

On Restrained 2-Domination Number of the Join and Corona of Graphs

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Abstract : Let G be a graph. A subset S of $V(G)$ is a restrained 2-dominating set if every vertex of $V(G) \setminus S$ is adjacent to at least two vertices of S and every vertex of $V(G) \setminus S$ is adjacent to at least one vertex in $V(G) \setminus S$. The restrained 2-domination number of G , denoted by $\gamma_{r_2}(G)$, is the smallest cardinality of a restrained 2-dominating set G . In this paper, we characterize the graphs obtained in the join of two graphs having $\gamma_{r_2}(G) = 2, 3, 4$. We also characterize restrained 2-dominating sets in the corona of two graphs and give a formula for computing the restrained 2-domination number.

I. Introduction

Given a connected graph $G = (V(G), E(G))$ a dominating $S \subseteq V(G)$ is a restrained dominating set if every vertex of $V(G) \setminus S$ is adjacent to a vertex in S and to a vertex in $V(G) \setminus S$. The cardinality of a minimum restrained dominating set is the restrained domination number of G and is denoted by $\gamma_r(G)$. A dominating $S \subseteq V(G)$ is a 2-dominating set if every vertex in $V(G) \setminus S$ is adjacent to at least two vertices in S . The 2-domination number of G , denoted by $\gamma_2(G)$, is the cardinality of a minimum 2-dominating set of G . A dominating set $S \subseteq V(G)$ is called a restrained 2-dominating set if every vertex in $V(G) \setminus S$ is adjacent to at least two vertices in S and to a vertex in $V(G) \setminus S$. The restrained 2-domination number of G , denoted by $\gamma_{r_2}(G)$, is the cardinality of a minimum restrained 2-domination set.

The *join* of any graphs G and H , denoted by $G+H$, is the graph with $V(G+H) = V(G) \cup V(H)$ and $E(G+H) = E(G) \cup E(H) \cup \{uv : u \in V(G) \text{ and } v \in V(H)\}$. The *corona* of G and H , denoted by $G \circ H$ is defined as the graph obtained by taking one copy of G (which has p vertices) and p copies of H , and then joining the i th vertex of G to every vertex in the i th copy of H . We denote the copy of H with respect to vertex $a \in V(G)$ by H_a . The neighbor of a vertex x in G , denoted by $N_G(x)$ is the set of all vertices adjacent to x , i.e., $N_G(x) = \{y \in G \mid xy \in E(G)\}$.

The concept of domination has been the subject of so many researches in the last few decades due to its application in many fields, such as network topologies, location problems, protection strategies, etc. Another domination variant was introduced in [1] which was derived from two existing dominations concepts independently introduced by Telle, et al [4] and Fink, et al [7]. In [1], the authors provided some characterizations and bounds for the restrained 2-domination of a graph. Furthermore, it contained the restrained 2-domination numbers of special graphs. This present work investigated intensively the concept of restrained 2-domination in some graphs resulting from binary operations, such as the join and corona of graphs, which were not tackled by Kelkar, et. al. Important results on characterization of restrained 2-domination sets were presented here and corresponding formulas for computing the restrained 2-domination were also explicitly given.

Throughout this paper, $G = (V(G), E(G))$ is a simple undirected and connected graph. The order of G , denoted by $|G|$, is the cardinality of $V(G)$. For other graph theoretic terms which are assumed here, readers are advised to refer to [5].

II. Join of two Graphs

Theorem 2.1 Let G and H be connected graphs. Then $2 \leq \gamma_{r_2}(G+H) \leq 4$.

Proof.

A minimum r_2 -dominating set must contain at least two vertices, hence, $2 \leq \gamma_{r_2}(G+H)$.

Suppose $|G|=2$. Let $S = \{a,b\} = V(G)$ and $x \in V(G+H) \setminus S$. Then $x \in V(H)$. It follows that $|N_S(x)|=2$. Since H is connected, there exists $y \in V(H) = V(G+H) \setminus S$ such that x and y are adjacent in H , and so x and y are adjacent in $G+H$. Thus, $\gamma_{r_2}(G+H) = 2$. Similarly, if $|H|=2$, $\gamma_{r_2}(G+H) = 2$.

Suppose $|G| \geq 3$ and $|H| \geq 3$. Let $S = \{a,b,c,d\}$ where $a,b \in V(G)$ and $c,d \in V(H)$. Let $x \in V(G+H) \setminus S$. If $x \in V(G) \setminus \{a,b\}$, then $|N_S(x)| \geq 2$ and there exists $w \in V(H)$ such that x and w are adjacent in $G+H$. If $x \in V(H) \setminus \{c,d\}$, then $|N_S(x)| \geq 2$ and there exists $u \in V(G)$ such that x and u are adjacent in $G+H$. Hence, S is a r_2 -dominating set in $G+H$. Accordingly, $\gamma_{r_2}(G+H) \leq 4$.
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Theorem 2.2 Let G and H be connected graphs of order at least two. Then $\gamma_{r_2}(G+H) = 2$ if and only if one of the following holds:

1. $|G|=1$ or $|H|=1$;
2. $\gamma(G)=1$ and $\gamma(H)=1$;
3. $\gamma_2(G)=2$ or $\gamma_2(H)=2$.

Proof. Suppose $\gamma_{r_2}(G+H) = 2$. If (i) or (iii) holds, then we are done. Suppose that instead of (i) and (ii) i.e., the following conditions hold:

1. $|G| \geq 3$ and $|H| \geq 3$; and
2. $\gamma_2(G) \geq 3$ or $\gamma_2(H) \geq 3$.

Let $S = \{a,b\}$ be a minimum r_2 -dominating set in $G+H$. Suppose $a,b \in V(G)$ and $x \in V(G) \setminus \{a,b\}$. Then $x \in V(G+H) \setminus S$. Since S is an r_2 -dominating set in $G+H$, $a,b \in N_G(x)$. This implies that $\{a,b\}$ is a 2-dominating set in G . Thus, $\gamma_2(G) \leq 2$. This contradicts (a). Similarly, if $a,b \in V(H)$, we will also obtain a contradiction. Hence, $a \in V(G)$ and $b \in V(H)$.

Let $x \in V(G) \setminus \{a\}$. Then $x \in V(G+H) \setminus S$. Since S is an r_2 -dominating set, $N_S(x) = \{a,b\}$. It follows that $N_G(x) = \{a\}$. Thus, $\{a\}$ is a dominating set in G . This implies that $\gamma(G) \leq 1$. Hence, $\gamma(G) = 1$. Similarly, $\gamma(H) \leq 1$. Hence, (ii) holds.

Conversely, suppose (i) holds. Then following the case in the proof of Theorem 2.1, $\gamma_{r_2}(G+H) = 2$.

Suppose (ii) holds. Assume $|G| \geq 3$ and $|H| \geq 3$. Let $S = \{a,b\}$ where $\{a\}$ and $\{b\}$ are dominating sets in G and H , respectively. Let $x \in V(G+H) \setminus S$. If $x \in V(G) \setminus \{a\}$, then x and a are adjacent in G . In effect, $N_S(x) = \{a,b\}$. Moreover, there exists $w \in V(H) \setminus \{b\} \subseteq V(G+H) \setminus S$ such that x and w are adjacent in $G+H$. The same conclusion is obtained if $x \in V(G) \setminus \{b\}$. Thus, S is an r_2 -dominating set in $G+H$. In effect, $\gamma_{r_2}(G+H) \leq 2$. By Theorem 2.1, $\gamma_{r_2}(G+H) = 2$.

Suppose (iii) holds. Assume $\gamma_2(G) = 2$. Let $S = \{a,b\}$ be a minimum 2-dominating set in G . Let $x \in V(G+H) \setminus S$. Suppose $x \in V(G) \setminus S$. Then $|N_S(x)| = 2$. Moreover, there exists $w \in V(H) \subseteq V(G+H) \setminus S$ such that x and w are adjacent in $G+H$. Thus, S is an r_2 -dominating set

in $G+H$ Hence, $\gamma_{r_2}(G+H) = 2$. Similarly, if $\gamma_2(H) = 2$, we have $\gamma_{r_2}(G+H) = 2$. +

Theorem 2.3 Let G and H be connected graphs such that $\gamma_2(G) \geq 3$ and $\gamma_2(H) \geq 3$. Then $\gamma_{r_2}(G+H) = 3$ if and only if one of the following holds:

1. $\gamma(G) = 1$ and $\gamma(H) \geq 2$;
2. $\gamma(G) \geq 2$ and $\gamma(H) = 1$;
3. $\gamma(G) = 2$ or $\gamma(H) = 2$;
4. $\gamma_2(G) = 3$ or $\gamma_2(H) = 3$.

Proof. Suppose $\gamma_{r_2}(G+H) = 3$. Then by Theorem 2.2, the following must hold:

1. $\gamma(G) \geq 2$ or $\gamma(H) \geq 2$;
2. $\gamma_2(G) \geq 3$ and $\gamma_2(H) \geq 3$.

Let $S = \{a, b, c\}$ be an r_2 -dominating set in $G+H$. Consider the following cases:

Case 1. $a, b \in V(G)$ and $c \in V(H)$. Then $\{a, b\}$ is a dominating set in G . Consequently, $\gamma(G) \leq 2$. Suppose $\gamma(G) = 1$. Then by (1), $\gamma(H) \geq 2$. Thus, (i) holds. If $\gamma(G) = 2$, then (iii) holds.

Case 2. $a \in V(G)$ and $b, c \in V(H)$. Following the argument in *Case 1*, (ii) or (iv) holds.

Case 3. $a, b, c \in V(G)$. Then $\{a, b, c\}$ is a 2-dominating set in G . In effect, $\gamma_2(G) \leq 3$. By Theorem 2.2, $\gamma_2(G) = 3$. Thus, (iv) holds.

Case 4. $a, b, c \in V(H)$. Following the argument in *Case 3*, we have $\gamma_2(H) = 3$. Thus, (iv) holds.

Conversely, assume (i) holds. Let $S = \{a, b, c\}$, where $a, b \in V(G)$ and $c \in V(H)$ and $\{a\}$ is a dominating set in G . Let $x \in V(G+H) \setminus S$. Suppose $x \in V(G) \setminus \{a, b\}$. Note that $|N_S(x)| \geq 2$. Moreover there exists $w \in V(H) \setminus \{c\}$ such that x and w are adjacent in $G+H$. Suppose $x \in V(H) \setminus \{c\}$. Then $|N_S(x)| \geq 2$ and there exists $u \in V(G)$ such that x and u are adjacent in $G+H$. Thus, S is an r_2 -dominating in $G+H$. Accordingly, $\gamma_{r_2}(G+H) \leq 3$. Following the proof of Theorem 2.2, we will obtain a contradiction if $\gamma_{r_2}(G+H) = 2$. Thus, $\gamma_{r_2}(G+H) = 3$.

Similarly, if (ii) holds, $\gamma_{r_2}(G+H) = 3$.

Suppose $\gamma(G) = 2$. Let $S = \{a, b, c\}$ where $\{a, b\}$ is a minimum dominating set in G and $c \in V(H)$. Let $x \in V(G+H) \setminus S$. Suppose $x \in V(G) \setminus \{a, b\}$. Then $|N_S(x)| \geq 2$ and there exists $w \in V(H) \setminus \{c\}$ such that x and w are adjacent. Suppose $x \in V(H) \setminus \{c\}$. Then $|N_S(x)| \geq 2$ and there exists $u \in V(G) \setminus \{a, b\}$ such that x and u are adjacent. Thus, S is an r_2 -dominating set. Accordingly, $\gamma_{r_2}(G+H) \leq 3$. Following the argument in the proof of Theorem 2.2, we will obtain a contradiction if $\gamma_{r_2}(G+H) = 2$. Thus, $\gamma_{r_2}(G+H) = 3$.

Similarly, if $\gamma(H) = 2$, $\gamma_{r_2}(G+H) = 3$.

Suppose $\gamma_2(G) = 3$. Let $S = \{a, b, c\}$ be a 2-dominating set in G . Let $x \in V(G+H) \setminus S$. Suppose $x \in V(G) \setminus S$. Then $|N_S(x)| \geq 2$ and there exists $w \in V(H) \subseteq V(G+H) \setminus S$ such that x and w are adjacent. Suppose $x \in V(H)$. Then $|N_S(x)| \geq 3$ and there exists $w \in V(H) \subseteq V(G+H) \setminus S$ such that x and w are adjacent. Thus, S is an r_2 -dominating set. Accordingly, $\gamma_{r_2}(G+H) \leq 3$. Following the argument in the proof of Theorem 2.2, we will obtain a contradiction if $\gamma_{r_2}(G+H) = 2$. Thus,

$$\gamma_{r_2}(G + H) = 3.$$

Similarly, if $\gamma_2(H) = 3$ we have $\gamma_{r_2}(G + H) = 3.$ +

III. Corona of Two Graphs

Theorem 3.1 Let G be a connected graph of order at least two. Then $\gamma_{r_2}(G \circ K_1)$.

Proof. Let $V(G) = \{a_i : 1 \leq i \leq n\}$ and $A = \{b_i : 1 \leq i \leq n\}$. Let $V(G \circ K_1) = V(G) \cup A$ and $E(G \circ K_1) = \{a_i b_i : 1 \leq i \leq n\} \cup E(G)$. Let $S = A \cup B$, where B is a minimum r -dominating set in G , and $x \in V(G \circ K_1) \setminus S$. Then $x \in V(G) \setminus B$. Note that $x = a_k$, for some k , with $1 \leq k \leq n$. Since B is r -dominating in G , there exist p and q , with $1 \leq p, q \leq n$, such that $a_p \in B$, $a_q \in V(G) \setminus B$, and x is adjacent to both a_p and a_q . Now x is adjacent to $b_k \in A \subseteq S$, so $|N_S(x)| \geq 2$ and $|N_{V(G \circ K_1) \setminus S}(x)| \geq 1$. Thus S is an r_2 -dominating set in $G \circ K_1$. Hence, $\gamma_{r_2}(G \circ K_1) \leq |S| = |A| + |B| = |G| + \gamma_r(G)$.

Let S' be a minimum r_2 -dominating set in $G \circ K_1$. Suppose there exists i , with $b_i \notin S'$, $1 \leq i \leq n$. Since S' is an r_2 -dominating set in $G \circ K_1$, $|N_{S'}(b_i)| \geq 2$. This is a contradiction since $N(b_i) = \{a_i\}$. Thus, $A \subseteq S'$.

Suppose $V(G) \cap S'$ is not an r -dominating set in $G \circ K_1$. This implies that there exists $x \in V(G) \setminus (V(G) \cap S')$ such that $N_{V(G) \cap S'}(x) = \emptyset$ or $N_{V(G) \setminus S'}(x) = \emptyset$. If $N_{V(G) \cap S'}(x) = \emptyset$, then $|N_{S'}(x)| = 1$. This contradicts the fact that S' is an r_2 -dominating set. If $N_{V(G) \setminus S'}(x) = \emptyset$, then $N_{V(G \circ K_1) \setminus S'}(x) = \emptyset$. This is also a contradiction. Thus, $V(G) \cap S'$ is an r -dominating set in G . Hence,

$$\begin{aligned} \gamma_{r_2}(G \circ K_1) &= |S'| = |A| + |V(G) \cap S'| \\ &\geq |G| + \gamma_r(G) \end{aligned}$$

Therefore, $\gamma_{r_2}(G \circ K_1) = |G| + \gamma_r(G)$.

Theorem 3.2 Let $n \geq 2$ and G be a connected graph of order at least 2. Then $\gamma_{r_2}(G \circ K_n) = 2|G|$

Proof. Let $V(G) = \{a_i : 1 \leq i \leq |G|\}$ and for each i with $1 \leq i \leq |G|$, let $A_i = \{b_{ij} : 1 \leq j \leq n\}$ such that $\langle A_i \rangle \cong K_n$. Let $V(G \circ K_n) = V(G) \cup (\bigcup_{i=1}^{|G|} A_i)$ and $E(G \circ K_n) = E(G) \cup [\bigcup_{i=1}^{|G|} \{a_i b_{ij} : 1 \leq j \leq n\}]$. Let $S = \bigcup_{i=1}^{|G|} \{b_{ij} : j = 1, 2\}$ and $x \in V(G \circ K_n) \setminus S$. Then $x = b_{ij}$ for some i and j , with $1 \leq i \leq |G|$ and $3 \leq j \leq n$, or $x = a_k$ for some k , with $1 \leq k \leq |G|$. If $x = b_{ij}$, then $N_S(x) = \{b_{i1}, b_{i2}\}$ and $a_i \in N_{V(G \circ K_n) \setminus S}(x)$. If $x = a_k$, then $N_S(x) = \{b_{k1}, b_{k2}\}$ and there exists $u \in V(G) \subseteq V(G \circ K_n) \setminus S$ such that x and u are adjacent. Thus, in any case, S is an r_2 -dominating set in $G \circ K_n$. Accordingly, $\gamma_{r_2}(G \circ K_n) \leq |S| = 2|G|$.

Let S' be a minimum r_2 -dominating set in $G \circ K_n$. Suppose there exists i with $1 \leq i \leq |G|$, such that $|S' \cap (A_i \cup \{a_i\})| \leq 1$. Let $b_{ik} \in A_i \setminus S' \subseteq V(G \circ K_n) \setminus S'$. Now, $|S' \cap (A_i \cup \{a_i\})| \leq 1$, so $|N_{S'}(b_{ik})| \leq 1$. This contradicts the fact that S' is an r_2 -dominating set. Thus, $|S' \cap (A_i \cup \{a_i\})| \geq 2$. Hence,

$$\gamma_{r_2}(G \circ K_n) = |S'| = \left| \bigcup_{i=1}^{|G|} (S' \cap (A_i \cup \{a_i\})) \right|$$

$$= \sum_{i=1}^{|G|} |S' \cap (A_i \cup \{a_i\})| \geq \sum_{i=1}^{|G|} 2 = 2|G|.$$

Therefore, $\gamma_{r_2}(G \circ K_n) = 2|G|$.

Lemma 3.3 Let H be a connected graph of order at least three. Then $\gamma_2(H) < |H|$.

Proof. Let H be a connected graph of order at least three. Suppose $deg(a) = 1$ for every $a \in V(H)$. Let $b \in V(H)$ such that $ab \in E(H)$. Let $c \in V(H) \setminus \{a, b\}$. Then $ac, bc \notin E(H)$. This means that H is disconnected, which is a contradiction. Thus, there exists $a' \in V(H)$ such that $deg(a') \geq 2$. Let $S = V(H) \setminus \{a'\}$ and $x \in V(H) \setminus S$, i.e., $x = a'$. Then $|N_S(x)| \geq 2$. Thus, S is a 2-dominating set in H . Hence, $c = |S| = |H| - 1 < |H|$. +

Theorem 3.4 Let G and H be connected graphs of order at least three. Then

$$\gamma_{r_2}(G \circ H) = \min\{|G| \cdot \gamma_2(H), |G| \gamma_r(H) + |G|\}$$

Proof. For each $a \in V(G)$, let $H_a \cong H$. Let $V(G \circ H) = V(G) \cup (\cup_{a \in V(G)} V(H_a))$ and $E(G \circ H) = E(G) \cup (\cup_{a \in V(G)} \{ax : x \in V(H_a)\})$. For each $a \in V(G)$, let S_a be a minimum 2-dominating set in H_a . By Lemma 3.3, $|S_a| < |H|$ for every $a \in V(G)$. Note that $|S_a| \geq 2$ for every $a \in V(G)$. Let $S_1 = \cup_{a \in V(G)} S_a$ and $x \in V(G \circ H) \setminus S_1$. If $x \in V(G)$, then $|N_{S_1}(x)| = |S_a| \geq 2$ and there exists $b \in V(G) \subseteq V(G \circ H) \setminus S_1$ such that b and x are adjacent. If $x \in V(H_a) \setminus S_a$ for some $a \in V(G)$, then $|N_{S_1}(x)| \geq |N_{S_a}| \geq 2$ and x and a are adjacent. Thus, S_1 is an r_2 -dominating set in $G \circ H$. Hence,

$$\gamma_{r_2}(G \circ H) \leq |S| = \sum_{a \in V(G)} |S_a| = |G| \cdot \gamma_2(H)$$

Suppose $\gamma_2(H) \leq \gamma_r(H) + 1$. Then

$$\gamma_{r_2}(G \circ H) \leq \min\{|G| \cdot \gamma_2(H), |G| \gamma_r(H) + |G|\}.$$

Suppose $\gamma_r(H) + 1 < \gamma_2(H)$. For each $a \in V(G)$, let T_a be a minimum r -dominating on H_a . By Lemma 3.3, $|T_a| + 1 < |H|$ for every $a \in V(G)$. Let $S_2 = \cup_{a \in V(G)} (T_a \cup \{a\})$ and $x \in V(G \circ H) \setminus S_2$. Then $x \in V(H_a) \setminus T_a$ for some $a \in V(G)$. Since T_a is an r -dominating set in H_a , there exist $u \in T_a$ and $x \in H_a \setminus T_a$ such that x is adjacent to both u and v . Thus, $|N_{S_2}(x)| \geq 2$ and $|N_{V(G \circ H) \setminus S_2}(x)| \geq 1$.

Consequently, S_2 is an r_2 -dominating set in $G \circ H$. Hence,

$$\begin{aligned} \gamma_{r_2}(G \circ H) \leq |S| &= \sum_{a \in V(G)} (|T_a| + 1) = |G|(\gamma_r(H) + 1) \\ &\leq \min\{|G| \cdot \gamma_2(H), |G| \gamma_r(H) + |G|\} \end{aligned}$$

Let S be a minimum r_2 -dominating set in $G \circ H$. Then

$$S = A \cup \left(\bigcup_{a \in A} (V(H_a) \cap S) \right) \cup \left(\bigcup_{a \in A \setminus A} (V(H_a) \cap S) \right)$$

where $S \subseteq V(G)$. Suppose $a \in V(G)$ and $x \in V(H_a) \setminus (V(H_a) \cap S)$. Since S is an r_2 -

dominating set, $|N_{V(G \circ H) \setminus S}(x)| \geq 1$. Since $N_S(x) = \{a\} \cup N_{V(H_a) \cap S}(x)$ and

$N_{V(G \circ H) \setminus S}(x) = N_{V(H_a) \cap S}(x)$, $|N_{V(H_a) \cap S}(x)| \geq 1$. Thus, $N_{V(H_a) \cap S}$ is r -dominating, if $a \in A$.

Suppose $a \in V(G) \setminus A$ and $x \in V(H_a) \setminus (V(H_a) \cap S)$. Since S is an $r2$ -dominating set, $|N_S(x)| \geq 2$.

Now, $N_S(x) = N_{V(H_a) \cap S}(x)$, so $|N_{V(H_a) \cap S}(x)| \geq 2$. Thus, $V(H_a) \cap S$ is 2-dominating if $a \in V(G) \setminus A$.

. Let $\lambda = \min\{\gamma_2(H), \gamma_r(H) + 1\}$. Hence,

$$\begin{aligned} \gamma_{r2}(G \circ H) &= |S| = |A| + \sum_{a \in A} |V(H_a) \cap S| + \sum_{a \in V(G) \setminus A} |V(H_a) \cap S| \\ &\geq |A| + \sum_{a \in A} (\gamma_r(H)) + \sum_{a \in V(G) \setminus A} (\gamma_2(H)) \\ &= |A| + |A| \cdot \gamma_r(H) + (|G| - |A|) \gamma_2(H) \\ &\geq |A| + |A| \cdot \lambda + (|G| - |A|) \lambda \\ &= |A| + |A| \cdot \lambda + |G| \lambda - |A| \lambda \\ &= |A| + |G| \lambda \\ &\geq |G| \lambda \end{aligned}$$

The result now follows. +

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