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Construction Smooth Hausdroff Space from Fuzzy Metric Space

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Abstract: *In this paper, concepts of smooth topology and fuzzy metric space have been introduced. We define smooth Hausdroff space and so fuzzy open ball in fuzzy metric space to get the smooth topological space which is called induced smooth topological space then we proved this fuzzy metric is smooth Hausdroof space.*
Keywords: *Smooth topology, Fuzzy metric space, quasi (q) and not quasi ($\leftharpoondown q$)-coincident.*

I. Introduction:

Many authors have introduced the concept of smooth topology on crisp set [1, 2, 4] and so fuzzy metric space [5, 7].There are several definitions of fuzzy Metric space, one is using fuzzy numbers to define metric in ordinary spaces [6] and the other using fuzzy scalars (fuzzy points defined on the real-valued space R) to measure the distances between fuzzy points [5] as in this paper. In this paper, fuzzy points are usually denoted by $x_\lambda$and the set of all the fuzzy points defined on X is denoted by $P_F(X)$. Particularly, when X = R, fuzzy points are also called fuzzy scalars and the set of all the fuzzy scalars is denoted by $S_F(R)$.

1-Preliminaries

Definition 1.1: [1, 2, And 4]: A smooth topological space (sts) is a pair (X,τ) where X is a nonempty set and $\tau: I^x \to I$ is mapping satisfy the following properties:

1- $\tau(X) = \tau(\emptyset) = 1$.
2- $\forall A, B \in I^x, \tau(A \cap B) \geq \tau(A) \wedge \tau(B)$.
3- For every subfamily$\{A_i : i \in J\} \subseteq I^x, \tau(\bigcup_{i\in J} A_i) \geq \bigwedge_{i\in J} \tau(A_i)$.

Definition 1.2: let X be a nonempty set and τ is st defines on it, the family$\tau_\alpha = \{A \in I^x : \tau(A) \geq \alpha\}$ and $\alpha \in$ (0,1], is called α-level.

Note: In the following when we say the set G is open or neighborhood then we meaning $\tau(G) \geq \alpha$ for some $\alpha \in$ (0,1].

Definition 1.3: Let (X,τ) be sts, (X, τ_α) is fuzzy Hausdroff if for each distinct fuzzy points x_t and y_r in X when $x \neq y$ there exist fuzzy neighborhoods G_1 of x_t and G_2 of y_r such that $G_{1\leftharpoondown}qG_2$, when $x = y$ and $t < r$ (say) x_t has a fuzzy neighborhoods G_1 and y_r has a fuzzy q-neighborhoods G_2 in which $G_{1\leftharpoondown}qG_2$.

Definition 1.4: An sts (X,τ) is called "smooth Hausdroff" iff for each $\propto \in (0,1]$, (X, τ_α) is fuzzy Hausdroff.

Definition 1.5, [7]: Suppose a_λ and b_γ are two fuzzy scalars then we say:

1- a_λ No less than b_γ if $a \geq b$ and denoted by $a_\lambda \succ b_\gamma$ or$b_\gamma \prec a_\lambda$.
2- a_λ Is nonnegative if $a \geq 0$. The set of all the nonnegative fuzzy scalars is denoted by $S_F^+(R)$.

Definition 1.6, [5]: Suppose X is a nonempty set and $d_F: P_F \times P_F \longrightarrow S_F^+(R)$ is a mapping. $(P_F(X), d_F)$ Is said to be a fuzzy metric space if for any fuzzy points x_t, y_r and z_p belong to $P_F(X)$, d_F satisfies the following three conditions:

1- Nonnegative: $d_F(x_t, y_r) = 0$ iff x=y and t=r=1.
2- Symmetric: $d_F(x_t, y_r) = d_F(y_r, x_t)$.
3- Triangle inequality: $d_F(x_t, z_p) \prec (x_t, y_r) + (y_r, z_p)$.

II. Smooth Metric Space.

Definition 2.1: The fuzzy open ball $B_{\varepsilon_\lambda}(x_t)$ with fuzzy center x_t and fuzzy radiusε_λ, $\varepsilon \in \mathbb{R}^+$ defines as following: $B_{\varepsilon_\lambda}(x_t)$= $\{y_r \in X : r = t$ and $d_F(x_t, y_r) \prec \varepsilon_\lambda\}$.

Proposition 2.1: Let $\tau: I^x \to I$ is mapping define as following:

$$\tau(B) = \begin{cases} 1 \; if \; for \; any \; x_t \in B \; there \; exist \; fuzzy \; open \; ball \; B_{\varepsilon_\lambda}(x_t) \subseteq B \\ 0 \; otherwise \end{cases}$$

Then τ is st on fuzzy metric space $(P_F(X), d_F)$, since:

1- ϕ Has no elements therefore $\tau(\phi) = 1$, and for any $x_t \in X$ there exist fuzzy open ball $B_{\varepsilon_\lambda}(x_t)$ belong to X therefore $\tau(X) = 1$.

2- Let $A_1, A_2 \in P(A)$ then:

a- If A_1 *and* A_2 are not open, if A_1 is open and A_2 is not open or conversely then it's clear $\tau(A_1 \cap A_2) \geq \tau(A_2) \wedge \tau(A_2)$.

b- If A_1 *and* A_2 are open then we want to prove $\tau(A_1 \cap A_2) \geq \tau(A_2) \wedge \tau(A_2)$:

Let $x_t \in A_1 \cap A_2 \Rightarrow x_t \in A_1 \, and \, x_t \in A_2$ then there exist two fuzzy open balls $B_{\varepsilon_\lambda}(x_t)$ and $B_{\delta_\gamma}(x_t)$ contain x_t, contained in A_1 *and* A_2, respectively, such that:

$B_{\varepsilon_\lambda}(x_t)=\{y_r \in X: r = t \text{ and } d_F(x_t, y_r) \prec \varepsilon_\lambda\}$

$B_{\delta_\gamma}(x_t) = \{z_s \in X: s = t \text{ and } d_F(x_t, z_s) \prec \delta_\gamma\}$

Let $l = \min\{r, s\}$ and $\Omega = \{\varepsilon, \delta\}$ then $B_{\Omega_\Lambda}(x_t) = \{h_l \in X: l = t \text{ and } d_F(x_t, h_l) \prec \Omega_\Lambda\}$ $\Lambda = \lambda \text{ or } \gamma$. Since $l = \min\{r, s\}$ and $\Omega_\Lambda \prec \varepsilon_\lambda, \delta_\gamma$ then $B_{\Omega_\Lambda}(x_t) \subseteq B_{\varepsilon_\lambda}(x_t) \text{ and } B_{\Omega_\Lambda}(x_t) \subseteq B_{\delta_\gamma}(x_t) \Rightarrow B_{\Omega_\Lambda}(x_t) \subseteq B_{\varepsilon_\lambda}(x_t) \cap B_{\delta_\gamma}(x_t) \Rightarrow \tau(A_1 \cap A_2) = 1$.

3- Let $\{A_i\}_{i\in I}$ be a collection of fuzzy subsets of A then if there exist $i \in I$, A_i is not open $\Rightarrow \tau(\bigcup_{i\in I} A_i) \geq \bigwedge_{i\in I} \tau(A_i)$. Let for each $i \in I$, A_i is open then we want to prove $\tau(\bigcup_{i\in I} A_i) \geq \bigwedge_{i\in I} \tau(A_i)$:

Let $x_t \in \bigcup_{i\in I} A_i \Rightarrow \exists i \in I \text{ such that } x_t \in A_i$ but A_i is open, then there exist fuzzy open ball $B_{\varepsilon_\lambda}(x_t) \subseteq A_i$ but $A_i \subseteq \bigcup_{i\in I} A_i \Rightarrow B_{\varepsilon_\lambda}(x_t) \subseteq \bigcup_{i\in I} A_i \Rightarrow \tau(\bigcup_{i\in I} A_i) \geq \bigwedge_{i\in I} \tau(A_i)$.

Definition 2.2: Let $(P_F(X), d_F)$ be a fuzzy metric space and τ be a fuzzy st on $(P_F(X), d_F)$ then $(P_F(X), \tau)$ is called induced smooth topological space.

Proposition 2.2: The induced smooth topological space $(P_F(X), \tau)$ is smooth Hausdroff.

Proof: Let $x_t, y_r \in P_F(X)$, then:

1. If $x \neq y$, let $d_F(x_t, y_r) = \varepsilon_\lambda$ and we define $B_{\frac{\varepsilon}{2_\lambda}}(x_t)= \{z_s \in X: s = t \text{ and } d_F(x_t, z_s) \prec \frac{\varepsilon}{2_\lambda}\}$ and so $B_{\frac{\varepsilon}{2_\lambda}}(y_r)= \{h_l \in X: l = r \text{ and } d_F(y_r, h_l) \prec \frac{\varepsilon}{2_\lambda}\}$.

Let $g_k \in B_{\frac{\varepsilon}{2_\lambda}}(x_t) \cap B_{\frac{\varepsilon}{2_\lambda}}(y_r) \Rightarrow k = t \text{ and } d_F(x_t, g_k) \prec \frac{\varepsilon}{2_\lambda}$ and so $k = r \text{ and } d_F(y_r, g_k) \prec \frac{\varepsilon}{2_\lambda} \Rightarrow d_F(x_t, y_r) \prec (x_t, g_k) + (g_k, y_r) \prec \frac{\varepsilon}{2_\lambda} + \frac{\varepsilon}{2_\lambda} = \frac{\varepsilon}{2_{2\lambda}} - \lambda^2$ [3], but $\frac{\varepsilon}{2_{2\lambda}} - \lambda^2 \prec \varepsilon_\lambda$ and this contradiction, therefore $B_{\frac{\varepsilon}{2_\lambda}}(x_t) \leftharpoondown q B_{\frac{\varepsilon}{2_\lambda}}(y_r)$.

2. Let x=y and $t < r$.let $B_{\varepsilon_\lambda}(x_t)=\{z_s \in X: s = t \text{ and } d_F(x_t, z_s) \prec \varepsilon_\lambda\}$ and let $h_l \in X$ s.t $l + t \leq 1$ and $l + r > 1$ then we will define the following ball $B_{\varepsilon_\lambda}(h_l)=\{g_k \in X: k = l \text{ and } d_F(h_l, g_k) \prec \varepsilon_\lambda\}$ s.t $y_r \in B_{\varepsilon_\lambda}(h_l) \Rightarrow B_{\varepsilon_\lambda}(h_l) q y_r$ and $B_{\varepsilon_\lambda}(h_l) \leftharpoondown q B_{\varepsilon_\lambda}(x_t)$.

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