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Some Properties of Fuzzy Soft Groups

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Abstract: *In this paper, the concept of fuzzy soft group is extended and some of their properties and structured characteristics are discussed and studied. Some important results are proved. Some important results are proved.*
Keywords*: Soft set, Fuzzy Soft Set, Soft Group, Fuzzy Soft Group, Fuzzy Subgroup, Soft Homomorphism.*

I. Introduction

Researchers in economics, engineering, environmental science, sociology, medical science and many other fields deal daily with the complexities of modeling uncertain data. Rosenfeld [10] proposed the concept of fuzzy groups in order to establish the algebraic structures of fuzzy sets.

The main purpose of this paper is a basic version of soft group theory, which extends the notion of a group to include the algebraic structures of soft sets.

In this paper begins by introducing the basic concepts of fuzzy soft set theory then we discuss a basic version of fuzzy soft group theory, which extends the notion of a group to the algebraic structures of fuzzy soft sets. A fuzzy soft group, on the other hand, is a parameterized family of fuzzy subgroups.

II. Preliminaries

In this section, we first recall the basic definitions related to fuzzy soft sets which would be used in the sequel.

Definition2.1. Soft set [7]

Suppose that U is an initial universe set and E is a set of parameters, let P(U) denotes the power set of U .A pair (F,E) is called a ***soft set*** over U where F is a mapping given by $F: E \rightarrow P(U)$.

Clearly, a soft set is a mapping from parameters to P(U), and it is not a set, but a parameterized family of subsets of the Universe.

Definition 2.2. Fuzzy soft set [7]

Let U be an initial Universe set and E be the set of parameters. Let $A \subset E$. A pair (F, A) is called ***fuzzy soft set*** over U where F is a mapping given by $F: A \rightarrow I^U$, where I^U denotes the collection of all fuzzy subsets of U.

Definition 2.3. Fuzzy soft class [9]

Let U be an initial Universe set and E be the set of attributes. Then the pair (U,E) denotes the collection of all fuzzy soft sets on U with attributes from E and is called a ***fuzzy soft class***.

Definition ***2.4. Union of fuzzy soft sets [9]***

Union of two fuzzy soft sets (F,A) and (G,B) in a soft class (U,E) is a fuzzy soft set (H,C) where
$C = A \cup B$ and

$$\forall e \in C, H(e) = \begin{cases} F(e) & , \text{if } e \in A-B \\ G(e) & , \text{if } e \in B-A \\ F(e) \cup G(e) & , \text{if } e \in A \cap B \end{cases}$$

and is written as $(F,A) \cup (G,B) = (H,C)$

Definition ***2.5***

Intersection of two fuzzy soft sets (F,A) and (G,B) in a soft class (U,E) is a fuzzy soft *set (H,C)* where $C = A \cap B$ *and*

$\forall$ e $\in$ C ,H(e) = F(e) or G(e)

and is written as

(F,A) $\cap$ (G,B) = (H,C)

Definition 2.6[1]

If (F,A) and (G,B) are two soft sets, then (F,A) **AND** (G,B) is denoted as (F,A) $\wedge$ (G,B). (F,A) $\wedge$ (G,B) is defined as (h, A x B) where h(a,b) = $h_{a,b} = f_a \wedge g_b$, $\forall$(a,b) $\in$ A x B

III. Fuzzy Soft Groups

Aktas andÇagman [3] introduced the notion of soft groups, which extends the notion of group to include the algebraic structures of soft sets. A soft group is a parameterized family of subgroups. Now in this section we introduce the definition of fuzzy soft groups and give some fundamental properties of fuzzy soft groups.

Definition 3.1: Soft Group[1]

Let X be a group and (F,A) be a soft set over X. Then (F,A) is said to be a ***soft group*** over X iff F(a) < X, for each a $\in$ A.

Definition 3.2: Fuzzy Soft Group [1]

Let X be a group and (f, A) be a fuzzy soft set over X. Then (f, A) is said to be a fuzzy soft group over X iff for each a$\in$A and x, y $\in$ X,

(i) $f_a(x \,.\, y) \geq T\,(f_a(x), f_a(y))$

(ii) $f_a(x^{-1}) \geq f_a(x)$

That is, for each a$\in$A, f_a is a fuzzy subgroup in Rosenfeld's sense [10]

Proposition 3.1

Let (f, A) be a fuzzy soft set. Then (f,A) is a fuzzy soft group if and only if for each a$\in$ A and x, y$\in$X, $f_a(x.y^{-1}) \geq T\,(f_a(x), f_a(y)$

Proof.

For each a $\in$ A and x, y $\in$ X, by monotonicity of T and (ii) we have

$f_a(x.y^{-1}) \geq T\,(f_a(x), f_a(y^{-1}))$

$\geq T\,(f_a(x), f_a(y))$

Conversely, first of all we have

$f_a(e) = f_a(x.x^{-1})$

$\geq T\,(f_a(x), f_a(x^{-1}))$

$\geq T\,(f_a(x), f_a(x)\,)$ by (ii)

$= f_a(x)$.

For each x$\in$ X, where e is the unit element of X.

Furthermore,

$f_a(x^{-1}) = f_a(e.x^{-1}) \geq T\,(f_a(e), f_a(x^{-1}))$

$\geq T\,(f_a(e), f_a(x)\,)$

$= f_a(x)$.

On the other hand, for each a $\in$ A and x,y$\in$ X,

$f_a(x.y) = f_a(x.(y^{-1})^{-1}) \geq T\,(f_a(x^{-1}), f_a(y^{-1}))$

$\geq T\,(f_a(x), f_a(y)\,)$

This completes the proof.□

Proposition 3.2.[1]

Let (f, A) be a fuzzy soft set and e is the unit element of X. Then for each a $\in$ A and for each x $\in$ X,

(1)$f_a(x^{-1}) \geq f_a(x)$

(2) $f_a(e) = f_a(x)$

Proposition 3.3. [6]

Let f_a and g_a be two fuzzy softgroups of G, then $f_a \cap g_a$ is fuzzy soft group of G.

Proposition 3.4. [6]

Let f_a and g_a be two fuzzy soft groups of G, then $f_a \cup g_a$ is fuzzy soft group of G.

Proposition 3.5.

Let f_a and g_a be two fuzzy soft groups over G, then $f_a \wedge g_a$ is fuzzy soft group of G.

Proof:

Let x, y $\in$G.

FSG(i)

$$(f_a \wedge g_a)(x.y) = f_a(x.y) \wedge g_a(x.y)$$
$$\geq T(f_a(x), f_a(y)) \wedge T(g_a(x), g_a(y))$$
$$\geq T(f_a(x) \wedge g_a(x)), T(f_a(y) \wedge g_a(y))$$
$$\geq T(f_a \wedge g_a)(x), (f_a \wedge g_a)(y))$$

$$\boxed{(f_a \wedge g_a)(x.y) \geq T(f_a \wedge g_a)(x), (f_a \wedge g_a)(y))}$$

FSG(ii)

$$(f_a \wedge g_a)(x^{-1}) \geq f_a(x^{-1}) \wedge g_a(x^{-1})$$
$$\geq f_a(x) \wedge g_a(x)$$
$$\geq (f_a \wedge g_a)(x)$$

$$\boxed{(f_a \wedge g_a)(x^{-1}) \geq (f_a \wedge g_a)(x)}$$

Proposition 3.6.

Let f_a be two fuzzy soft groups over G, then $f_a(x\,y)^2 = f_a(x^2\,y^2)$ is fuzzy soft group of G.

Proof:

Let x, y $\in$G.

FSG(i)

$$f_a(x\,y)^2 = f_a((x.y)(x.y))$$
$$\geq T(f_a(x,y), f_a(x.y))$$
$$\geq T(T(f_a(x),f_a(y)),T(f_a(x),f_a(y)))$$
$$\geq T(T(f_a(x),f_a(x)),T(f_a(y), f_a(y)))$$
$$\geq T(f_a(x.x), f_a(y.y))$$
$$\geq T(f_a(x^2), f_a(y^2))$$
$$= f_a(x^2\,y^2)$$

FSG(ii) $f_a(x\,y)^{-1} \geq T(f_a(x^{-1}), f_a(y^{-1}))$

$$\geq T(f_a(x), f_a(y))$$
$$= f_a(x\,y)$$

Proposition 3.7:

If $\{f_{ai}\}_{i \in fa}$ is a family of fuzzy soft groups of G, then $\cup f_{ai}$ is fuzzy soft group G whose $\cup f_{ai} = \{(x, \vee f_{ai}(x) / x \in G\}$, where i $\in f_a$.

Proof:

Let x, y $\in$G, then for i $\in f_a$, it follows that

(FSG(i))

$$\cup f_{ai}(x.y) = \vee f_{ai}(xy)$$
$$\geq \vee T(f_{ai}(x), f_{ai}(y))$$
$$\geq T((\vee f_{ai}(x)), (\vee f_{ai}(y))$$
$$\geq T((\cup f_{ai}(x), (\cup f_{ai}(y))$$

$$\boxed{\cup f_{ai}(x.y) \geq T((\cup f_{ai}(x), (\cup f_{ai}(y))}$$

(FSG(ii))

$$\cup f_{ai}(x^{-1}) = \vee f_{ai}(x^{-1})$$
$$\geq \vee f_{ai}(x)$$
$$\geq (\cup f_{ai})(x)$$

$$\boxed{(\cup f_{ai})(x^{-1}) \geq (\cup f_{ai})(x)}$$

Hence $(\cup f_{ai})$ is fuzzy soft group of G.

Proposition 3.8:[6]

If $\{f_{ai}\}i \in_{fa}$ is a family of fuzzy soft groups of G, then $\cap f_{ai}$ is fuzzy soft group G whose $\cap f_{ai} = \{ (x, \wedge f_{ai}(x) / x \in G\}$, where $i \in f_a$.

Proposition 3.9:[6]

Let G and G' be two groups and θ: G →G' be a soft homomorphism. If f_a is fuzzy soft group of G then the pre-image $\theta^{-1}(f_a)$ is fuzzy soft group of G.

Proposition 3.10 [6]

Let θ: G → G' be an epimorphism and f_a be fuzzy soft set in G'. If $\theta^{-1}(f_a)$ is fuzzy soft group of G, f_a is fuzzy soft group of G'.

Proposition 3.11

Let f_a be two fuzzy soft groups over G,and θ is endomorphism of G, then $f_a[\theta]$ is fuzzy soft group of G.

Proof:

Let $x, y \in G$

FSG(i)

$$\begin{aligned} f_a([\theta](x\,y)) &= f_a(\theta(x.y)) \\ &= f_a(\theta(x).\theta(y)) \\ &\geq T(f_a(\theta x), f_a(\theta y)) \\ &\geq T(f_a[\theta](x), f_a[\theta](y)) \end{aligned}$$

$$\boxed{f_a([\theta](x\,y)) \geq T(f_a[\theta](x), f_a[\theta](y))}$$

FSG(ii)

$$\begin{aligned} f_a([\theta](x)^{-1}) &= f_a(\theta(x^{-1})) \\ &\geq f_a(\theta(x)) \\ &\geq f_a([\theta](x)) \end{aligned}$$

$$\boxed{f_a([\theta](x)^{-1}) \geq f_a([\theta](x))}$$

$\therefore f_a[\theta]$ is fuzzy soft group of G

Definition 3.3 [11]

Let X be a nonempty set. A ***fuzzy subset*** of X is a function μ from X into [0,1].

Definition 3.4 [12]

A fuzzy subset of G is called a ***fuzzy subgroup*** of G if

$\mu(xy) \geq \min\{\mu(x), \mu(y)\}, \forall x, y \in G$

$\mu(x-1) \geq \mu(x), \forall x \in G$. element of a group G by e .

Definition 3.5 [11]

Let $\mu \in SP(G)$. Then μ is called a ***soft subgroup*** of G , if

(1) $\mu(xy) \supseteq \mu(x) \cap \mu(y), \forall x, y \in G$ and

(2) $\mu(x-1) \supseteq \mu(x)\ \forall x \in G$.

Where SP(G) is denoted by soft power of G, We denote the set of all soft subgroups of G by S(G) .

Definition 3.6 [12]

Let f_a be a fuzzy soft group G. let $C(x) = \{ x \in G / f_a([xy]) = f_a(e)\ \forall x \in G \}$. Then C(x) is called fuzzy centralizer of f_a, where [x y] is the commutator of two element x,y in G

ie,. $[x\,y] = x^{-1}y^{-1}x\,y$

Proposition 3.12:

Let f_a be a fuzzy soft group G. and n be a natural number then $f_a((x\,y)^n) = f_a(x^n y^n)\quad \forall x, y \in C(x)$

Proof :

For every $x, y \in C(x)$

$$\begin{aligned} f_a((x\,y)^n) &= f_a(xy\ldots\ldots xyxyxy) \\ &= f_a(xy\ldots\ldots xyxyxy\,.\,[1]) \\ &= f_a(xy\ldots\ldots xy\,xy^2x\,[x^{-1}y^{-1}x\,y]) \\ &= f_a(xy\ldots\ldots xy\,xy^2x\,[x,y]) \end{aligned}$$

$\geq T\,(f_a\,(xy\ldots\ldots xy\;xy^2x, f_a\,([x,y]))$
$\geq T\,(f_a\,(xy\ldots\ldots xy\;xy^2x, f_a\,(e))$
$\geq f_a\,(xy\ldots\ldots xy\;xy^2x)$
$\geq\; f_a\,(x^2y\ldots\ldots xy^3x\,[x,y]\,)$
$\geq f_a\,(x^3y\ldots xy^3) \geq \ldots.. \geq f_a\,(x^{n-1}yxy^{n-1})$
$= f_a\,(x^{n-1}y^n\;x\,[x,y^{\,n-1}])$
$= f_a\,(x^{n-1}y^n\;x)$
$= f_a\,(x^ny^n)$

and

$f_a\,(x^ny^n) = f_a\,(x^{n-1}y^n\;x)$
$= f_a\,(x^{n-1}y\;x\;y^{\,n-1}\,[y^{\,n-1}, x])$
$\geq f_a\,(x^{n-1}\,y\;x\;y^{n-1}) \geq \ldots.. \geq f_a(xy\ldots\ldots xyxy^2x)$
$= f_a\,(xy\ldots\ldots xyxyxy\,[x,y]\,)$
$\geq f_a(\,(x\;y)^n)$

Hence the result

IV. Conclusion

In this paper we studied the algebraic properties of fuzzy soft sets in group structures. This work focused on fuzzy soft groups, homomorphism of fuzzy soft groups. To extend this work one could the properties of fuzzy soft sets in other algebraic structures such as rings, field and ideals.

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