin LOB is due to the volume of Limit Orders from other traders in the LOB, relative to average volume of Market Orders.

Balanced Limit Order Books with uniformly distributed volume

This group of Limit Order Books (LOBs) includes two cases: Thick Bid, Thin Ask, and Thin Bid, Thick Ask. Both cases are unbalanced, with volume uniformly distributed across all price levels.

The figures demonstrate that Method 1 recommends placing the optimal bid and ask at mirrored positions, with the optimal placement depth for the bid at the -5 position being equivalent to that of the ask at the +5 position, as it is unable to detect any imbalance in the Limit Order Book (LOB).

Methods 2 and 3, which can detect the imbalance of the LOB suggest the optimal Limit Order placement depth different from Method 1. In the Thick Bid, Thin Ask case, Methods 2 and 3 recommend a shallower optimal depth for the bid side and a deeper optimal depth for the ask side. This is because the bid side has a lower matching probability, while the ask side has a higher matching probability due to LOB imbalance. Conversely, in the Thin Bid, Thick Ask case, the optimal placement depth for the bid and ask sides is reversed, with the bid side placed deeper and the ask side placed shallower.

Methods 2 and 3 suggest the same optimal placement depth, as the uniform distribution of volumes across all price levels makes these methods mathematically equivalent.

Balanced Limit Order Books with not uniformly distributed volume

This group of Limit Order Books (LOBs) includes two cases: Decreasing Bid, Decreasing Ask, and Increasing Bid, Increasing Ask. Both cases are balanced, but volume are not uniformly distributed across all price levels.

Methods 1 and 2 suggest the same optimal placement depth because the Limit Order Book (LOB) is balanced in this case. In contrast, Method 3 recommends a different optimal placement depth, as it uses a different approach for capturing changes in volume at deeper price levels within the LOB. Specifically, for a decreasing Bid and Ask LOB, Method 3 suggests a deeper optimal placement depth due to a higher probability of matching at deeper price levels. Conversely, for an increasing Bid and Ask LOB, Method 3 suggests a shallower optimal placement depth, as the likelihood of matching at deeper price levels is lower.

Unbalanced Limit Order Books with not uniformly distributed volume

This group of Limit Order Books (LOBs) consists of Thick Bid Decreasing Ask, Thin Bid Increasing Ask, Thick Bid Increasing Ask, and Thin Bid Decreasing Ask.

In summary, the analysis of the four figures indicates that Method 2 suggests deeper or shallower levels compared to Method 1, due to the Limit Order Book (LOB). Furthermore, Method 3 presents deeper or shallower levels than Method 2, which can be attributed to the increasing or decreasing volume at deeper price levels within the LOB.

IV. Limitations And Future Works

In our future work, we plan to simulate the Limit Order Book (LOB) in different trading environments and apply the optimal limit order placement depths derived from Methods 1, 2, and 3. We will then evaluate and compare the performance of each method to determine whether any method significantly outperforms the others.

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