IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN:2319-765X. Volume 10, Issue 1Ver. I. (Jan. 2014), PP 08-12
www.iosrjournals.org

Adomian Decomposition Method for Solving the Kuramoto – Sivashinsky Equation

Saad A. Manaa[1], Fadhil H. Easif[2], Majeed A. Yousif[3]
[1,2,3] *Department of Mathematics, Faculty of Science, University of Zakho,Duhok, Kurdistan Region, Iraq*

Abstract: *The approximate solutions for the Kuramoto –Sivashinsky Equation are obtained by using the Adomian Decomposition method (ADM). The numerical example show that the approximate solution comparing with the exact solution is accurate and effective and suitable for this kind of problem.*
Keywords: *Adomian decomposition method (ADM); Kuramoto –Sivashinsky Equation .*

I. INTRODUCTION

The ADM was first introduced by Adomian in the beginning of 1980's. The method is useful obtaining both a closed form and the explicit solution and numerical approximations of linear or nonlinear differential equations and it is also quite straight forward to write computer codes. This method has been applied to obtain a formal solution to a wide class of stochastic and deterministic problems in science and engineering involving algebraic, differential, integro-differential, differential delay, integral and partial differential equations. In the present study, Adomian decomposition method (ADM) has been applied to solve the Kuramoto–Sivashinsky equations. The numerical results are compared with the exact solutions. It is shown that the errors are very small. [1]

II. II.1 Mathematical Model

The Kuramoto–Sivashinsky equation is a non-linear evolution equation and has many applications in a variety of physical phenomena such as reaction diffusion systems (Kuramoto and Tsuzuki, 1976)[2], long waves on the interface between two viscous fluids (Hooper and Grimshaw, 1985)[3], and thin hydrodynamics films (Sivashinsky, 1983)[4]. The Kuramoto-Sivashinsky equation has been studied numerically by many authors (Akrivis and Smyrlis, 2004; Manickam et al.,1998; Uddin et al., 2009)[5-7].

Consider the Kuramoto–Sivashinsky equation

$$u_t + uu_x + \alpha\, u_{xx} + \gamma\, u_{xxx} + \beta\, u_{xxxx} = 0 \qquad (1)$$

Subject to the initial condition

$$u(x,0) = f(x) \qquad a \le x \le b. \qquad (2)$$

And boundary conditions

$$\left.\begin{aligned} &u(a,t) = g_1(t), \quad u(b,t) = g_2(t) \qquad t > 0 \\ &\frac{\partial^2 u}{\partial x^2} = h_1\,, \quad \text{at } x = a \text{ and } x = b \qquad \text{where } h_1 \ge 0. \end{aligned}\right\} \quad (3)$$

II.2 Basic idea of Adomian Decomposition Method (ADM)

Consider the differential equation:-

$$Lu + Ru + Nu = g \qquad (4)$$

$$u(x,0) = f(x) \qquad (5)$$

where L is the operator of the highest-ordered derivatives and R is the remainder of the linear operator. The nonlinear term is represented by$N(u)$.
Thus we get:

$$Lu = g - Ru - Nu \qquad (6)$$

Where $$L = \frac{\partial}{\partial t}\ ,\ Define L^{-1} = \int_0^t (.)dt \qquad (7)$$

operating with the operator L^{-1} on both sides of Eq. (6) we have

$$u = f(x) - L^{-1}(Ru) - L^{-1}(Nu)\ , \text{where} L^{-1}(g) = f(x) \qquad (8)$$

The standard Adomian decomposition method define the solution $u(x,t)$an infinite series of the form:

$$u(x,t) = \sum_{k=0}^{\infty} u_k(x,t) \qquad (9)$$

Where $\qquad u_0 = f(x) \qquad (10)$

And $\quad u_1, u_2, \ldots$ aredetermined by

$$u_{k+1} = -L^{-1}(Ru_k) - L^{-1}(Nu_k) \;, k \geq 0 \qquad (11)$$

and the nonlinear operator $N(u)$can be decomposed by an infinite series ofpolynomials given by

$$N(u) = \sum_{k=0}^{\infty} A_k \qquad (12)$$

Where $\qquad A_k = \frac{1}{k!}\frac{d^k}{d\lambda^i}\left[F\left(\sum_{i=0}^{k}\lambda^i u_i\right)\right]_{\lambda=0} \;, k = 0,1,\ldots \qquad (13)$

It is now well known in the literature that these polynomials can be constructed for all classes of nonlinearity according to algorithms set by Adomian [8,9] and recently developed by an alternative approach in [8-10].

II.3 Derivative of (ADM) for Kuramoto-Sivashinsky equation

We consider Kuramoto-Sivashinsky equation

$$u_t + uu_x + \alpha\, u_{xx} + \gamma\, u_{xxx} + \beta\, u_{xxxx} = 0\,, \qquad (14)$$

with the initial condition of

$$u(x,0) = f(x) \qquad (15)$$

Applying the operator L^{-1}on both sides of Eq. (14) and using the initial condition we find

$$u(x,t) = f(x) - L^{-1}(uu_x + \alpha\, u_{xx} + \gamma\, u_{xxx} + \beta\, u_{xxxx}) \qquad (16)$$

Where $\qquad L = \frac{\partial}{\partial t} \;,\; DefineL^{-1} = \int_0^t (.)dt \qquad (17)$

$$u(x,t) = \sum_{k=0}^{\infty} u_k(x,t) \qquad (18)$$

$$\sum_{k=0}^{\infty} u_k(x,t) = f(x) - L^{-1}\left(\sum_{k=0}^{\infty} A_k + \alpha\left(\sum_{k=0}^{\infty} u_k\right)_{xx} + \gamma\left(\sum_{k=0}^{\infty} u_k\right)_{xxx} + \beta\left(\sum_{k=0}^{\infty} u_k\right)_{xxxx}\right) \qquad (19)$$

Identifying the zeros component $u_0(x,t)$by $f(x)$, the remaining components$k \geq 1$ can be determined by using the recurrence relation

$$u_0(x,t) = f(x) \qquad (20)$$

$$u_{k+1}(x,t) = -L^{-1}(A_k + \alpha(u_k)_{xx} + \gamma(u_k)_{xxx} + \beta(u_k)_{xxxx}),\quad k \geq 0, \qquad (21)$$

where$A_k$are Adomian polynomials that represent the nonlinear term (uu_x) and given by

$$A_k = \frac{1}{k!}\frac{d^k}{d\lambda^i}\left[F\left(\sum_{i=0}^{k}\lambda^i u_i\right)\right]_{\lambda=0} \;, k = 0,1,\ldots \qquad (22)$$

When $\qquad F(u) = uu_x\,, \qquad (23)$

Then

$$A_0 = \frac{1}{0!}\frac{d^0}{d\lambda^0}\,[F(\lambda^0 u_0)]_{\lambda=0} = F(u_0) = u_0 u_{0x} \qquad (24)$$

$$A_1 = \frac{1}{1!}\frac{d}{d\lambda}\,[F(u_0 + \lambda u_1)]_{\lambda=0} = \frac{d}{d\lambda}\,[(u_0 + \lambda u_1)(u_0 + \lambda u_1)_x]_{\lambda=0}$$
$$= \frac{d}{d\lambda}\,[(u_0 u_{ox} + u_0 u_{1x}\lambda + u_1 u_{0x}\lambda + u_1 u_{1x}\lambda^2)]_{\lambda=0}$$
$$= u_0 u_{1x} + u_1 u_{0x} \qquad (25)$$

$$A_2 = \frac{1}{2!}\frac{d^2}{d\lambda^2}\,[F(u_0 + u_1\lambda + u_2\lambda^2)]_{\lambda=0}$$
$$= \frac{1}{2}\frac{d^2}{d\lambda^2}[(u_0 + u_1\lambda + u_2\lambda^2)(u_0 + u_1\lambda + u_2\lambda^2)_x]_{\lambda=0}$$
$$= \frac{1}{2}\frac{d^2}{d\lambda^2}[(u_0 u_{0x} + u_0 u_{1x}\lambda + u_0 u_{2\,x}\lambda^2 + u_1 u_{0\,x}\lambda + u_1 u_{1\,x}\lambda^2 + u_1 u_{2x}\lambda^3 u_2 u_{0\,x}\lambda^2 + u_2 u_{1x}\lambda^3 u_2 u_{2x}\lambda^4)]_{\lambda=0}$$
$$= u_0 u_{2x} + u_1 u_{1x} + u_2 u_{0x} \qquad (26)$$

Other polynomials can be generated in a similar way. The first few components of $u_k(x,t)$follows immediately upon setting

$$u_0(x,t) = f(x) \quad (27)$$
$$u_1(x,t) = -L^{-1}(A_0 + \alpha(u_0)_{xx} + \gamma(u_0)_{xxx} + \beta(u_0)_{xxxx}) \quad (28)$$
$$u_2(x,t) = -L^{-1}(A_1 + \alpha(u_1)_{xx} + \gamma(u_1)_{xxx} + \beta(u_1)_{xxxx}) \quad (29)$$
$$u_3(x,t) = -L^{-1}(A_2 + \alpha(u_2)_{xx} + \gamma(u_2)_{xxx} + \beta(u_2)_{xxxx}) \quad (30)$$
$$\cdots$$

Then $$u(x,t) = \sum_{k=0}^{\infty} u_k(x,t) = u_0(x,t) + u_1(x,t) + u_2(x,t) + u_3(x,t) + \cdots \quad (31)$$

III. Figures And Tables

III.1 Numerical Example

In this section, we apply the technique discussed in the previous section to find numerical solution of the Kuramoto–Sivashinsky equations and compare our results with exact solutions.

Example: [11]

$$u_t + uu_x + u_{xx} + u_{xxxx} = 0 \quad , \qquad x \in [0,32\pi], \quad (32)$$

With the initial condition of

$$u(x,0) = \cos\left(\frac{x}{16}\right)\left(1 + \sin\frac{x}{16}\right); \quad (33)$$

Exact solution of problem is given by:

$$u(x,t) = \cos\left(\frac{x}{16} - t\right)\left(1 + \sin\left(\frac{x}{16} - t\right)\right) \quad (34)$$

$$A_0 = u_0 u_{0x} = -\cos\left(\frac{x}{16}\right)\left(1 + \sin\left(\frac{x}{16}\right)\right)\left(\frac{\sin\left(\frac{x}{16}\right)\left(1 + \sin\left(\frac{x}{16}\right)\right) - \cos^2\left(\frac{x}{16}\right)}{16}\right) \quad (35)$$

$$u_1(x,t) = t\cos\left(\frac{x}{16}\right)\left(\frac{9200\sin\left(\frac{x}{16}\right) - 8192\cos^2\left(\frac{x}{16}\right)\sin\left(\frac{x}{16}\right) - 12288\cos^2\left(\frac{x}{16}\right) + 8447}{65536}\right) \quad (36)$$

$$A_1 = u_0 u_{1x} + u_1 u_{0x} = -\left(t\cos\left(\frac{x}{16}\right)\right)$$
$$\left(\frac{\left(35294\sin\left(\frac{x}{16}\right) - 118720\cos^2\left(\frac{x}{16}\right)\sin\left(\frac{x}{16}\right) + 49152\cos^2\left(\frac{x}{16}\right)\sin\left(\frac{x}{16}\right) - 134861\cos^2\left(\frac{x}{16}\right) + 102400\cos^4\left(\frac{x}{16}\right) + 35294\right)}{1048576}\right) \quad (37)$$

$$u_2(x,t) = \left(t^2\cos\left(\frac{x}{16}\right)\right)$$
$$\left(\left(\frac{165437696\sin\left(\frac{x}{16}\right) - 517734400\cos^2\left(\frac{x}{16}\right)\sin\left(\frac{x}{16}\right) + 201326592\cos^4\left(\frac{x}{16}\right)\sin\left(\frac{x}{16}\right)}{8589934592}\right)\right.$$
$$\left. + \left(\frac{-579706880\cos^2\left(\frac{x}{16}\right) + 419430400\cos^4\left(\frac{x}{16}\right) + 164855297}{8589934592}\right)\right) \quad (38)$$

$$A_2 = u_0 u_{2x} + u_1 u_{1x} + u_2 u_{0x}$$

$$= -\left(t^2\cos\left(\frac{x}{16}\right)\right)\left(\left(\frac{972569604\sin\left(\frac{x}{16}\right) - 6823380992\cos^2\left(\frac{x}{16}\right)\sin\left(\frac{x}{16}\right)}{137438953472}\right)\right.$$
$$+ \left(\frac{9043968000\cos^4\left(\frac{x}{16}\right)\sin\left(\frac{x}{16}\right) - 2147483648\cos^6\left(\frac{x}{16}\right)\sin\left(\frac{x}{16}\right)}{137438953472}\right)$$
$$\left. + \left(\frac{-7304897891\cos^2\left(\frac{x}{16}\right) + 12242182144\cos^4\left(\frac{x}{16}\right) - 5754585088\cos^6\left(\frac{x}{16}\right) + 971435586}{137438953472}\right)\right) \quad (39)$$

$$u_3(x,t) = \left(t^3 \cos\left(\frac{x}{16}\right)\right)\left(\left(\frac{4859359014912 \sin\left(\frac{x}{16}\right) - 30758081134592 \cos^2\left(\frac{x}{16}\right)\sin\left(\frac{x}{16}\right)}{1688849860263936}\right) + \left(\frac{38638599536640 \cos^4\left(\frac{x}{16}\right)\sin\left(\frac{x}{16}\right) - 8796093022208 \cos^6\left(\frac{x}{16}\right)\sin\left(\frac{x}{16}\right) - 33071821131776 \cos^2\left(\frac{x}{16}\right)}{1688849860263936}\right) + \left(\frac{52566188621824 \cos^4\left(\frac{x}{16}\right) - 23570780520448 \cos^6\left(\frac{x}{16}\right) + 4826353967871}{1688849860263936}\right)\right) \quad (40)$$

Then approximation solution of Eq.(32) is $u(x,t) = u_0 + u_1 + u_2 + u_3$ with third-order approximation.

$x * \pi$	t	EXACT SOLUTION	APPROXIMATION	ABSOLUTE ERROR
0	0	1.000000000000000	1.000000000000000	0
	0.0002	0.999799980005333	0.999988278219568	1.882982142341616e-004
	0.0004	0.999599920042668	0.999976556481800	3.766364391323274e-004
	0.0006	0.999399820144005	0.999964834786718	5.650146427130798e-004
	0.0008	0.999199680341350	0.999953113134345	7.534327929941131e-004
	0.001	0.998999500666708	0.999941391524700	9.418908579912344e-004
6.4	0	0.602909620521184	0.602909620521184	0
	0.0002	0.603261605525986	0.602924029951888	3.375755740976372e-004
	0.0004	0.603613531113795	0.602938440031289	6.750910825057410e-004
	0.0006	0.603965397251127	0.602952850759426	1.012546491701349e-003
	0.0008	0.604317203904505	0.602967262136334	1.349941768170049e-003
	0.001	0.604668951040459	0.602981674162053	1.687276878405974e-003
12.8	0	-1.284545252522524	-1.284545252522524	0
	0.0002	-1.284489444647476	-1.284551820710811	6.237606333447943e-005
	0.0004	-1.284433528322049	-1.284558388250076	1.248599280267992e-004
	0.0006	-1.284377503541076	-1.284564955140284	1.874515992073000e-004
	0.0008	-1.284321370299405	-1.284571521381397	2.501510819914454e-004
	0.001	-1.284265128591897	-1.284578086973377	3.129583814804882e-004
19.2	0	-0.333488736227371	-0.333488736227371	0
	0.0002	-0.333668118536193	-0.333484164639820	1.839538963727683e-004
	0.0004	-0.333847544554259	-0.333479593175463	3.679513787964717e-004
	0.0006	-0.334027014266963	-0.333475021834294	5.519924326690129e-004
	0.0008	-0.334206527659685	-0.333470450616308	7.360770433769148e-004
	0.001	-0.334386084717798	-0.333465879521502	9.202051962963198e-004
25.6	0	0.015124368228711	0.015124368228711	0
	0.0002	0.015095977652350	0.015123677789543	2.770013719346869e-005
	0.0004	0.015067621719845	0.015122987353990	5.536563414442614e-005
	0.0006	0.015039300412906	0.015122296922050	8.299650914345494e-005
	0.0008	0.015011013713235	0.015121606493723	1.105927804877852e-004
	0.001	0.014982761602528	0.015120916069009	1.381544664819934e-004
32	0	1.000000000000000	1.000000000000000	0
	0.0002	0.999799980005333	0.999988278219567	1.882982142346057e-004
	0.0004	0.999599920042668	0.999976556481800	3.766364391322163e-004
	0.0006	0.999399820144005	0.999964834786718	5.650146427135239e-004
	0.0008	0.999199680341350	0.999953113134344	7.534327929941131e-004
	0.001	0.998999500666708	0.999941391524699	9.418908579915675e-004

Table (1) comparison exact with Adomian Decomposition Method (ADM)

Now we compare exact solution with Adomian Decomposition Method (ADM) solution in **Fig.1,Fig.2.**

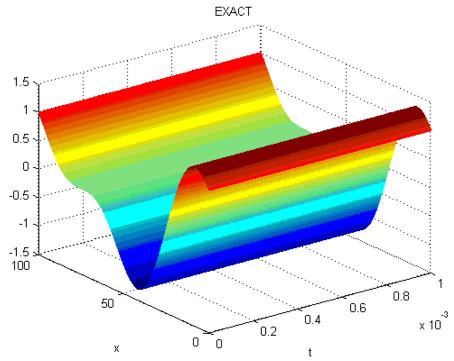

Fig.(1) Exact solution of Kuramoto-Sivashinsky equation

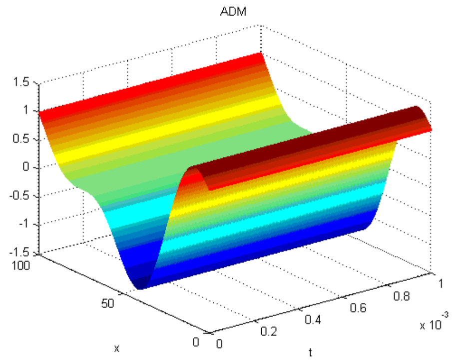

Fig.(2) (ADM) solution of Kuramoto-Sivashinsky equation

IV. Conclusion

The (ADM) applied to Kuramoto-Sivashinsky equation and by comparing with the exact solution **Fig.(1) , Fig.(2)** and **Table(1)** shows that the absolute error is so small and the approximate solution is so closed to the exact solution.

References

[1] S.T. Mohyud-Din, On Numerical Solutions of Two-Dimensional Boussinesq Equations by Using Adomian Decomposition and He's Homotopy Perturbation Method,*J. Applications nd Applied Mathematics,1932-9466*, 2010, 1-11.

[2] Kuramoto, Y., Tsuzuki, T., Persistent propagation of concentrationwaves in dissipative media far from thermal equilibrium,*Pron. Theor. Phys. ,55*, 1976, 356.

[3] Hooper, A.P., Grimshaw, R., Nonlinear instability at theinterface between two viscous fluids. *Phys. Fluids ,28*, 1985, 37–54.

[4] Sivashinsky, G.L., Instabilities, pattern-formation, and turbulencein flames. Ann. Rev. *Fluid Mech. ,15*, 1983, 179–199.

[5] Akrivis, Georgios, Smyrlis, Yiorgos-sokratis, Implicit-explicitBDF methods for the Kuramoto-Sivashinsky equation,*Appl.Numer. Math. 51*, 2004, 151–169.

[6] Manickam, A.V., Moudgalya, K.M., Pani, A.K., Second-ordersplitting combined with orthogonal cubic spline collocation methodfor the Kuramoto–Sivashinsky equation, *Comput. Math. Appl. 35*, 1998, 5–25.

[7] Uddin, M., Haq, S., Siraj-ul-Islam, A mesh-free numericalmethod for solution of the family of Kuramoto–Sivashinskyequations, *Appl. Math. Comput. 212*, 2009, 458–469.

[8] G. Adomian, Solving Frontier Problems of Physics: the Decomposition Method,(Kluwer, Boston, 1994).

[9] G. Adomian, A review of the decomposition method in applied mathematics, *J. Math. Anal. Appl. 135*, 1988,501–544.

[10] G. Adomian, New approach to nonlinear partial differential equations, *J. Math. Anal. Appl. 102*,1984, 420–434.

[11] Ling lin, *High Order Collocation Software for the Numerical Solution of Fourth Order Parabolic PDEs.*, Master Thesis ,Halifax, Nova.Scotia, University of Saint Mary's, 2009.