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Common Fixed Point Theorems of Multivalued Operators in Generalized Metric Spaces

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***Abstract**: The purpose of this article is to obtain common fixed point theorems of multivalued operators in generalized metric spaces.*
***Keywords**: multivalued mapping, fixed point, common fixed point, generalized metric space.*
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I. Introduction:

The concept of D-metric spaces was initiated by B C Dhage. The study was further enhanced by B E Rhoades, B C Dhage, A M Pathan.

Definition1.1: A non-empty set X together with a function $D:X \times X \times X \rightarrow [0 \ \infty)$ is called a D-metric space, denoted $\langle X \ D \rangle$ if D satisfies

i) $D(x,y,z) = 0$ *if and only if* $x = y = z$ *(coincidence)*

ii) $D(x,y,z) = D(p\{x, y, z\})$ *where* p *is a permutation of* x, y, z (*symmetry*)

iii)
$D(x,y,z) \le D(x,y,a) + D(x,a,z) + D(a,y,z)$ *for* $x,y,z,a \in X$ (*tetrahedral inequality*)

The non-negative real function D is called a D-metric on X . A D-metric is called generalized metric on X and the pair $\langle X \ D \rangle$ is called Generalized metric space.

Generally the usual ordinary metric is called the distance function. D-metric is called diameter function of the points of X .

The common fixed point theorems for multivalued mappings in metric spaces have been obtained by Alina Sintamarian[1] which improve and generalize a result given by A. Latif and I. Beg in [2]. Here we make use of the following theorem to obtain common fixed point theorems of multivalued operators in generalized metric spaces.

Theorem 2.1 Let $\langle X \ D \rangle$ be a metric space and $S, T : X \rightarrow P(X)$ be two multivalued operators. We suppose that at least one of the following condition is satisfied:

(i) there exists $\varphi : R_+ \rightarrow R_+$ *a function with the property that* $\varphi(0) = 0$ *and such that for each* $x \in X$ *, any* $u_x \in S(x)$ *and for all* $y \in X$ *, there exists* $u_y \in T(y)$ *so that we have*

$$d(u_x, u_y) \le \varphi(d(x,y))$$

ii) there exists $a_1, a_2 ....., a_5 \in R_+$*, with* $a_3 + a_4 < 1$ *such that for each* $x \in X$ *, any* $u_x \in S(x)$ *and for all* $y \in X$ *, there exists* $u_y \in T(y)$ *so that we have*

$$d(u_x,u_y) \le a_1 d(x,y) + a_2 d(x,u_x) + a_3 d(y,u_y) + a_4 d(x,u_y) + a_5 d(y,u_x)$$

iii) there exists $a \in R_+$*, with* $a < 1$ *such that for each* $x \in X$ *, any* $u_x \in S(x)$ *and for all* $y \in X$ *, there exists* $u_y \in T(y)$ *so that we have*

$$d(u_x,u_y) \le a \max\{ d(x,y), d(x,u_x), d(y,u_y), \tfrac{1}{2}[d(x,u_y) + d(y,u_x)]\}$$

iv) there exists $\varphi : R_+^5 \to R_+$ *a function with the property that* $\varphi(0,0,t,t,0) < t$, *for all* $t>0$ *and such that for each* $x \in X$, *any* $u_x \in S(x)$ *and for all* $y \in X$,*there exists* $u_y \in T(y)$ *so that we have*

$$d(u_x,u_y) \le \varphi(d(x,y), d(x,u_x), d(y,u_y), d(x,u_y), d(y,u_x))$$

Then $F_S \subseteq F_T$.

Definition 1.2 Let X be a non empty set. By $P(X)$ we shall understand the set of all non empty subsets of X . A correspondence $T : X \to P(X)$ is called a multivalued mapping on X .

Definition 1.3 A fixed point of multivalued mapping $T : X \to P(X)$ is a point $x \in X$ such that $x \in T(x)$.

We denote by F_T the set of the fixed points of T .

Let $\{T_n\}_{n\in N}$ be a sequence of multivalued operators with nonempty values that is

$T_n : X \to P(X)$ *for* $n \in N$.

We denote by $ComFP(T)$ the set of the common fixed points of the multivalued operators T_n , *for* $n \in N$

That is $ComFP(T) = \{x \in X | x \in T_n(x),\ for\ all\ n \in N\} = \bigcap_{n\in N} F_{T_n}$

Main Result

Theorem 2.2 Let $\langle X\ \ D\rangle$ be a generalized metric space and $S, T : X \to P(X)$ be two multivalued mappings. We suppose that at least one of the following condition is satisfied.

(i) there exists $\varphi : R_+ \to R_+$ *a function with the property that* $\varphi(0) = 0$ *and such that for each* $x \in X$, *any* $u_x \in S(x)$ *and for all* $y \in X$,*there exists* $u_y \in T(y)$ *so that we have*

$$D(u_x, u_y, u_y) \le \varphi(D(x,y,y))$$

ii) there exists $a_1, a_2 ....., a_5 \in R_+$, *with* $a_3 + a_4 < 1$ *such that for each* $x \in X$, *any* $u_x \in S(x)$ *and for all* $y \in X$,*there exists* $u_y \in T(y)$ *so that we have*

$$D(u_x,u_y u_y) \le a_1 D(x,y,y) + a_2 D(x,u_x,u_x) + a_3 D(y,u_y,u_y) + a_4 D(x,u_y,u_y) + a_5 D(y,u_x,u_x)$$

iii) there exists $a \in R_+$,*with* $a < 1$ *such that for each* $x \in X$, *any* $u_x \in S(x)$ *and for all* $y \in X$,*there exists* $u_y \in T(y)$ *so that we have*

$$D(u_x,u_y,u_y) \le a \max\{ D(x,y,y), D(x,u_x,u_x), D(y,u_y,u_y), \tfrac{1}{2}[D(x,u_y,u_y) + D(y,u_x,u_x)]\}$$

iv) there exists $\varphi : R_+^5 \to R_+$ *a function with the property that* $\varphi(0,0,t,t,0) < t$, *for all* $t>0$ *and such that for each* $x \in X$, *any* $u_x \in S(x)$ *and for all* $y \in X$,*there exists* $u_y \in T(y)$ *so that we have*

$$D(u_x,u_y,u_y) \le \varphi(D(x,y,y), D(x,u_x,u_x), D(y,u_y,u_y), D(x,u_y,u_y), D(y,u_x,u_x))$$

Then $F_S \subseteq F_T$.

Proof: We assume that condition (i) is satisfied

Let $x^* \in F_S$ *Then* $x^* \in S(x^*)$ and it follows that there exists $u \in T(x^*)$ such that

$$D(x^*,u,u) \le \varphi(D(x^*,x^*,x^*)) = \varphi(0) = 0$$

This implies that $x^* = u$

Therefore, $x^* \in T(x^*)$ and hence $F_S \subseteq F_T$

Now suppose that the condition (ii) is satisfied.

Let $x^* \in F_S$ so $x^* \in S(x^*)$ and there exists $u \in T(x^*)$ such that

$$D(x^*, u, u) \le a_1 D(x^*, x^*, x^*) + a_2 D(x^*, x^*, x^*) + a_3 D(x^*, u, u) + a_4 D(x^*, u, u) + a_5 D(x^*, x^*, x^*)$$
$$= (a_3 + a_4) D(x^*, u, u)$$

This implies $x^* = u$

Therefore $x^* \in T(x^*)$ *that is* $x^* \in F_T$

For the case when condition (iii) is fulfilled, the demonstration is made similarly with the proof from the second case.

Finally, we assume that the condition (iv) is verified.

Let $x^* \in F_S$ *then there exists* $u \in T(x^*)$ such that

$$D(x^*, u, u) \le \varphi(D(x^*, x^*, x^*), D(x^*, x^*, x^*), D(x^*, u, u), D(x^*, u, u), D(x^*, x^*, x^*))$$

Introducing the notation $t = D(x^*, u, u)$ we obtain

$$t \le \varphi(0, 0, t, t, o)$$

If we suppose that $t \neq 0$, then we reach the condition $t \le \varphi(0, 0, t, t, o) < t$

Thus $t = 0$ which means that $u = x^*$. It follows that $x^* \in T(x^*)$ and so $F_S \subseteq F_T$. This completes the proof of the theorem.

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