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# Solution of Non-Linear Differential Equations by Using Differential Transform Method

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***Abstract:*** *In this paper, we apply the differential transform method to solve some nonlinear differential equation. The nonlinear terms can be easily handled by the use of differential transform method.*
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## I. Introduction

Many problems of physical interest are described by ordinary or partial differential Equations with appropriate initial or boundary conditions, these problems are usually formulated as initial value Problems or boundary value problems, differential transform method [1, 2, 3] is particularly useful for finding solutions for these problems.

The other known methods are totally incapable of handling nonlinear Equations because of the difficulties that are caused by the nonlinear terms. This paper is using differential transforms method[4 ,5 ,6] to decompose the nonlinear term, so that the solution can be obtained by iteration procedure .The main aim of this paper is to solve nonlinear differential Equations by using of differential transform method. The main thrust of this technique is that the solution which is expressed as an infinite series converges fast to exact solutions.

### 1.1Differential Transform:

Differential transform of the function y(x) is defined as follows:

$$\mathrm{Y(k)} = \frac{1}{\mathrm{k\,!}}\left[\frac{\mathrm{d^k\,y\,(x)}}{\mathrm{dx^k}}\right]_{\mathrm{x=0}} \tag{1}$$

And the inverse differential transform of Y (k) is defined as:

$$\mathrm{y(x)} = \sum_{k=0}^{\infty} \mathrm{Y(k)}x^k$$

The main theorems of the one –dimensional differential transform are:

Theorem (1) If $\mathrm{w(x)} = \mathrm{y(x)} \pm \mathrm{z(x)}, then\ \mathrm{W(k)} = \mathrm{Y(k)} \pm \mathrm{Z(k)}$

Theorem (2) If $\mathrm{w(x)} = \mathrm{c\ y(x)}\ , then\ \mathrm{W(k)} = c\mathrm{Y(k)}$

Theorem (3) If $\mathrm{w(x)} = \frac{d\mathrm{y(x)}}{\mathrm{dx}}\ , then\ W(\mathrm{k}) = (\mathrm{k}+1)Y(\mathrm{k}+1)$

Theorem (4) If $\mathrm{w(x)} = \frac{\mathrm{d^n\,y\,(x)}}{\mathrm{dx^n}}$, then $\mathrm{W(k)} = \frac{\mathrm{(k+\,n)\,!}}{\mathrm{k\,!}}Y\,(\mathrm{k}+\mathrm{n})$

Theorem (5) If w(x) = y (x) z (x), then W (k) = $\sum_{r=0}^{k} \mathrm{Y\,(r)}Z\,(k-r)$

Theorem (6) If w (x) =$x^n$, then W (k) = δ (k-n) = $\begin{cases} 1, & \mathrm{k=n} \\ 0, & \mathrm{k} \neq \mathrm{n} \end{cases}$

Note that *c* is constant and *n* is a nonnegative integer.

### 1.2 Analysis of Differential transform:

In this section, we will introduce a reliable and efficient algorithm to calculate the differential transform of nonlinear functions.

**1.2.1 Exponential non-linearity: *f(y) = $e^{ay}$***

From the definition of transform:

$$F(0) = [e^{ay(x)}]_{x=0} = e^{ay(0)} = e^{aY(0)} \tag{2}$$

By differentiation f (y) = $e^{ay}$ with respect to x, we get:

$$\frac{df(y)}{dx} = ae^{ay}\frac{dy(x)}{dx} = af(y)\frac{dy(x)}{dx} \tag{3}$$

Application of the differential transform to Eq. (3) gives:

$$(k+1)F(k+1) = a\sum_{m=0}^{k}(m+1)Y(m+1)F(k-m) \tag{4}$$

Replacing $k+1$ by k gives:

$$F(k) = a\sum_{m=0}^{k-1}\left(\frac{m+1}{k}\right)Y(m+1)F(k-1-m) \quad , \; k \geq 1 \tag{5}$$

Then From Eqs. (2) and (5), we obtain the recursive relation:

$$F(k) = \begin{cases} e^{aY(0)} & , \; k = 0 \\ a\displaystyle\sum_{m=0}^{k-1}\frac{m+1}{k}Y(m+1)F(m-1-m) & , \; k \geq 1 \end{cases} \tag{6}$$

**1.2.2. Logarithmic non-linearity:**

f (y) = ln(a + b), a+by>0.

Differentiating f (y) = ln(a + by), with respect to *x*, we get:

$$\frac{df(y(x))}{dx} = \frac{b}{a+by}\frac{dy(x)}{dx} \; , \quad or \quad a\frac{df(y)}{dx} = b\left[\frac{dy(x)}{dx} - y\frac{df(y)}{dx}\right] \tag{7}$$

By the definition of transform:

$$F(0) = [\ln(a+by(x))]_{x=0} = \ln[a+by(0)] = \ln[a+bY(0)] \tag{8}$$

Take the differential transform of Eq. (7) to get:

$$aF(k+1) = b\left[Y(k+1) - \sum_{m=0}^{k}\frac{m+1}{k+1}F(m+1)Y(k-m)\right] \tag{9}$$

Replacing k + 1 by k yields:

$$aF(k) = b\left[Y(k) - \sum_{m=0}^{k-1}\frac{m+1}{k}F(m+1)Y(k-1-m)\right] \quad , k \geq 1 \tag{10}$$

Put k=1into Eq. (10) to get:

$$F(1) = \frac{b}{a+by(0)}Y(1) \tag{11}$$

For k ≥2, Eq. (10) can be rewritten as:

$$F(k) = \frac{b}{a+by(0)}\left[Y(k) - \sum_{m=0}^{k-2}\frac{m+1}{k}F(m+1)Y(k-1-m)\right] \tag{12}$$

Thus the recursive relation is:

$$F(k) = \begin{cases} \ln[a+bY(0)] & , k = 0 \\ \dfrac{b}{a+by(0)}Y(1) & , k = 1 \\ \dfrac{b}{a+by(0)}\left[Y(k) - \displaystyle\sum_{m=0}^{k-2}\frac{m+1}{k}F(m+1)Y(k-1-m)\right] & , \; k \geq 2 \end{cases}$$

**1.3 Application:**

Example (1)

Solve $f'(r) = [f(r)]^2$ , $f(0) = 1$

Solution:

$f' = f^2$ (13)

$f(0) = 1$ (14)

By taking differential transform on both sides of Eq. (13), we get:

$$(k+1)!\,F(k+1) = \sum_{i=0}^{k} \frac{k!}{i!\,(k-i)}(i!)F(i)(k-i)!\,F(k-i)$$

$$i.e, \qquad F(k+1) = \frac{1}{k+1}\sum_{i=0}^{k} F(i)F(k-i) \tag{15}$$

Using Eq. (15) as a recurrence relation and putting coefficient of the k = 0, 1, 2, 3, 4….. in Eq. (15),
We can get the coefficient of the power series:

When *k = 0*, $F(1) = [F(0)]^2$

$F(1) = 1$

When *k = 1*, $F(2) = \frac{1}{2}[F(0)F(1) + F(1)F(0)]$

$F(2) = 1$

When *k = 2* , $F(3) = \frac{1}{3}\sum_{i=0}^{2} F(i)F(2-i)$

$F(3) = \frac{1}{3}[F(0)F(2) + F(1)F(1) + F(2)F(0)]$

$F(3) = 1$

When *k= 3, 4,5* ....proceeding in this way, we can find the rest of the coefficients in the power series:

$$f(r) = F(0) + F(1)r^1 + F(2)r^2 + F(3)r^3 + \cdots.. = 1 + r + r^2 + r^3 + \cdots.. = \frac{1}{1-r}$$

Example (2): Solve

We consider the following nonlinear differential Equation:

$$\frac{df(r)}{dr} = f(r) - [f(r)]^2 \qquad f(0)=2$$

Solution:

$f'(r)=f(r)-[f(r)]^2$ (16)

$f(0) = 2$ (17)

Applying differential transform on both sides of Eq. (16), we get:

$$(k+1)!\,F(k+1) = k!\,F(k) - \sum_{i=0}^{k} \frac{k!}{i!\,(k-i)!}(i!)F(i)(k-i)!\,F(k-i)$$

$$i.e, \qquad F(k+1) = \frac{1}{k+1}F(k) - \frac{1}{k+1}\sum_{i=0}^{k} F(i)F(k-i) \tag{18}$$

Using Eq. (18) as a recurrence relation and using Eq. (17) in Eq. (18), we can obtain the coefficients of the power series as follows:

We put k = 0,1,2,3 …..

When k = 0 , $F(1) = F(0) - [F(0)]^2$

$F(1) = -2$

When $k = 1$ , $F(2) = \frac{1}{2}F(1) - \frac{1}{2}\sum_{i=0}^{k} F(i)F(k-i)$

$$F(2) = \frac{1}{2}F(1) - \frac{1}{2}[F(0)F(1) + F(1)F(0)]$$

$$F(2) = \frac{1}{2}(-2) - \frac{1}{2}(-8)$$

$F(2) = 3$

When *k = 2*,

$$F(3) = \frac{1}{3}F(2) - \frac{1}{3}[F(0)F(2) + F(1)F(1) + F(2)F(0)]$$

$$F(3) = \frac{1}{3}(3) - \frac{1}{3}[(2)(3) + (-2)(-2) + (3)(2)]$$

$$F(3) = 1 - \frac{8}{3}$$

$$F(3) = \frac{-5}{3}$$

Then we have the following approximate solution to initial value problem:

$$f(r) = F(0) + F(1)r^1 + F(2)r^2 + F(3)r^3 + \cdots ...$$

$$f(r) = 2 - 2r + 3r^2 - \frac{5}{3}r^3 + \cdots = \frac{2}{2 - e^{-r}}$$

Example (3)

Solve: $y'(t) = 2\sqrt{y(t)}$ , $y(0) = 1$

$f'(r) = 2\sqrt{f(r)}$ , at $r = 0$ , $f=1$

Solution:

$$[f'(r)]^2 = 4 f(r) \tag{19}$$

$$f(0)=1 \tag{20}$$

Applying differential transform method on both sides (19), we get:

$$k!F(k) = \frac{1}{4}\sum_{i=0}^{k} \frac{k!}{i!(k-i)!}(i+1)!F(i+1)(k+1-i)!F(k+1-i)$$

$$F(k) = \frac{1}{4}\sum_{i=0}^{k}(i+1)F(i+1)(k+1-i)F(k+1-i) \tag{21}$$

Using Eq.(21) as a recurrence relation and using Eq.(20) in Eq.(21), we can obtain the coefficient of power series as follows, by putting k = 0, 1, 2, 3 …

When $k = 0$

$$F(0) = \frac{1}{4}[F(1)]^2$$

$$F(1) = 2$$

When $k = 1$

$$F(1) = \frac{1}{4}[2F(1)F(2) + 2F(2)F(1)]$$

$$F(1) = F(1)F(2)$$

$$F(2) = 1$$

When $k = 2$

$$F(2) = \frac{1}{4}[3F(1)F(3) + 4F[(2)]^2 + 3F(3)F(1)]$$

$$1 = \frac{3}{2}F(3)F(1) + 4$$

$$1 = 3F(3) + 4$$

$$F(3) = -1$$

When $k = 3$

$$F(3) = \frac{1}{4}[F(1)4F(4) + 2F(2)3F(3) + 3F(3)2F(2) + 4F(4)F(1)]$$

$$-1 = \frac{1}{4}[8F(4) - 6 - 6 + 8F(4)]$$

$$-4 = [16F(4) - 12]$$

$$8 = 16F(4)$$

$$F(4) = \frac{1}{2}$$

Proceeding in this way we can obtain the following power series as a solution:

$$f(r) = F(0) + F(1)r^1 + F(2)r^2 + F(3)r^3 + \cdots$$
$$f(r) = 1 + 2r + r^2 - r^3 + \frac{1}{2}r^4 + \cdots$$

## II. Conclusion

In this paper, the series solutions of nonlinear differential Equations are obtained by Differential transform methods. This technique is useful to solve linear and nonlinear differential Equations.

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