IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN:2319-765X. Volume 10, Issue 2 Ver. IV (Mar-Apr. 2014), PP 23-28
www.iosrjournals.org

Availability Analysis of a Standby System with Three Types of Failure Categories

[1] Upasana Sharma, [2]Jaswinder kaur
[1,2]Department of Statistics Punjabi University Patiala

Abstract: *The present paper is an attempt to analyze the availability of an operating compressor unit working in a milk plant. In a milk plant's refrigeration system compressor plays an important role. Any major failure or annual maintenance brings the operating unit to a complete halt. It has been observed that the unit can fail due to various types of failures which can be categorized as- serviceable type, repairable type and replaceable type. For availability analysis of the unit real failure as well as repair time data from a milk plant have been collected and measures of unit effectiveness i.e. availability and mean time to unit failure has been computed graphically as well as numerically by using semi-Markov process and regenerative point technique.*
Keywords: *Compressor unit, Regenerative point technique, Refrigeration system, Semi-Markov process .*

I. Introduction

Standby systems are commonly used in many industries and therefore, researchers [1-3] have spent a great deal of efforts in analyzing such systems to get the optimized reliability results which are useful for effective equipment/plant maintenance. For graphical study, they have taken assumed values for failure and repair rates, and not used the observed values. However, some researchers including [4-7] studied some reliability models collecting real data on failure and repair rates of the units used in such systems.

A potential application of the reliability concepts has been recently explored in terms of developing a specific probabilistic model for desalination unit considering Nine Failure Categories and thereby achieving some reliability measures of the unit effectiveness which in turn are meaningful in understanding the plant/unit performance by S M Rizwan ,N Padmavathi, G Taneja, AG Mathew and Ali Mohammed Al-Balushi [8].

Getting inspiration from the above concept the present paper is thus, an attempt to analyze a compressor unit probabilistically and availability of the unit is obtained. In present paper a three unit standby model is developed Initially there are two operating and one standby compressor unit and atleast two compressor units are needed to keep the system functioning state . In the present model real failure situations are used as depicted in the data for analysis.For this purpose, a refrigeration system used in milk plant is identified. In a milk plant's refrigeration system compressor plays an important role. Any major failure or annual maintenance brings the operating unit to a complete halt. It has been observed that the unit can fail due to various types of failures which can be categorized as- serviceable type, repairable type and replaceable type .

For availability analysis of the unit real failure as well as repair time data from a milk plant have been collected and measures of unit effectiveness i.e. mean time to unit failure has been computed graphically as well as numerically while availability has been computed numerically only by using semi-Markov process and regenerative point technique .

Notations

OI,OII ,OIII First , Second and Third Compressor are in Operative State
SII, SIII Second and Third Compressors are in Standby state
$\lambda_{11},\lambda_{21},\lambda_{31}$ Failure rate when failure is of serviceable type for first ,second and third compressor respectively.
$\lambda_{12},\lambda_{22},\lambda_{32}$ Failure rate when failure is of repairable type for first,second and third compressor respectively.
$\lambda_{13},\lambda_{23},\lambda_{33}$ Failure rate when failure is of replaceable type for first, second and third compressor respectively.
$\alpha_{11},\alpha_{12},\alpha_{13}$ Repair rates when failure is of serviceable , repairable and replaceable type for first compressor.
$\alpha_{21},\alpha_{22},\alpha_{23}$ Repair rates when failure is of serviceable, repairable and replaceable type for second compressor.
$\alpha_{31},\alpha_{32},\alpha_{33}$ Repair rates when failure is of serviceable , repairable and replaceable type for third compressor.
FsI,FsII ,FsIII Failure category of serviceable type for first, Second and third Compressor.
FrI,FrII,FrIII Failure category of repairable type for first ,second and third compressor.
FrepI,FrepII,FrepIII Failure category of replaceable type for First, Second and third compressor.
FwrI,FwsI,FwrepI First compressor is waiting for Repair, Service, Replacement respectively.

$G_{11}(t), g_{11}(t), G_{21}(t), g_{21}(t), G_{31}(t), g_{31}(t)$ c.d.f and p.d.f of time for service when failure is of serviceable type for first ,second and third compressor respectively.

$G_{12}(t), g_{12}(t) G_{22}(t), g_{22}(t), G_{32}(t), g_{32}(t)$ c.d.f and p.d.f. of time for repair when failure is of repairable type for first , second and third compressor respectively.

$G_{13}(t), g_{13}(t), G_{23}(t), g_{23}(t), G_{33}(t), g_{33}(t)$ c.d.f and p.d.f of time for replacement when failure is of replaceable type for first ,second and third compressor respectively.

$Q_{ij}(t)$ cumulative distribution function (c.d.f) of first passage time from a regenerative state i to j or to a failed state j in (0, t].

Model Description and Assumptions

1) The unit is initially operative at state 0 and its transition depends upon the type of failure category to any of the three states 1 to 3 with different failure rates.
2) Priority of repair is given to recently failed unit.
3) When two units are failed then the third unit automatically go to the standby state.
4) All failure times are assumed to have exponential distribution .
5) After each servicing/ repair/replacement at states the unit works as good as new.

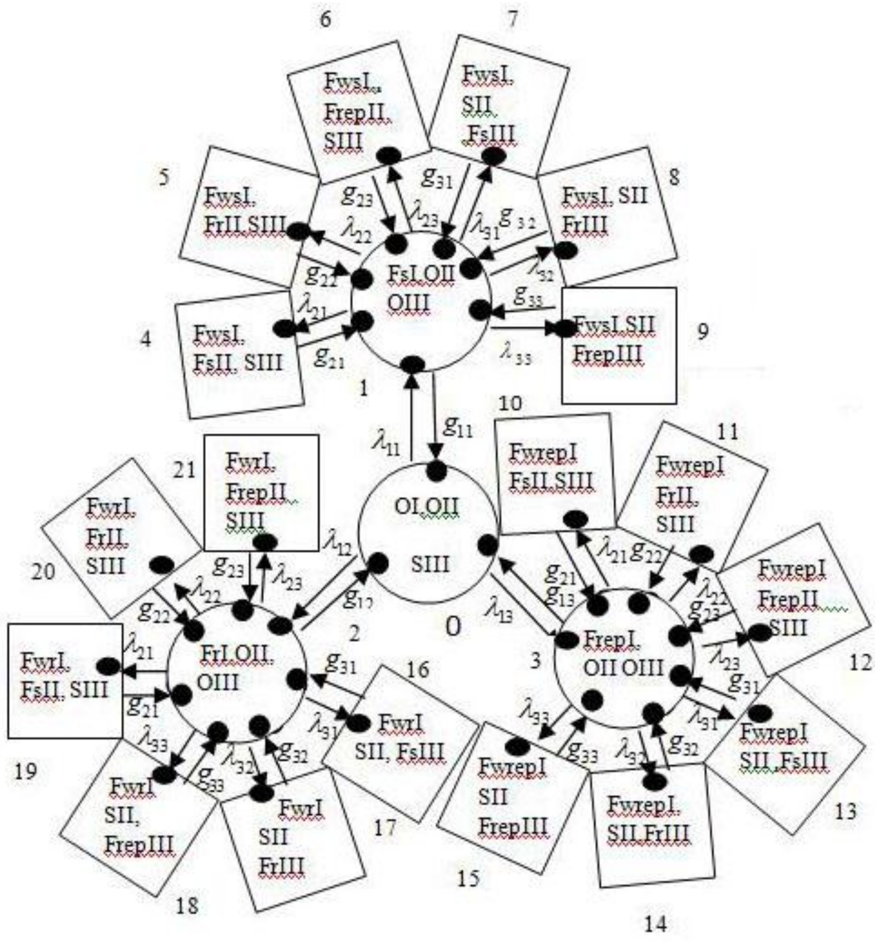

fig 1 **State Transition diagram**

Transition Probabilities and Mean Sojourn Times

A state transition diagram showing the various states of transition of the system is shown in fig.1.The epochs of entry into states0,1,2,3,4,5,6,7,8,9,10,11,1213,14,15,16,17,18,19,20and21 are regenerative states. States 4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21 are down states and 0,1,2,3 are upstates. The non zero elements p_{ij} are given below:

$$p_{01} = \frac{\lambda_{11}}{\lambda *}, p_{02} = \frac{\lambda_{12}}{\lambda *}, p_{03} = \frac{\lambda_{13}}{\lambda *}, p_{10} = g_{11}^{*}(\lambda), p_{20} = g_{12}^{*}(\lambda), p_{30} = g_{13}^{*}(\lambda)$$

Where

$$\lambda^{*} = \lambda_{11} + \lambda_{12} + \lambda_{13}$$

$$\lambda = \lambda_{21} + \lambda_{22} + \lambda_{23} + \lambda_{31} + \lambda_{32} + \lambda_{33}$$

$$p_{2,16}, p_{22}^{16} = \frac{\lambda_{31}}{\lambda}(1 - g_{12}^{*}(\lambda)), p_{2,17}, p_{22}^{17} = \frac{\lambda_{32}}{\lambda}(1 - g_{12}^{*}(\lambda)), p_{2,18}, p_{22}^{18} = \frac{\lambda_{33}}{\lambda}(1 - g_{12}^{*}(\lambda)) p_{2,19}, p_{22}^{19} = \frac{\lambda_{21}}{\lambda}(1 - g_{12}^{*}(\lambda))$$

$$p_{2,20}, p_{22}^{20} = \frac{\lambda_{22}}{\lambda}(1 - g_{12}^{*}(\lambda)), p_{2,21}, p_{22}^{21} = \frac{\lambda_{23}}{\lambda}(1 - g_{12}^{*}(\lambda)) p_{14}, p_{11}^{4} = \frac{\lambda_{21}}{\lambda}(1 - g_{11}^{*}(\lambda)), p_{15}, p_{11}^{5} = \frac{\lambda_{22}}{\lambda}(1 - g_{11}^{*}(\lambda))$$

$$p_{16}, p_{11}^{6} = \frac{\lambda_{23}}{\lambda}(1 - g_{11}^{*}(\lambda)), p_{17}, p_{11}^{7} = \frac{\lambda_{31}}{\lambda}(1 - g_{11}^{*}(\lambda)) p_{18}, p_{11}^{8} = \frac{\lambda_{32}}{\lambda}(1 - g_{11}^{*}(\lambda)), p_{19}, p_{11}^{9} = \frac{\lambda_{33}}{\lambda}(1 - g_{11}^{*}(\lambda))$$

$$p_{3,10}, p_{33}^{10} = \frac{\lambda_{21}}{\lambda}(1 - g_{13}^{*}(\lambda)), p_{3,11}, p_{33}^{11} = \frac{\lambda_{22}}{\lambda}(1 - g_{13}^{*}(\lambda)) p_{3,12}, p_{33}^{12} = \frac{\lambda_{23}}{\lambda}(1 - g_{13}^{*}(\lambda))$$

$$p_{3,13}, p_{33}^{13} = \frac{\lambda_{31}}{\lambda}(1 - g_{13}^{*}(\lambda)), p_{3,14}, p_{33}^{14} = \frac{\lambda_{32}}{\lambda}(1 - g_{13}^{*}(\lambda)), p_{3,15}, p_{33}^{15} = \frac{\lambda_{33}}{\lambda}(1 - g_{13}^{*}(\lambda))$$

$$p_{01} + p_{02} + p_{03} = 1$$

$$p_{10} + p_{14} + p_{15} + p_{16} + p_{17} + p_{18} + p_{19} = 1, p_{10} + p_{11}^{4} + p_{11}^{5} + p_{11}^{6} + p_{11}^{7} + p_{11}^{8} + p_{11}^{9} = 1$$

$$p_{20} + p_{2,16} + p_{2,17} + p_{2,18} + p_{2,19} + p_{2,20} + p_{2,21} = 1, p_{20} + p_{22}^{16} + p_{22}^{17} + p_{22}^{18} + p_{22}^{19} + p_{22}^{20} + p_{22}^{21} = 1$$

$$p_{30} + p_{3,10} + p_{3,11} + p_{3,12} + p_{3,13} + p_{3,14} + p_{3,15} = 1, p_{30} + p_{33}^{10} + p_{33}^{11} + p_{33}^{12} + p_{33}^{13} + p_{33}^{14} + p_{33}^{15} = 1$$

The mean sojourn time (μ_i) in the regenerative state 'i' is defined as time of stay in that state before transition to any other state:

$$\mu_0 = \frac{1}{\lambda *}$$

$$\mu_1 \mu_2, \mu_3 = \frac{1}{\lambda}$$

$$\mu_4, \mu_{10}, \mu_{16} = \int_0^{\infty} \overline{G}_{21}(t)dt, \mu_5, \mu_{11}, \mu_{17} = \int_0^{\infty} \overline{G}_{22}(t)dt, \mu_6, \mu_{12}, \mu_{18} = \int_0^{\infty} \overline{G}_{23}(t)dt$$

$$\mu_7, \mu_{13}, \mu_{19}, = \int_0^{\infty} \overline{G}_{31}(t)dt, \mu_8, \mu_{14}, \mu_{20} = \int_0^{\infty} \overline{G}_{32}(t)dt, \mu_9, \mu_{15}, \mu_{21} = \int_0^{\infty} \overline{G}_{33}(t)dt$$

The unconditional mean time taken by the system to transit for any regenerative state 'j' when it (time) is counted from the epoch of entrance into state 'i' is mathematically state as:

$$mij = \int_0^{\infty} t d\mathrm{Q}_{\mathrm{ij}}(t) = -q_{ij}{'}{*}(0)$$

$$m_{01} + m_{02} + m_{03} = \frac{1}{\lambda *} = \mu_0$$

$$m_{10} + m_{11}^{4} + m_{11}^{5} + m_{11}^{7} + m_{11}^{8} + m_{11}^{9} = \mu_1(1 - g_{11}^{*}(\lambda))$$

$$m_{20} + m_{22}^{16} + m_{22}^{17} + m_{22}^{18} + m_{22}^{19} + m_{22}^{20} + m_{22}^{21} = \mu_2(1 - g_{12}^{*}(\lambda))$$

$$m_{20} + m_{2,16} + m_{2,17} + m_{2,18} + m_{2,19} + m_{2,20} + m_{2,21} = \mu_2(1 - g_{12}^{*}(\lambda))$$

$$m_{30} + m_{3,10} + m_{3,11} + m_{3,12} + m_{3,13} + m_{3,14} + m_{3,15} = \mu_3(1 - g_{13}^{*}(\lambda))$$

$$m_{30} + m_{33}^{10} + m_{33}^{11} + m_{33}^{12} + m_{33}^{13} + m_{33}^{14} + m_{33}^{15} = \mu_3(1 - g_{13}^{*}(\lambda))$$

II. Mean Time to System Failure

To determine the mean time to system failure (MTSF) of the system, we regard the failed states of the system absorbing. By probabilistic arguments ,we obtain the following recursive relation for $ø_i(t)$:

$$Ø_0(t) = \mathrm{Q}_{01}(t)(s)Ø_1(t) + \mathrm{Q}_{02}(t)(s)Ø_2(t) + \mathrm{Q}_{03}(t)(s)Ø_3(t)$$

$$Ø_1(t) = \mathrm{Q}_{14}(t) + \mathrm{Q}_{15}(t) + \mathrm{Q}_{16}(t) + \mathrm{Q}_{17}(t) + \mathrm{Q}_{18}(t) + \mathrm{Q}_{19}(t) + \mathrm{Q}_{10}(s)Ø_0(t)$$

$$Ø_2(t) = \mathrm{Q}_{2,16}(t) + \mathrm{Q}_{2,17}(t) + \mathrm{Q}_{2,18}(t) + \mathrm{Q}_{2,19}(t) + \mathrm{Q}_{2,20}(t) + \mathrm{Q}_{2,21}(t) + Q_{20}(s)Ø_0(t)$$

$$Ø_3(t) = \mathrm{Q}_{3,10}(t) + \mathrm{Q}_{3,11}(t) + \mathrm{Q}_{3,12}(t) + \mathrm{Q}_{3,13}(t) + \mathrm{Q}_{3,14}(t) + \mathrm{Q}_{3,15}(t) + \mathrm{Q}_{30}(s)Ø_0(t)$$

Taking Laplace –Stieltjes Transforms(L.S.T) of above relations and solving for $\text{Ø}_0^{**}(s)$ .Now the mean time to system failure(MTSF) when system starts from the state 0.

$$\text{MTSF} = T_0 = \lim_{S\to 0}\frac{1-\text{Ø}_0{}^{**}(s)}{s} = N/D$$

$$N = m_{01}p_{10} + m_{10}p_{01} + m_{02}p_{20} + m_{20}p_{02} + m_{03}p_{30} + m_{30}p_{03} + m_{01} + m_{02} + m_{03} - m_{01}p_{10} - m_{02}p_{20} - m_{03}p_{30} - m_{30}p_{03}$$
$$-m_{20}p_{02} - m_{10}p_{01} + p_{03}\mu_3(1-g_{13}^*(\lambda)) + p_{02}\mu_2(1-g_{12}^*(\lambda)) + p_{01}\mu_1(1-g_{11}^*(\lambda)), D = 1 - p_{01}p_{10} - p_{02}p_{20} - p_{03}p_{30}$$

Where

$N = 8887.14323, D = .631107281$

$MTSF = 14081.82649 hrs$

III. Availability Analysis

Let $A_i(t)$ be the probability that the system is in up state at instant t given that the system entered regenerative state i at t=0. The availability $A_i(t)$ is to satisfy the following recursive relations:

$$A_0(t) = M_0(t) + q_{01}©A_1(t) + q_{02}©A_2(t) + q_{03}©A_3(t)$$
$$A_1(t) = M_1(t) + q_{10}©A_0(t) + q_{11}{}^{(4)}©A_1(t) + q_{11}{}^{(5)}©A_1(t) + q_{11}{}^{(6)}©A_1(t) + q_{11}{}^{(7)}©A_1(t) + q_{11}{}^{(8)}©A_1(t) + q_{11}{}^{(9)}©A_1(t)$$
$$A_2(t) = M_2(t) + q_{20}©A_0(t) + q_{22}{}^{(16)}©A_1(t) + q_{22}{}^{(17)}©A_1(t) + q_{22}{}^{(18)}©A_1(t) + q_{22}{}^{(19)}©A_1(t) + q_{22}{}^{(20)}©A_1(t) + q_{22}{}^{(21)}©A_1(t)$$
$$A_3(t) = M_3(t) + q_{30}©A_0(t) + q_{33}{}^{(10)}©A_1(t) + q_{33}{}^{(11)}©A_1(t) + q_{33}{}^{(12)}©A_1(t) + q_{33}{}^{(13)}©A_1(t) + q_{33}{}^{(14)}©A_1(t) + q_{33}{}^{(15)}©A_1(t)$$
$$A_4 = q_{41}©A_1(t), A_5 = q_{51}©A_1(t), A_6 = q_{61}©A_1(t)$$
$$A_7 = q_{71}©A_1(t), A_8 = q_{81}©A_1(t), A_9 = q_{91}©A_1(t)$$
$$A_{10} = q_{10,3}©A_3(t), A_{11} = q_{11,3}©A_3(t), A_{12} = q_{12,3}©A_3(t)$$
$$A_{13} = q_{13,3}©A_3(t), A_{14} = q_{14,3}©A_3(t), A_{15} = q_{15,3}©A_3(t)$$
$$A_{16} = q_{16,2}©A_2(t), A_{17} = q_{17,2}©A_2(t), A_{18} = q_{18,2}©A_2(t)$$
$$A_{19} = q_{19,2}©A_2(t), A_{20} = q_{20,2}©A_2(t), A_{21} = q_{21,2}©A_2(t)$$

where

$$M_0(t) = e^{-(\lambda_{11}+\lambda_{12}+\lambda_{13})t}, M_1(t) = e^{-(\lambda_{21}+\lambda_{22}+\lambda_{23}+\lambda_{31}+\lambda_{32}+\lambda_{33})t}\bar{G}_{11}(t)$$
$$M_2(t) = e^{-(\lambda_{21}+\lambda_{22}+\lambda_{23}+\lambda_{31}+\lambda_{32}+\lambda_{33})t}\bar{G}_{12}(t), M_3(t) = e^{-(\lambda_{21}+\lambda_{22}+\lambda_{23}+\lambda_{31}+\lambda_{32}+\lambda_{33})t}\bar{G}_{13}(t)$$

Insteady state availability of the system is given as

$$A_0 = \lim_{s\to 0}(sA_0^*(s)) = N_1/D_1$$

where

$$N_1 = \mu_0 p_{10}p_{20}p_{30} + p_{01}\mu_1(1-g_{11}{}^*)p_{20}p_{30} + p_{02}\mu_2(1-g_{12}{}^*)p_{10}p_{30} + p_{03}\mu_3(1-g_{13}{}^*)p_{10}p_{20}$$
$$D_1 = (\mu_1(1-g_{11}{}^*) - m_{10})p_{20}p_{30} + (\mu_2(1-g_{12}{}^*) - m_{20})p_{10}p_{30} + (\mu_3(1-g_{13}{}^*) - m_{30})p_{10}p_{20} + +m_{01}p_{01}p_{20}p_{30} + p_{10}m_{01}p_{20}p_{30}$$
$$-(\mu_2(1-g_{12}{}^*) - m_{20})p_{10}p_{30}p_{01} - (\mu_3(1-g_{13}{}^*) - m_{30})p_{10}p_{20}p_{01} + p_{10}m_{02}p_{20}p_{30} + p_{02}m_{20}p_{10}p_{30} - (\mu_1(1-g_{11}{}^*) - m_{10})p_{20}p_{30}p_{02}$$
$$-(\mu_3(1-g_{13}{}^*) - m_{30})p_{10}p_{20}p_{02} + p_{10}m_{02}p_{20}p_{30} + p_{02}m_{20}p_{10}p_{30} - (\mu_1(1-g_{11}{}^*) - m_{10})p_{03}p_{30}p_{20} - (\mu_2(1-g_{12}{}^*) - m_{20})p_{10}p_{30}p_{03}$$

A_0=.999999999

Particular Cases

For graphical representation ,let us suppose that

$$g_{11}(t) = \alpha_{11}e^{-\alpha_{11}t}, g_{12}(t) = \alpha_{12}e^{-\alpha_{12}t}, g_{13}(t) = \alpha_{13}e^{-\alpha_{13}t}$$

using the above particular case, the following values are estimated as

$\alpha_{11} = 0.006896, \alpha_{12} = 0.000586$

$\alpha_{13} = 0.04166, \alpha_{21} = 0.0000983$

$\alpha_{22} = 0.0001347, \alpha_{23} = 0.00015873$

$\lambda_{11}, \lambda_{12}, \lambda_{13} = 0.00003868, \lambda_{21}, \lambda_{22}, \lambda_{23} = 0.0007352, \lambda_{31}, \lambda_{32}, \lambda_{33} = 0.0000456071$

IV. Conclusion

The measures of system effectiveness are obtained as:
Mean time to unit/compressor MTSF =14081.82649 hrs.
Availability of the unit/compressor (A_0) =0.999999999

It has been achieved that the expected time for which the unit/compressor is in operation before it completely fails is about 14081.82649 hours. Also, the probability that the unit/compressor will be able to operate within the tolerances for a specified period of time is 0.99999999 which certainly would meet the annual maintenance norms fixed for the plant.

V. Graphical Interpretation

Graph in fig 2 represents the behaviour of MTSF and failure rate λ_{12} with variation in λ_{22} and λ_{32}.It is clear that as failure rate λ_{12} increases MTSF decreases. As the variation is taken in failure rate λ_{22} and λ_{32} for MTSF , it can be concluded that as the failure rate λ_{22} , λ_{32} increases MTSF decreases.

Graph between MTSF and λ_{12}(variation in λ_{22} ,λ_{32})

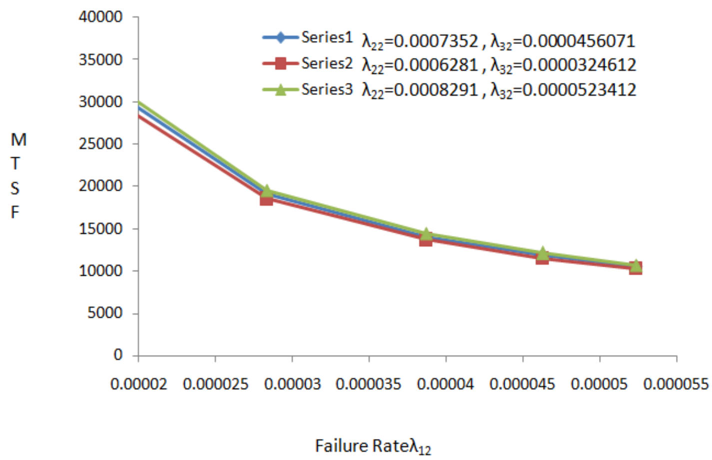

fig2

Graph in fig 3 represents the behaviour of MTSF and failure rate λ_{21} with variation in λ_{11} and λ_{31}.It is clear that as failure rate λ_{21} increases MTSF decreases. As the variation is taken in failure rate λ_{11} and λ_{31} for MTSF , it can be concluded that as the failure rate λ_{11}, λ_{31} increases MTSF decreases.

Graph between MTSF and λ_{21}(variation in λ_{11} ,λ_{31})

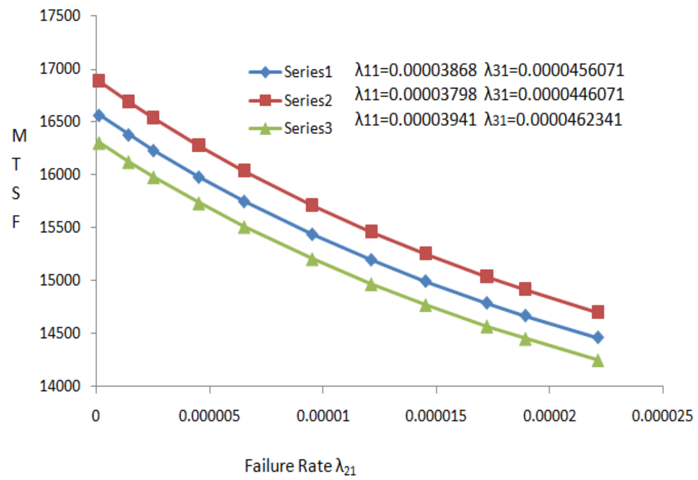

fig 3

References

[1] R.K. Tuteja, and G.Taneja, "Profit analysis of one server one unit system with partial failure and subject to random inspection",Microelectron. Reliab,vol 33,pp 319-322.1993.
[2] R.K.Tuteja, G.Taneja, and U.Vashistha, " Two-dissimilar units system wherein standby unit in working state may stop even without failure", International Journal of Management and Systems, vol 17 no 1,pp 77-100,2001
[3] M.E.S Khaled, . and S.E.S Mohammed, "Profit evaluation of two unit cold standby systemwith preventive and random changes in units."Journal of Mathematics and Statistics. vol 1, pp. 71-77. 2005
[4] G.Taneja, V.K.Tyagi, and P. Bhardwaj,. "Profit analysis of a single unit programmable logic controller (PLC)", Pure and Applied Mathematika Sciences, vol. LX no (1-2),pp 55-71,2004
[5] G.Taneja, "Reliability and profit analysis of a system with PLC used as hot standby". Proc.INCRESE Reliability Engineering Centre, IIT, Kharagpur India, pp: 455-464.,2005(Conference proceedings)
[6] B. Parasher, and G.Taneja,. "Reliability and profit evaluation of a standby system based on a master-slave concept and two types of repair facilities". IEEE Trans. Reliabil., 56: pp.534-539, 2007
[7] A Goyal and Gulshan Taneja, D.V Singh, "Analysis of a 2-unit cold standby system working in a sugar mill with operating and rest periods. Caledon. J. Eng., 5: 1-5. ,2009
[8] S M Rizwan, N Padmavathi, G Taneja, A G Mathew and Ali Mohammed Al-Balushi "Probabilistic Analysis of a Desalination unit with Nine Failure Categories", Proceedings of the World Congress on Engineering 2010 Vol III WCE 2010, June 30 - July 2, 2010, London, U.K.pp.1877-1880