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ABC index on subdivision graphs and line graphs

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Recently introduced Atom-bond connectivity index (ABC Index) is defined as ABC(G) = $\sum\sqrt{\frac{d_i+d_j-2}{d_i.d_j}}$, where d_i and d_j are the degrees of vertices v_i and v_j respectively. In this paper we present the ABC index of *subdivision graphs* of some connected graphs.We also provide the ABC index of the *line graphs* of some subdivision graphs.

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1 Introduction and Terminologies

Topological indices have a prominent place in Chemistry, Pharmacology etc.[9] The recently introduced Atom-bond connectivity (ABC) index has been applied up until now to study the stability of alkanes and the strain energy of cycloalkanes. Furtula et al. (2009) [4] obtained extremal ABC values for chemical trees, and also, it has been shown that the star $K_{1,n-1}$, has the maximal ABC value of trees. In 2010, Kinkar Ch Das present the lower and upper bounds on ABC index of graphs and trees, and characterize graphs for which these bounds are best possible[1].

In [8], Ranjini and lokesha studied about Zagreb indices of the subdivision graphs of Tadpole graphs and wheel graphs. Motivated by their work, in this study, we selected the subdivision graphs of three simple, connected graphs : Helm graph, Ladder graph and Lollipop graph to study about ABC index. We also produce the ABC index of line graph of the subdivision graph of the above mentioned graphs.

Let $G = (V,E)$ be a simple connected graph with vertex set $V(G) = v_1, v_2, ..., v_n$ and edge set $E(G)$. Let d_i be the degree of vertex v_i, where i=1,2,3,...n. The *ABC* index, proposed by Ernesto Estrada et al, is defined as follows.

$$ABC(G) = \sum\sqrt{\frac{d_i+d_j-2}{d_i.d_j}}, (\mathrm{v_i}, \mathrm{v_j}) \in \mathrm{E(G)}$$

We refer the reader to [3] for the proof of this fact.

The *wheel graph* W_{n+1} [7] is defined as the graph $K_1 + C_n$, where K_1 is the singleton graph and C_n is the cycle graph.

The *subdivision graph* [12] $S(G)$ is the graph obtained from G by replacing each of its edge by a path of length 2 or equivalently, by inserting an additional vertex into each edge of G. The *line graph* $L(G)$ is the graph whose vertices correspond to the edges of G with two vertices being adjacent if and only if the corresponding edges in G have a vertex in common [12].

The H_n *helm graph* [11] is the graph obtained from a n - *wheel graph* by adjoining a pendant edge at each node of the cycle. The *lollipop graph* $L_{n,k}$ is the graph obtained by joining a complete graph K_n to a path graph P_k with a bridge. The *ladder graph* L_n can be defined as $P_2 \mathrm{W} P_n$, where P_n is a path graph.

This paper is organized as follows: In section 2, we calculated the ABC index of *subdivision graphs* of *Helm graph* H_n, *lollipop graph* $L_{n,k}$ and *ladder graph* L_n. In the last section, ABC index of *line graphs* of *subdivision graphs* of *Helm graph* H_n, *lollipop graph* $L_{m,n}$ and *ladder graph* L_n are computed.

2 ABC index on the subdivision graphs of Helm graph, Lollipop graph and Ladder graph

In this section, we derive an expression for *ABC index* on *subdivision graphs* of *Helm graph* H_n, *Lollipop Graph* $L_{n,k}$ and *Ladder graph* L_n

Theorem 2.1 *For the subdivision graph of a Helm graph, the Atom-bond Connectivity index is*

$$ABC(G) = \frac{6n}{\sqrt{2}}$$

.

Proof. Subdivision graph of *Helm graph* $S(H_n)$ contains one vertex of degree n, n vertices of degree 4, n pendent vertices and $3n$ *subdivision vertices* of degree 2.

In $S(H_n)$, n edges are formed by joining vertices of degrees $(n,2)$ and $(2,1)$ and $4n$ edges are formed by joining vertices of degrees $(4,2)$. Each of these edges make the sum

$$\sum \sqrt{\frac{d_i + d_j - 2}{d_i . d_j}} = \frac{1}{\sqrt{2}}$$

.

Hence in $S(H_n)$

$$ABC(G) = \frac{6n}{\sqrt{2}}$$

.

Theorem 2.2 *For the subdivision graph of a* $Lollipop\ Graph$*, the Atom-bond Connectivity index is*

$$ABC(G) = \frac{1}{\sqrt{2}}(n^2 + 2k - n)$$

.

Proof. The subdivision graph of $L_{n,k}$ contain a cycle graph C_n and path graph P_k. C_n contains one vertex of degree n, $n-1$ vertices of degree $n-1$ and $\frac{n(n-1)}{2}$ subdivision vertices of degree 2. Path P_k contains $k-1$ vertices of degree 2, one pendent vertex and k subdivisional vertices of degree 2.

The cycle C_n of $S(L_{n,k})$ contains $n(n-1)$ edges and the path P_k contains $2k$ edges.

The edges in the cycle C_n are formed by joining vertices of degrees $(n,2)$, $(n-1,2)$ and $(2,2)$ and the

edges in the path P_k are formed by joining vertices of degrees $(n,1)$, $(2,2)$ and $(2,1)$.
Each of the above edges make the sum

$$\sum\sqrt{\frac{d_i+d_j-2}{d_i.d_j}}=\frac{1}{\sqrt{2}}$$

.

Hence in $S(L_{n,k})$

$$ABC(G)=\frac{1}{\sqrt{2}}(n^2+2k-n)$$

.

Theorem 2.3 *For the subdivision graph of a* $Ladder\quad Graph$, *the Atom-bond Connectivity index is*

$$ABC(G)=\frac{6n-4}{\sqrt{2}}$$

.

Proof. The subdivision graph of the ladder graph $S(L_n)$ contains 4 vertices of degree 2, $2n-4$ vertices of degree 3 and $3n+2$ subdivision vertices of degree 2.
In $S(L_n)$, 8 edges are formed by joining vertices of degrees $(2,2)$ and $6n-12$ edges are formed by joining vertices of degrees $(3,2)$. Each of the above edges make the sum

$$\sum\sqrt{\frac{d_i+d_j-2}{d_i.d_j}}=\frac{1}{\sqrt{2}}$$

.

Hence in $S(L_n)$

$$ABC(G)=\frac{6n-4}{\sqrt{2}}$$

3 ABC index on the Line graphs of subdivision graphs of Helm graph, Lollipop graph and Ladder graph

In this section, we derive expressions for *ABC index* on the $line\quad graphs$ of *subdivision graphs* of *Helm graph* H_n, *Lollipop Graph* $L_{n,k}$ and *Ladder graph* L_n.

Theorem 3.1 *For the line graph of the subdivision graph of* $Helm\quad graph\quad H_n$

$$ABC(G)=\sqrt{\frac{3}{4}}n+\sqrt{\frac{2(n-1)}{n^2}}\frac{n(n-1)}{2}+\sqrt{\frac{n+2}{4n}}n+\sqrt{\frac{3}{8}}.$$

Proof. The line graph of the subdivision graph of the helm graph $L(S(H_n))$ contains $\frac{1}{2}(n^2+17n)$ edges.

Out of these, n edges, formed by joining vertices of degrees $(1,4)$ makes the sum $\sum\sqrt{\frac{d_i+d_j-2}{d_i.d_j}}=\frac{3}{\sqrt{4}}$,

$\frac{1}{2}n(n-1)$ edges formed by joining vertices of degree (n,n) makes the sum $\sqrt{\frac{2(n-1)}{n^2}}$, the edges formed by joining vertices of degree $(n,4)$ makes the sum $\sqrt{\frac{n+2}{4n}}$ and the edges formed by joining vertices of degree $(4,4)$ makes the sum $\sqrt{\frac{3}{8}}$. Hence

$$ABC(G)=\sqrt{\frac{3}{4}}n+\sqrt{\frac{2(n-1)}{n^2}}\frac{n(n-1)}{2}+\sqrt{\frac{n+2}{4n}}n+\sqrt{\frac{3}{8}}.$$

Theorem 3.2 *For the line graph of the subdivision graph of Lollipop graph* $L_{n,k}$

$$ABC(G)=(n-1)\sqrt{\frac{n-1}{2}}+\sqrt{\frac{(n-1)(2n-3)}{n}}+\frac{n^3-4n^2+9n-12}{n-1}\sqrt{\frac{n-2}{2}}+\frac{2k-1}{\sqrt{2}}$$

Proof. The line graph of the subdivision graph of $L_{n,k}$ contains a cycle graph C_n and a path graph P_k.
The cycle graph C_n contains n vertices of degree n and n^2-2n+1 vertices of degree $n-1$. The path graph P_k contains a pendent vertex and $2k-2$ vertices of degree 2.

$\sum\sqrt{\frac{d_i+d_j-2}{d_i.d_j}}$ with respect to the cycle graph C_n.

The edges of the cycle graph C_n of $L(S(L_{n,k}))$ are formed by joining vertices of degree (n,n), $(n,n-1)$ and $(n-1,n-1)$.

In $L(S(L_{n,k}))$, $\frac{n(n-1)}{2}$ edges, formed by joining vertices of degree (n,n), makes the sum $\sqrt{\frac{2(n-1)}{n^2}}$.

The $n-1$ edges, formed by joining vertices of degree $(n,n-1)$ makes the sum $\sqrt{\frac{2n-3}{n(n-1)}}$ and $\frac{n^3-4n^2+9n-12}{2}$ edges, formed by joining vertices of degree $(n-1,n-1)$, makes the sum $\sqrt{\frac{2(n-2)}{(n-1)^2}}$.

Hence in the cycle graph C_n of $L(S(L_{n,k}))$

$$\sum\sqrt{\frac{d_i+d_j-2}{d_i.d_j}}=\frac{n(n-1)}{2}\sqrt{\frac{2(n-1)}{n^2}}+(n-1)\sqrt{\frac{2n-3}{n(n-1)}}+\frac{n^3-4n^2+9n-12}{2}\sqrt{\frac{2(n-2)}{(n-1)^2}} \qquad (1)$$

$\sum\sqrt{\frac{d_i+d_j-2}{d_i.d_j}}$ with respect to the path P_k

The path graph P_k of $L(S(L_{n,k}))$ contains edges , formed by joining vertices of degrees $(n,2)$, $(2,2)$ and $(2,1)$. The $2k-3$ edges are formed by joining vertices of degrees $(2,2)$ and the vertex pairs $(n,2)$ and

$(2,1)$ makes a single edge each. All the above edges make the sum $\frac{1}{\sqrt{2}}$ in P_K.

Hence in the path P_k,

$$\sum\sqrt{\frac{d_i+d_j-2}{d_i.d_j}}=(2k-1)\frac{1}{\sqrt{2}} \tag{2}$$

Adding the above two equations,

$$ABC(G)=(n-1)\sqrt{\frac{n-1}{2}}+\sqrt{\frac{(n-1)(2n-3)}{n}}+\frac{n^3-4n^2+9n-12}{n-1}\sqrt{\frac{n-2}{2}}+\frac{2k-1}{\sqrt{2}}$$

Theorem 3.3 *For the line graph of the subdivision graph of* $ladder\quad graph\quad L(S(L_n))$

$$ABC(G)=5\sqrt{2}+\frac{2(9n-20)}{3}$$

Proof. The line graph of the subdivision graph of ladder graph $L(S(L_n))$ contains 8 vertices of degree 2 and $6n-12$ vertices of degree 3.

In $L(S(L_n))$, the 6 edges, formed by joining vertices of degrees $(2,2)$ and the 4 edges, formed by joining edges of degrees $(2,3)$ makes the sum, $\sum\sqrt{\frac{d_i+d_j-2}{d_i.d_j}}=\frac{1}{\sqrt{2}}$. The remaining $9n-20$ edges, formed by joining edges of degrees $(3,3)$ makes the sum $\frac{2}{3}$.

Hence in $L(S(L_n))$,

$$ABC(G)=5\sqrt{2}+\frac{2(9n-20)}{3}$$

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