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Open Cyclic Grid Graphs Are Graceful

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Abstract : *In this paper, we present the graceful labeling of open cyclic grid graph and vertex cordial labeling of generalized open cyclic grid graph.*
Keywords:*Graph labeling, Graceful labeling, Vertex Cordial labeling*
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I. Introduction

Let $G(V,E)$ be any finite simple graph. For any graph theoretic notation we follow West [4]. A function f is called a graceful labeling of a graph G with m edges, if f is an injection from the vertices of $G$to the set $\{0,1,2,\ldots,m\}$such that when edge $uv$is assigned the label, $|f(u)-f(v)|$the resulting labels are distinct. In the field of graph labeling there are very few results deals with the generation of bigger graceful graphs from the smaller ones by using standard operations like join, union etc., For detailed survey refer Gallian[1]. In this direction we introduce new method of combining cycles and proved that the resulting graph is graceful. Let f be a function from the vertices of G to $\{0, 1\}$ and for each edgexyassign the label$|f(x)-f(y)|$. Call fa vertex cordial labeling of G if the number of vertices(edges) labeled 0 and the number of vertices(edges) labeled 1 differs by at most 1. The graph that admits vertex cordial labeling is called vertexcordial graph. The concept of vertexcordial labeling was introduced by Cahit [1].

A vertex $v \in V$ of a connected graph $G$is said to be an attachment vertex, if $deg(v)=2$. Let C_m and C_n be any cycles of length $4k, k \geq 1$. Then $OG(1,n)=C_m \oplus C_n$ is the graph obtained by attaching a copy of C_n with all attachment vertices of C_m. In the same way, define$OG(i,n)=OG(i-1,t) \oplus C_n$, that is, $OG(i,n)$ is the graph obtained by attaching a copy of C_n with all attachment vertices of $OG(i-1,t)$.

An open cyclic grid graph is the graph $OG(1,n)[=C_m \oplus C_n]$ as defined above. Refer figure.1.An generalized open cyclic grid graph is the graph $OG(i,n), i \geq 2$.

In this paper, for $n \equiv 0(mod\ 4)$ we prove that the open cyclic grid graph $OG(1,n)$ admits graceful labeling and generalized open cyclic grid graph $OG(i,n), i \geq 2$ admits vertex cordial labeling.

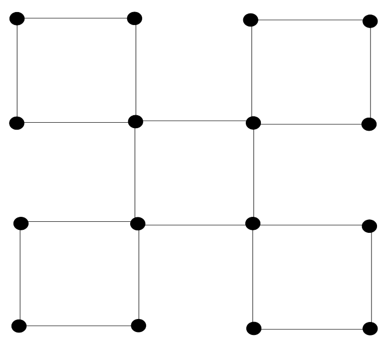


Fig.1.Open Cyclic Grid Graph $OG(1,4)=C_4 \oplus C_4$

II. Main Results

Theorem. 2.1 Open Cyclic grid graph$OG(1,n)=C_{n_0} \oplus C_n$is graceful for $n_0, n \equiv 0(mod\ 4)$.

Let $C_{n_0}: a_1 e_1 a_2 e_2 \ldots e_{n_0-1} a_{n_0} e_{n_0} a_1$and $C_n: a_{i,1} e_{i,1} a_{i,2} e_{i,2} \ldots e_{i,n-1} a_{i,n} e_{i,n} a_{i,1}$ be any two cycles of length $n_0, n \equiv 0(mod\ 4)$. Then $OG(1)=C_{n_0} \oplus C_n$ is the graph obtained as described above having $p=n_0+n_0(n-1)$ vertices and $q=n_0+n_0 n$ edges. In $OG(1)$, For$1 \leq i \leq n_0, i-even$ let $a_i=a_{i,1}$and $a_{i,n}=a_i, i-odd$.

Define the labeling $\varphi: V(OG(1,n)) \rightarrow \{0,1,2,\ldots,q\}$ as follows,

$$\varphi(a_{1,1})=q, \qquad \varphi(a_{1,2})=0$$

$$\varphi(a_{1,j}) = \begin{cases} (a_{1,1}) - \left(\frac{j-1}{2}\right), & j-odd, j < \frac{n}{2} \\ (a_{1,1}) - \left(\frac{j+1}{2}\right), & j-odd, j \geq \frac{n}{2} \\ \frac{j}{2} - 1, & j-even \end{cases}$$

For $3 \leq i \leq n_0$ and $i - odd, \varphi(a_{i,1}) = \varphi(a_{i-1,n-1}) - 1$

$\varphi(a_{i,2}) = \varphi(a_{i-1,n}) + 1$,except$i = \frac{n_0}{2} + 1$

If $i = \frac{n_0}{2} + 1, \varphi(a_{i,2}) = \varphi(a_{i-1,n}) + 2$

For $3 \leq j \leq n, \varphi(a_{i,j}) = \begin{cases} (a_{i,1}) - \left(\frac{j-1}{2}\right), & j-odd, j < \frac{n}{2} \\ \varphi(a_{i,1}) - \left(\frac{j+1}{2}\right), & j-odd, j \geq \frac{n}{2} \\ \varphi(a_{i,2}) + \frac{j}{2} - 1, & j-even \end{cases}$

For $2 \leq i \leq n_0$ and $i - even, \varphi(a_{i,1}) = \varphi(a_{i-1,n-1}) - 1$

$\varphi(a_{i,2}) = \varphi(a_{i-1,n}) + 1$,except,$i = \frac{n_0}{2} + 1$

If $i = \frac{n_0}{2} + 1,\ \varphi(a_{i,2}) = \varphi(a_{i-1,n}) + 2$

For $3 \leq j \leq n, \varphi(a_{i,j}) = \begin{cases} \varphi(a_{i,1}) - \left(\frac{j-1}{2}\right), & j-odd \\ \varphi(a_{i,1}) + \frac{j}{2} - 1, & j-even, 4 \leq j \leq \frac{n}{2} \\ \varphi(a_{i,2}) + \frac{j}{2}, & j-even, j > \frac{n}{2} \end{cases}$

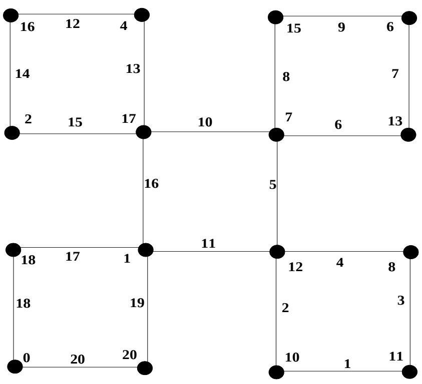
From the above vertex labeling we observe that for $1 \leq i \leq n_0, 1 \leq j \leq n$ the following set $\{\varphi(a_{i,j}) :\ j\ odd\}$ is a monotically decreasing sequence and for $1 \leq i \leq n_0$and $2 \leq j \leq n$, the set $\{\varphi(a_{i,j}) :\ j\ even\}$ is a monotically increasing sequence. From the above sequence it is clear that, for $1 \leq i \leq n_0$,

$$min\{(a_{i,j}) : 1 \leq j \leq n, j\ odd\} > max\{(a_{i,j}) : 2 \leq j \leq n, j\ even\}$$

Thus all the vertex labels are distinct and Henceφ is injective. From the above vertex labels it is clear that all the edge values are distinct and hence the graph $OG(1, n)$ is graceful.

Conjecture 2.2.For $i \geq 2$,Generalized Open Cyclic grid graph $OG(i, n)$is graceful.

2.3 Illustrations to Theorem. 2.1



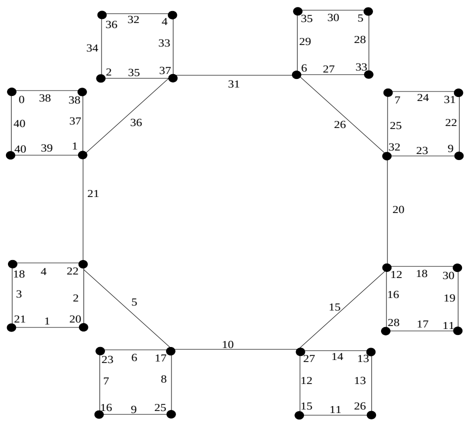


Fig.2. $OG(1) = C_4 \oplus\ C_4$Fig.3.$OG(1) = C_8 \oplus\ C_4$

Theorem.2.4. Open Cyclic grid graph $OG(1, n)$admits vertex cordial labeling for $n \equiv 0(mod\ 4)$.

Let $C_m : a_1 a_2 \ldots a_m a_1$be a cycle of length$m \equiv 0(mod\ 4)$.

Let V_0 and V_1denote the set of all vertices assigned the label 0 and 1 respectively. In the same way, let $E_0$and E_1denote the set of all edges assigned the label 0 and 1 respectively.

Define the labeling $\varphi : V(C_m) \rightarrow \{0,1\}$ as follows,

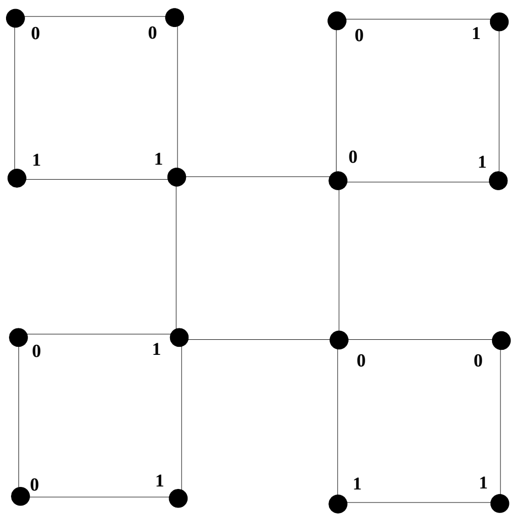
For $1 \leq i \leq m$, let $\varphi(a_i) = \begin{cases} 1, & if\ i\ is\ odd \\ 0, & if\ i\ is\ even \end{cases}$

From the graph C_m it is clear that the vertices $a_1, a_2, \ldots, a_m$ are the attachment vertices.

Let C_n be acycle of length $n \equiv 0(mod\ 4)$. Consider $m$copies of C_n and let it be $C^i: b_{i,1}b_{i,2} \ldots b_{i,n}b_{i,1}$ for $1 \leq i \leq n$. Now construct the graph $OG(1,n) = C_m \oplus C_n$ by merging the vertex $b_{i,1}$ of a copy C^i with all the attachment vertices a_i of C_m, for $1 \leq i \leq m$.
It is observed that for $1 \leq i \leq m$,the vertex $b_{i,1} = a_i$ and further the vertex $b_{i,1}$ has the labeling either 1 or 0. For the convenience of the labeling, arrange the edges $b_{i,j}$, $1 \leq i \leq m$ and $1 \leq j \leq n$ in a sequence of the form $b_{i,1}b_{i,2} \ldots b_{i,n}b_{i,1}$ where as the vertex $b_{i,1}$ has the label either 1 or 0.
Define the labeling $\varphi : V(\mathrm{OG}(1,n)) \rightarrow \{0,1\}$ as follows,
For $1 \leq i \leq m$ and $1 \leq j \leq n$, if the vertex $\varphi(b_{i,1}) = 1$, then $\varphi(b_{i,j}) = (1100)^{\frac{n}{4}}$ and if $\varphi(b_{i,1}) = 0$,then $\varphi(b_{i,j}) = (0011)^{\frac{n}{4}}$.
Clearly $|V_0| = |V_1|$ and $|E_1| = |E_0|$ and hence $OG(1,n)$ is vertex cordial.☐

In the next theorem, we prove that the generalized open cyclic grid graph admitsvertexcordial labeling. Recall thatfor$i \geq 2$, $OG(i,n) = OG(i-1,n) \oplus C_n$, is the graph obtained by attaching a copy of C_n with all attachment vertices of $OG(i-1,n)$, where $n \equiv 0(mod\ 4)$.
Theorem.2.5.For $i \geq 2$,GeneralizedOpen Cyclic grid graph $OG(i,n)$admits vertex cordiallabelingfor$n \equiv 0(mod\ 4)$.
For $i \geq 2$, consider the vertex cordial graph $OG(i-1,n)$ having the attachment vertices $a_1, a_2, \ldots, a_k$.
Let C_n be acycle of length $n \equiv 0(mod\ 4)$. Consider $k$copies of C_n and let it be $C^j: b_{j,1}b_{j,2} \ldots b_{j,n}b_{j,1}$ for $1 \leq j \leq k$. Now construct the generalized open cyclic grid graph $OG(i,n) = OG(i-1,n) \oplus C_n$ by merging the vertex $b_{j,1}$ of a copy C^j with all the attachment vertices a_j of $OG(i-1,n)$, for $1 \leq j \leq k$.
The remaining part of proof follows similarly as done in Theorem. 2.4.

2.6 Illustrations to Theorem. 2.5



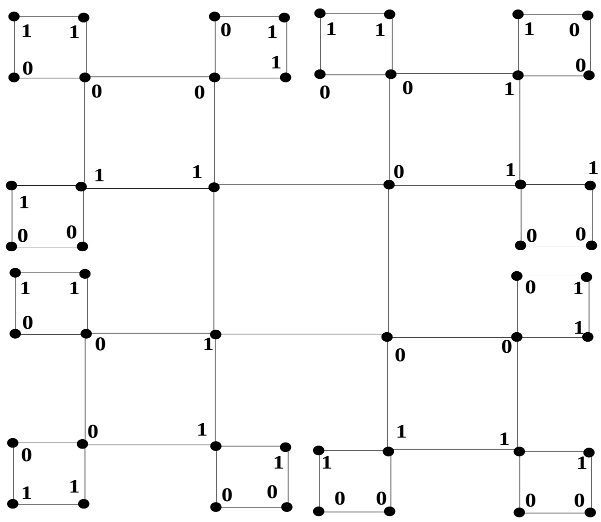


Fig.4. Vertex cordial labeling of $OG(1,4)$Fig.5.Vertex cordial labeling of $OG(2,4)$

III. Conclusion

In this paper the graceful labeling of open cyclic grid graph and vertex cordial labeling of generalized open cyclic grid graph are investigated.Further it is conjectured that generalized open cyclic grid graph is graceful and it has a potential to provide motivation to investigate analogous results for different types of labeling.

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