IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN:2319-765X. Volume 10, Issue 3 Ver. III (May-Jun. 2014), PP 17-20
www.iosrjournals.org

Pythagorean way of Proof for the segmental areas of one square with that of rectangles of adjoining square

R. D. Sarva Jagannadha Reddy
19-9-73/D3, Sri Jayalakshmi Colony, S.T.V. Nagar, Tirupati – 517 501, A.P., India

Abstract: *It is universally accepted that 3.14159265358… as the value of π. It is thought an approximation, at its last decimal place. It is a transcendental number and squaring a circle is an unsolved problem with this number. A new, exact, algebraic number $\frac{14-\sqrt{2}}{4}$ = 3.14644660942… is derived and verified with a proof that is followed for Pythagorean theorem. It is proved here squaring of circle and rectification of circumference of a circle are possible too.*
Keywords: *Pythagorean thorem, circle, square, π.*

I. Introduction

Official π value is 3.14159265358… It is obtained from the Exhaustion method, which is a geometrical method. This method involves the inscription of a polygon in a circle and increased the sides of the polygon, until the inscribed polygon touches the circle, leaving no gap between them. The value 3.14159265358… is actually the length of the perimeter of the inscribed polygon. And it is **not** the value of circle. There was no method till yesterday to measure the circumference of a circle, directly or indirectly.

3.14159265358… has four characteristics: 1) It represents polygon, 2) It is an approximation, 3) It is a transcendental number and 4) It says squaring a circle is impossible. And such a number is attributed and followed as π of the circle based on limitation principle, because of the impossibility of calculating, the length of the circumference of circle and in such a situation this field of mathematics has been thriving for the last 2000 years.

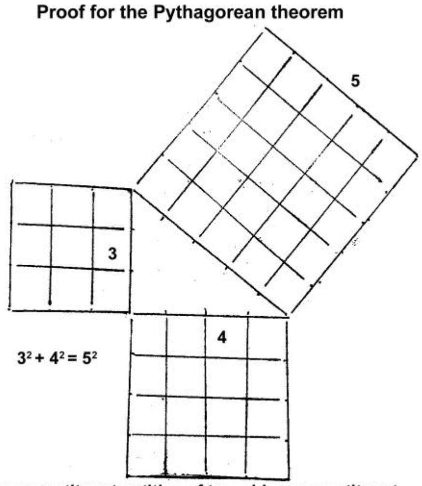
From 1450 **Madhavan** of South India and a galaxy of later generations of mathematicians have discarded geometrical construction and have introduced newly, the concept of **infinite series**.

In this paper geometrical constructions are approached again, for the derivation of π value. New π value has been derived. It is $\frac{14-\sqrt{2}}{4}$ = 3.14644660942… It is an exact value, an algebraic number and makes squaring of circle possible and done too.

II. Procedure

Siva method for the area of the circle of 1st square ABCD

Proof for the Pythagorean theorem



Thus constituent entities of two sides = constituent of entities of hypotenuse

Construction procedure

Draw a square ABCD. Draw two diagonals. 'O' is the centre. Inscribe a circle with centre 'O' and radius ½. E, F, H and J are the mid points of four sides. Join EH, FJ, FH, HJ, JE and EF. Draw four arcs taking A, B, C and D as centres and radius ½. Now the circle square nexus is divided into 32 segments. Number them 1 to 32. 1 to 16 segments are called S_1 segments. 17 to 32 segments are called S_2 segments. 17 to 24, S_2 segments are outside the circle. 25 to 32, S_2 segments are inside the circle.

Draw KP, a parallel line to the side DC which intersects diagonals at M and N.

Square = ABCD

Side = AB = 1 = EH = diameter

Areas of S_1 and S_2 segments $\quad S_1 = \frac{6-\sqrt{2}}{128}$; $S_2 = \frac{2+\sqrt{2}}{128}$

$16S_1 + 16S_2$ = Area of square $\quad 16\left(\frac{6-\sqrt{2}}{128}\right)+16\left(\frac{2+\sqrt{2}}{128}\right)=1$

$16S_1 + 8S_2$ = Area of circle $\quad 16\left(\frac{6-\sqrt{2}}{128}\right)+8\left(\frac{2+\sqrt{2}}{128}\right)=\frac{14-\sqrt{2}}{16}$

Square has 32 constituent parts

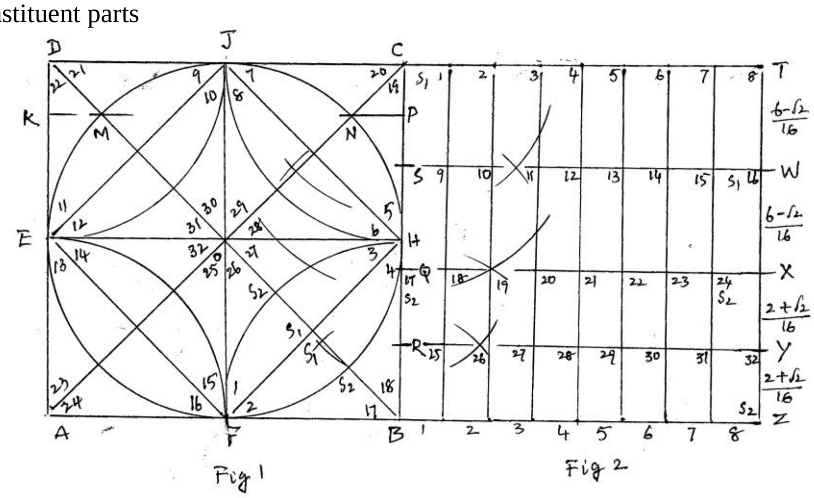


Fig-1: Segmental areas calculated; Fig-2: Areas of Rectangles are calculated
Both values are same

This method is taken from the book **Pi of the Circle** of this author (available at www.rsjreddy.webnode.com).

In this method there are two squares of same sides. First square has an inscribed circle divided into 32 segments of two dimensions called S_1 and S_2 segments, each category of 16 in number. And areas of these segments are calculated using the following two formulas

$$S_1 = \frac{a^2}{32}(\pi-2) \quad \text{and} \quad S_2 = \frac{a^2}{32}(4-\pi)$$

which are obtained by solving two equations (in Square 1)

$$16\ S_1 + 16\ S_2 = a^2 = \text{Area of the square (Eq.1)}$$
$$16\ S_1 + 8\ S_2 = \pi a^2/4 = \text{Area of the inscribed circle (Eq.2)}$$

This method is called as Siva method. In the present method: Siva Kesava method, second square is joined to the 1st square. One side CB is common to both the squares.

The second square is similarly divided, as in the case of 1st square, into 32 rectangles. Rectangles are also of two dimensions each category of 16 numbers. The areas of each type of rectangle is equal to S_1 and S_2 segments of the 1st square. **These rectangles are formed, based on the division of common side of the both the squares.** The areas of rectangles agree **cent percent** with the above two formulas of Siva method, where π value is $\frac{14-\sqrt{2}}{4}$. Thus, the division of 1st square is **exactly duplicated** in the second square, except for the difference, in the 1st square, 32 segments are curvy linear, and in the 2nd square, 32 segments are rectangles, naturally, of straight lines.

Now let us see how the common side CB is divided.

1. Squares 1 = ABCD, 2 = BZTC
2. Side = diameter of the inscribed circle = 1
3. KP = Parallel side to the side DC
4. OM = ON = radius ½
5. MON = triangle; MN = hypotenuse = $\frac{\sqrt{2}}{2}$
6. DK = KM = NP = PC = $\frac{KP-MN}{2}$ $\quad =\left(1-\frac{\sqrt{2}}{2}\right)\frac{1}{2}=\frac{2-\sqrt{2}}{4}$
7. So, CP = $\frac{2-\sqrt{2}}{4}$, PB = CB – CP $\quad = 1-\left(\frac{2-\sqrt{2}}{4}\right)=\frac{2+\sqrt{2}}{4}$

8. Bisect PB. PB $\rightarrow$ PQ + QB $\rightarrow$ QR + RB $= \frac{2+\sqrt{2}}{4} \rightarrow \frac{2+\sqrt{2}}{8} \rightarrow \frac{2+\sqrt{2}}{16}$

9. QB = $\frac{2+\sqrt{2}}{8}$, CB = 1, CQ = CB – QB $= CQ = \left(1-\frac{2+\sqrt{2}}{8}\right) = \frac{6-\sqrt{2}}{8}$

10. Bisect CQ $\rightarrow$ CS + SQ $= \frac{6-\sqrt{2}}{8} \rightarrow \frac{6-\sqrt{2}}{16}$

11. We have started with side = 1, divided next, into CP = $\frac{2-\sqrt{2}}{4}$, and PB = $\frac{2+\sqrt{2}}{4}$. In the second step PB is bisected into PQ = $\frac{2+\sqrt{2}}{8}$ and QB = $\frac{2+\sqrt{2}}{8}$. In the third step CQ = $\frac{6-\sqrt{2}}{8}$ is bisected into CS = $\frac{6-\sqrt{2}}{16}$ and SQ = $\frac{6-\sqrt{2}}{16}$.

12. After divisions, finally we have CB Side divided into 4 parts

CS = $\frac{6-\sqrt{2}}{16}$, SQ = $\frac{6-\sqrt{2}}{16}$, QR = $\frac{2+\sqrt{2}}{16}$ and RB = $\frac{2+\sqrt{2}}{16}$

13. 2^{nd} Square BZTC is divided horizontally into four parts: CS, SQ, QR and RB.

14. Now BZ side of 2^{nd} square is divided into 8 parts. So, each length is 1/8.

15. Finally, the 2^{nd} square is divided into 16 rectangles of one dimension equal in area to S_1 segments of 1^{st} square and 16 rectangles of 2^{nd} dimension, equal in area to S_2 segments of 1^{st} square.

16. Square BZTC consists of first two rows are of S_1 and 3^{rd} & 4^{th} rows are of S_2 segments.

17. Area of each rectangle = S_1 segments of 1^{st} square = $\frac{6-\sqrt{2}}{16} \times \frac{1}{8} = \frac{6-\sqrt{2}}{128}$

Sides of rectangles of 1^{st} & 2^{nd} rows=TW=WX=$\frac{6-\sqrt{2}}{16}$ and other side = $\frac{1}{8}$

18. Area of each rectangle = S_2 segment of 1^{st} square = $\frac{2+\sqrt{2}}{16} \times \frac{1}{8} = \frac{2+\sqrt{2}}{128}$

Sides of rectangles of 3^{rd} & 4^{th} rows=XY=YZ= $\frac{2+\sqrt{2}}{16}$ and other side = $\frac{1}{8}$

19. The areas of $16S_1$ and $16S_2$ segments of 1^{st} square

$16\left\{\frac{a^2}{32}\times(\pi-2)\right\}+16\left\{\frac{a^2}{32}\times(4-\pi)\right\}$ = 1 = Area of the 1^{st} square; where a = 1

20. The area of all the 32 rectangles.

$16\left(\frac{6-\sqrt{2}}{128}\right)+16\left(\frac{2+\sqrt{2}}{128}\right)$ = 1 = Area of the 2^{nd} square

21. The area of the inscribed circle in the 1^{st} square = $16S_1$ segments + $8S_2$ segments

$= 16\left(\frac{6-\sqrt{2}}{128}\right)+8\left(\frac{2+\sqrt{2}}{128}\right) = \frac{14-\sqrt{2}}{16}$ = Area of the circle in the 1^{st} square

22. The areas of the 1^{st}, 2^{nd} and 3^{rd} rows of rectangles of 2^{nd} square.

$8\left(\frac{6-\sqrt{2}}{128}\right)+8\left(\frac{6-\sqrt{2}}{128}\right)+8\left(\frac{2+\sqrt{2}}{128}\right) = \frac{14-\sqrt{2}}{16}$

23. Thus area of the circle from 1^{st} and 2^{nd} squares is = $\frac{14-\sqrt{2}}{16} = \frac{\pi d^2}{4}$

$\therefore \pi = \frac{14-\sqrt{2}}{4}$ where side = diameter = d = 1

24. When π is equal to $\frac{14-\sqrt{2}}{4}$, the length of the inscribed circle in the 1st square is = πd = πa = $\frac{14-\sqrt{2}}{4}\times 1=\frac{14-\sqrt{2}}{4}$ where side = diameter = 1

25. Perimeter of the rectangle QXWTCS is equal to the circumference of the inscribed circle in the 1st square.

QX = 1 = TC; XW = WT = CS = SQ = $\frac{6-\sqrt{2}}{16}$

QX + XW + WT + TC + CS + SQ = $1+\frac{6-\sqrt{2}}{16}+\frac{6-\sqrt{2}}{16}+1+\frac{6-\sqrt{2}}{16}+\frac{6-\sqrt{2}}{16}=\frac{14-\sqrt{2}}{4}$

In the first square we have seen that the length of the circumference of the inscribed circle is the **outer edge** of the 16 S_1 segments. In the 2nd square also the outer edges of the 1st and 2nd rows of 16 rectangles are equal to $\frac{14-\sqrt{2}}{4}$.

26. Thus, Siva Kesava Method supports the π value $\frac{14-\sqrt{2}}{4}$ obtained by earlier Gayatri, Siva, Jesus methods.

27. And also, the curvy linear $16S_1$ and $16S_2$ segments of 1st square are all squared in the 2nd square.

III. Conclusion

Two squares of same sides are drawn with one common side. Circle is inscribed in one square. Areas of square and its inscribed circle are calculated from their constituent curvy linear segments. The correctness of areas of constituent segments are verified with that of the areas of rectangles of the adjoining square. All the values thus are proved correct.