IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN:2319-765X. Volume 10, Issue 4 Ver. I (Jul-Aug. 2014), PP 21-31
www.iosrjournals.org

Heat Transfer on Unsteady MHD Oscillatory Visco-elastic flow through a Porous medium in a Rotating parallel plate channel taking Hall current effects

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Abstract: *We consider the unsteady flow of a conducting optically thin visco-elastic fluid through a rotating channel filled with saturated porous medium and non-uniform walls temperature has been discussed taking hall current effects. It is assumed that the fluid has small electrical conductivity and the electromagnetic force produced is very small. The analytical solutions are obtained for the problem making use of perturbation technique. The effects of the radiation and the magnetic field parameters on velocity profile and shear stress for different values of the visco-elastic parameter with the combination of the other flow parameters are illustrated graphically, and physical aspects of the problem are discussed.*

Keywords: *Radiation effects, heat transfer, visco-elastic fluids, MHD flows, porous medium, rotating parallel plate channels*

I. Introduction

Heat transfer problem through a porous medium has important application in geothermal reservoirs and geothermal energy extractions. MHD has attracted the attention of many scholars due to its diverse application in geophysics and astrophysics. It is applied to study the stellar and solar structure, interstellar matter, radio propagation through the ionosphere, design of MHD generators and accelerators in geophysics, in design of underground water energy storage system, soil-sciences, astrophysics, nuclear power reactors and so on. The phenomena of mass transfer is very common in theory of stellar structure, burning a pool of oil, spray drying, adsorption, leaching and mass transport process in animal and plant life. The effect of Hall current on the fluid flow with variable concentration has many applications in MHD power generation, in several astrophysical and meteorological studies as well as in plasma flow through MHD power generators. It is of great practical importance in view of several physical problems such as seepage of water in river beds, porous heat exchangers, cooling of nuclear reactors, filtration and purification processes. Because of its industrial importance, problem of flow and heat transfer in porous medium in the presence of magnetic field has been the subject of many experimental and analytical studies. McWhirter et al. [1], and Geindreau and Auriault [2] discussed in detail magnetohydrodynamic flow through porous medium. The investigations considering rotational effects are also very important, and the reason for studying flow in a rotating porous medium or rotating flow of a fluid overlying a porous medium in the presence of a magnetic field is fundamental because of its numerous applications in industrial, astrophysical and geophysical problems. The problem of MHD Couette flow and heat transfer between parallel plates is a classical one that has several applications in MHD accelerators, MHD pumps and power generators, and in many other industrial engineering designs. Thus such problems have been much investigated by researchers such as, Seth et al. [3], Singh et al. [4], Chauhan and Vyas [5], Attia [6,7], Attia and Ewis [8], Seth et al. [9], and Attia et al. [10], Al-Hadhrami et al. [31]. In most of the investigations, as above, we notice that, the Hall term is neglected for small or moderate values of the magnetic field in applying Ohm's law in the analysis. When a strong magnetic field is applied, the influence of electromagnetic force is noticeable, and the strong magnetic field induces many complex phenomenon in an electrically conducting flow regime including Hall currents, Joule's heating etc. as stated by Cramer and Pai, [11]. Infact, in an ionized gas under strong magnetic field when the density is low, the Hall current is induced which is mutually perpendicular to both electrical and magnetic fields. It has significant effect on the current density and hence on the electromagnetic force. Sato [12] and Sutton and Sherman [13] investigated the hydromagnetic flow of a viscous ionized gas between two parallel plates taking Hall effects into account. It was the first significant study to include Hall effect in the analysis and indicated that the fluid flow in the parallel plate channel becomes secondary in nature. Hall currents can have strong influence on the fluid flow distributions in MHD flow systems, e.g. in MHD power generators, electrically conducting aerodynamics and atmospheric science. Hall effects on MHD flow in a rotating channel have been investigated by Ghosh and Bhattacharjee [14] in the presence of inclined magnetic field. Further studies of Hall effects onMHDflow in parallel plate channel with perfectly conducting walls with heat transfer characteristics have been presented by Ghosh et al. [15] who analyzed the asymptotic behaviour of the solution. Hall currents in MHDCouette flow and heat transfer effects have been investigated in parallel plate channels with or without ion-slip effects by Soundalgekar et al. [16], Soundalgekar and Uplekar [17], and Attia [18]. Hall effects on MHD couette flow between arbitrarily conducting parallel plates have been investigated in a rotating system by Mandal and Mandal [19]. The same problem of MHD couette flow rotating flow in a rotating system with Hall current was examined by Ghosh [20] in the presence of an arbitrary magnetic field. The study of hydro magnetic couette flow in a porous channel has become important in the applications of fluid engineering and geophysics. Krishna et al. [21] investigated convection flow in a rotating porous medium channel. Bég et al. [22] investigated unsteady magneto hydro dynamic couette flow in a porous medium channel with Hall current and heat transfer. When the viscous fluid flows adjacent to porous medium, choa-Tapia [23,24] suggested stress jump conditions at the fluid porous interface when porous medium is modeled by Brinkman equation. Using these jump conditions, Kuznetsov [25] analytically investigated the couette flow in a composite channel partially filled with a porous medium and partially with a clear fluid. Chauhan and Rastogi [26], heat transfer effects on MHD conducting flow with Hall current in a rotating channel partially filled with a porous material using jump conditions at the fluid porous interface. Chauhan and Agrawal [27] investigated Hall current effects in a rotating channel partially filled with a porous medium using continuity of velocity components and stresses at the porous interface. Chauhan and Agrawal [28] further studied effects of Hall current on couette flow in similar geometry and matching conditions at the fluid porous interface. Recently Dileep Singh Chauhan and Priyanka Rastogi [32] discussed with the rotating magneto hydro dynamic Couette flow of electrically conducting fluid and heat transfer in a channel partially filled by a porous medium, with Hall effects. The jump condition is applied at the fluid porous interface suggested by Ochoa Tapia and Whitaker [23, 24]. N. Ahmed and S. Talukdar [33] made to present a theoretical analysis of a transient magnetohydrodynamic (MHD) flow of a visco-elastic fluid (Walter's B') past an infinite vertical porous plate embedded in a porous medium with constant permeability in a rotating system in the presence of Hall current when the plate temperature varies periodically with time. Nayak, Anita Dash, G. C.,[34] analysed the effect of injection/suction on an oscillatory flow of an incompressible electrically conducting viscous fluid in a porous channel. The channel with constant injection/suction and variable temperature rotates about an axis perpendicular to the plates of the channel. A magnetic field of uniform strength is also applied normally to the plates. The upper plate is allowed to oscillate in its own plane whereas the lower plate is kept at rest. The heat transfer is studied under two conditions, the temperature of the upper plate is allowed to oscillate whereas the temperature of the lower plate is kept constant. Morteza Abbasi et al. [35] studied, the problem of two-dimensional magneto hydro dynamic (MHD) boundary layer flow of steady, laminar flow of an incompressible, viscoelastic fluid in a parallel plate channel with slip at the boundaries. In this paper, the combined effect of hall current and radiative heat transfer on unsteady flow of a conducting optically thin visco-elastic fluid through a rotating channel filled with saturated porous medium and non-uniform walls temperature has been discussed. It is assumed that the fluid has small electrical conductivity and the electromagnetic force produced is very small. The analytical solutions are obtained for the problem making use of perturbation technique. The effects of the radiation and the magnetic field parameters on velocity profile and shear stress for different values of the visco-elastic parameter with the combination of the other flow parameters are illustrated graphically, and physical aspects of the problem are discussed

II. Mathematical Formulation and Solution of the Problem

Consider the flow of a conducting optically thin fluid in a channel filled with saturated porous medium in a rigidly rotating parallel plate channel with upper plate bounding the clean fluid and the lower plate bounding below on sparsely packed porous bed subjected to a uniform transverse magnetic field (an externally applied homogeneous magnetic field) normal to the channel and taking hall current into account and radiative heat transfer as shown in Figure 1.

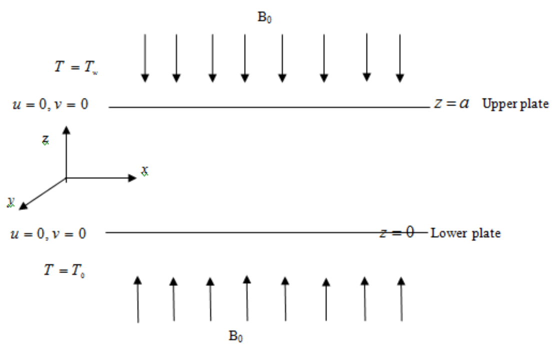

Figure 1: Geometry of the problem.

It is assumed that the fluid has small electrical conductivity and the electromagnetic force produced is very small. The x-axis is taken along the centre of the channel, and the z-axis is taken normal to it. The analytical solutions are obtained for the problem making use of perturbation technique. The constitutive equation for the incompressible second order fluid is of the form

$$\sigma = pI + \mu_1 A_1 + \mu_2 A_2 + \mu_3 (A_1)^2 \tag{1}$$

where σ is the stress tensor, p is the hydrostatic pressure, I is the unit tensor, $A_n (n = 1,2)$ are the kinematic Rivlin-Ericksen tensors, μ_1, μ_2 and μ_3 are the material coefficients describing viscosity, elasticity, and cross-viscosity, respectively. The material coefficients μ_1, μ_2 and μ_3 have taken constants with μ_1 and μ_3 as positive and μ_2 as negative (Markovitz and Coleman [9]). The equation (1) was derived by Coleman and Noll [10] from that of the simple fluids by assuming that stress is more sensitive to the recent deformation than to the deformation that occurred in the distant past.

The unsteady hydro magnetic flow in a rotating co-ordinate system is governed by the equation of motion, continuity equation and the Maxwell equations in the form.

$$\rho\left(\frac{\partial V}{\partial t} + (V.\nabla)V + 2\Omega \times V + \Omega \times (\Omega \times r)\right) = \nabla.T + J \times B \tag{2}$$

$$\nabla.V = 0 \tag{3}$$

$$\nabla.B = 0 \tag{4}$$

$$\nabla \times B = \mu_m J \tag{5}$$

$$\nabla \times E = -\frac{\partial B}{\partial t} \tag{6}$$

Where,

J is the current density, B is the total magnetic field, E is the total electric field, μ_m is the magnetic permeability and r is radial co-ordinate given by $r^2 = x^2 + y^2$.

In the initial undisturbed state both the fluid and the plates are in rigid rotation with the same angular velocity Ω about the normal to the plates and at $t > 0$ the fluid is driven by a constant pressure gradient parallel to the channel walls. In the equation of motion along x-direction the x-component current density $\mu_e J_y H_o$ and the x-component current density $-\mu_e J_x H_o$. Then, assuming a Boussinesq incompressible fluid model, the equations governing the motion are given by

$$\frac{\partial u}{\partial t} - 2\Omega v = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu_1 \frac{\partial^2 u}{\partial z^2} + \nu_2 \frac{\partial^3 u}{\partial z^2 \partial t} + \frac{\mu_e J_y H_o}{\rho} - \nu_1 \frac{u}{K} + g\beta(T - T_0) \tag{7}$$

$$\frac{\partial v}{\partial t} + 2\Omega u = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu_1 \frac{\partial^2 v}{\partial z^2} + \nu_2 \frac{\partial^3 v}{\partial z^2 \partial t} - \frac{\mu_e J_x H_o}{\rho} - \nu_1 \frac{v}{K} \tag{8}$$

When the strength of the magnetic field is very large, the generalized ohm's law is modified to include the hall current so that

$$J + \frac{\omega_e \tau_e}{B_O}(J \times B) = \sigma\left[E + V \times B + \frac{1}{e\eta_e}\nabla P_e\right] \tag{9}$$

Where ω_e the cyclotron frequency of the electrons, τ_e is the electron collision time, σ is the electrical conductivity, e is the electron charge and p_e is the electron pressure. The ion-slip and thermo electric effects are not included in equation (9). Further it is assumed that $\omega_e \tau_e \sim 0$ (1) and $\omega_i \tau_i << 1$, where ω_i *and* τ_i are the cyclotron frequency and collision time for ions respectively. In equation (9) the electron pressure gradient, the ion-slip and thermo-electric effects are neglected. We also assume that the electric field E=0 under assumptions reduces to

$$J_x + m\, J_y = \sigma \mu_e H_0 v \tag{10}$$

$$J_y - m\, J_x = -\sigma \mu_e H_0 u \tag{11}$$

Where $m = \omega_e \tau_e$ is the hall parameter. On solving equations (10) and (11) we obtain

$$J_x = \frac{\sigma \mu_e H_0}{1+m^2}(v+mu) \tag{12}$$

$$J_y = \frac{\sigma \mu_e H_0}{1+m^2}(mv-u) \tag{13}$$

Using the equations (12) and (13), the equations of the motion with reference to rotating frame are given by

$$\frac{\partial u}{\partial t} - 2\Omega v = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu_1 \frac{\partial^2 u}{\partial z^2} + \nu_2 \frac{\partial^3 u}{\partial z^2 \partial t} + \frac{\sigma \mu_e^2 H_0^2}{\rho(1+m^2)}(v+mu) - \nu_1 \frac{u}{K} + g\beta(T-T_0) \tag{14}$$

$$\frac{\partial v}{\partial t} + 2\Omega u = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu_1 \frac{\partial^2 v}{\partial z^2} + \nu_2 \frac{\partial^3 v}{\partial z^2 \partial t} - \frac{\sigma \mu_e^2 H_0^2}{\rho(1+m^2)}(mv-u) - \nu_1 \frac{v}{K} \tag{15}$$

$$\frac{\partial T}{\partial t} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial z^2} - \frac{1}{\rho C_p}\frac{\partial q_1}{\partial z} \tag{16}$$

Corresponding boundary conditions

$$u = 0, v = 0, T = T_W \quad \text{on} \quad z = 1 \tag{17}$$

$$u = 0, v = 0, T = T_0 \quad \text{on} \quad z = 0 \tag{18}$$

Where u is the axial velocity, t is the time, T is the fluid temperature, P is the pressure, g is the gravitational force, q_1 is the radiative heat flux, β is the coefficient of volume expansion due to temperature, C_p is the specific heat at constant pressure, k is the thermal conductivity, K is the porous medium permeability co-efficient, $B_0 (= \mu_e H_0)$ is electromagnetic induction, μ_e is the magnetic permeability, H_0 is the intensity of the magnetic field, σ_e is the conductivity of the fluid, ρ is fluid density, and $\nu_i = \mu_i / \rho, (i = 1, 2)$. It is assumed that both walls of temperature T_0, T_w are high enough to induce radiative heat transfer. Following Cogley et.al [11], it is assumed that the fluid is optically thin with a relatively low density and the radiative heat flux is given by

$$\frac{\partial q_1}{\partial z} = 4\alpha_1^2 (T_0 - T), \tag{19}$$

Where α_1 is the mean radiation absorption co-efficient.

Combining equations .1) and (2) and let $q = u + iv$ and $\xi = x - iy$, we obtain

$$\frac{\partial q}{\partial t} + 2i\Omega q = -\frac{1}{\rho}\frac{\partial p}{\partial \xi} + \nu_1 \frac{\partial^2 q}{\partial z^2} + \nu_2 \frac{\partial^3 q}{\partial z^2 \partial t} - \frac{\sigma_e \mu_e^2 H_0^2}{\rho(1+m^2)} q - \frac{\nu_1}{K} q + g\beta(T-T_0) \tag{20}$$

The following non-dimensional quantities are introduced:

$$x^* = \frac{x}{a},\ y^* = \frac{y}{a},\ \xi^* = \frac{\xi}{a},\ z^* = \frac{z}{a},\ u^* = \frac{u}{U},\ v^* = \frac{v}{U},$$

$$q^* = \frac{q}{U},\ t^* = \frac{tU}{a},\ p^* = \frac{ap}{\rho \nu_1 U},\ \theta = \frac{T - T_0}{T_w - T_0}$$

Making use of non-dimensional variables, the dimensionless governing equations together with appropriate boundary conditions (dropping asterisks) are

$$\mathrm{Re}\frac{\partial q}{\partial t} = -\frac{\partial p}{\partial \xi} + \frac{\partial^2 q}{\partial z^2} + \alpha \frac{\partial^3 q}{\partial z^2 \partial t} - \left(\frac{M^2}{1+m^2} + 2iE^{-1} + S^2\right) q + Gr\,T \tag{21}$$

$$Pe\frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial z^2} + R^2 \theta \tag{22}$$

with

$q = 0, \quad \theta = 1$ on $z = 1$ (23)

$q = 0, \quad \theta = 0$ on $z = 0$ (24)

Where

$\mathrm{Re} = \dfrac{Ua}{\nu_1}$ is the Reynolds number,

$M^2 = \dfrac{\sigma_e B_0^2 a^2}{\nu_1 U}$ is the Hartmann number, $E = \dfrac{\nu_1 U}{\Omega a^2}$ is the Eckmann number

$D = \dfrac{k}{a^2}$ is the Darcy number (or) $S = \dfrac{1}{D}$ is the porous medium shape factor parameter,

$\alpha = \dfrac{\nu_2 \mathrm{Re}}{a^2}$ is the visco-elastic parameter,

$Gr = \dfrac{g\beta(T_w - T_0)a^2}{\nu_1 U}$ is the Grashoff number,

$Pe = \dfrac{Ua\rho C_p}{k}$ is Peclet parameter,

$R^2 = \dfrac{4\alpha_1^2 a^2}{k}$ is the Radiation parameter and $m = \omega_e \tau_e$ is the hall parameter.

Solving the equations (8) and (9) for purely oscillatory flow, Let

$$-\frac{\partial P}{\partial x} = \lambda e^{i\omega t}, \; q(z,t) = q_0(z)\, e^{i\omega t}, \; \theta(z,t) = \theta_0(z)\, e^{i\omega t} \tag{25}$$

Where, λ is constant and ω is the frequency of oscillation.

Substituting the above expressions (25) into the equations (21) and (22), and making use of the corresponding boundary conditions (23) and (24), we obtain

$$(1 + i\alpha\omega)\frac{d^2 q_0}{\partial z^2} - m_1^2 q_0 = -\lambda - Gr\theta_0 \tag{26}$$

$$\frac{d^2\theta_0}{\partial z^2} + m_2^2\theta_0 = 0 \tag{27}$$

Subjected to the boundary conditions

$q_0 = 0, \quad \theta_0 = 1$ on $z = 1$ (28)

$q_0 = 0, \quad \theta_0 = 0$ on $z = 0$ (29)

Where $m_1 = \sqrt{\left(\dfrac{M^2}{1+m^2} + 2iE^{-1} + S^2\right) + i\omega \mathrm{Re}}$ and $m_2 = \sqrt{R^2 - i\omega Pe}$

Equations (26) and (27) are solved; we obtained the solution for the fluid velocity and temperature as follows

$$q(z,t) = \left\{ -a_1\, e^{-(m_1 z)/b} + \left(a_1 - \frac{\lambda}{m_1^2}\right) e^{(m_1 z)/b} + \frac{\lambda}{m_1^2} + \frac{Gr\sin(m_2 z)}{(m_2^2 b - m_1^2)\sin(m_2)} \right\} e^{i\omega t} \tag{30}$$

$$\theta(z,t) = \frac{\sin(m_2 z)}{\sin(m_2)}\; e^{i\omega t} \tag{31}$$

Where $a_1 = \dfrac{\left(\left(\dfrac{\lambda}{m_1^2}\right) e^{-m_1/b} - \left(\dfrac{\lambda}{m_1^2}\right) - \dfrac{Gr}{(m_2^2 b - m_1^2)}\right)}{\left(e^{m_1/b} - e^{-m_1/b}\right)}$, $b = \sqrt{1 + i\alpha\omega}$

The non-dimensional shear stress σ at the wall $z = 0$ is given by

$$\sigma=\frac{\sigma^{*}}{\left(\mu_1 U/a\right)}=\left(\frac{\partial u}{\partial z}+\alpha\frac{\partial^2 u}{\partial z\partial t}\right)_{z=0}$$

$$=\left[\frac{a_1 m_1}{b}+\left(a_1-\frac{\lambda}{m_1^2}\right)+\frac{Gr}{(m_2^2 b-m_1^2)\sin(m_2)}\right](1+i\omega\alpha)\,e^{i\omega t} \tag{32}$$

The rate of heat transfer across the channel wall $z=1$ is given as

$$Nu=-\left(\frac{\partial\theta}{\partial z}\right)_{z=1}=\frac{m_2\cos(m_2)}{\sin(m_2)}\,e^{i\omega t} \tag{33}$$

III. Results and Discussion:

The flow governed by the non dimensional parameters, Re is the Reynolds number, M is the Hartmann number, E Eckmann number, D is the Darcy number (or) S is the porous medium shape factor parameter, α is the visco-elastic parameter, R is the Radiation parameter, m hall parameter with fixed values of Gr the Grashoff number, Pe Peclet parameter. We have considered the real and imaginary parts of the results u and v throughout for numerical validation. The velocity profiles for the components against z is plotted in Figures (2–15) while figure (16-17) to observe temperature profiles on the visco-elastic effects and other parameters for various sets of values of Hartmann number H, porous parameter S and radiation parameter R, with fixed values of other flow parameters, namely, Pe = 0.7, t = 0.1, Gr = 2, λ = 1, and ω = 1.

It is evident from Figures (2–15) that the velocity profiles is parabolic in nature, and the magnitude of velocity u and v increase with the increasing values of the Reynolds number Re, Porous parameter S, the visco-elastic parameter $|\alpha|$, Radiation parameter R and m hall parameter (Figure 2, 3, 6-15). It is also noted from the figures (4-5) that the magnitude of the velocity component u experiences retardation and the behaviours of the velocity component v remains the same with the increasing values of the Hartmann number. We observe that lower the permeability of the porous medium lesser the fluid speed in the entire fluid region. The resultant velocity q enhances with increasing the parameters Re, D, α , R and experiences retardation with increasing the intensity of the magnetic field. It is evident that the temperature profiles exhibit the nature of the flow on governing parameters. The magnitude of the temperature increases with increasing R and experiences retardation with increasing the Peclet number Pe (Figures 16-17).

The shear stress on the wall and the rate of heat transfer evaluated analytically and computationally discussed with reference to various governing parameters (Tables 1-2). It is evident that the shear stress reduces with increasing Re, M, E, m and α . We also noted that it is increases firstly and then decreases with the increasing values of radiation parameter R and S. Table. 2 depict that the Nusselt number (Nu) decreases with the increasing values of R and the magnitude of the heat transfer increases with increasing Pe . It has also been observed that the temperature field is not significantly affected by the visco-elastic parameter.

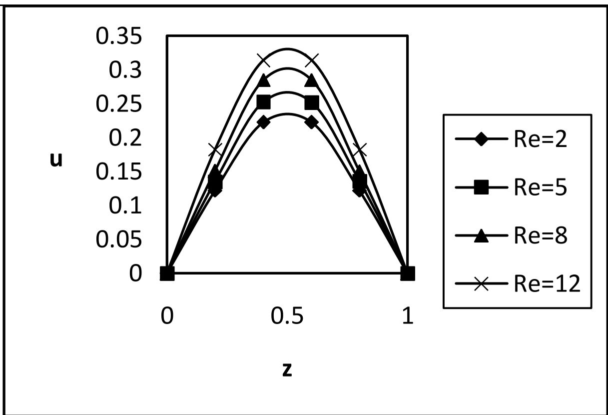

Fig. 2 The velocity profile for u on Reynolds number Re with $M=2,\ S=1,\ \alpha=-0.1, E=0.01, R=1.5, m=1$

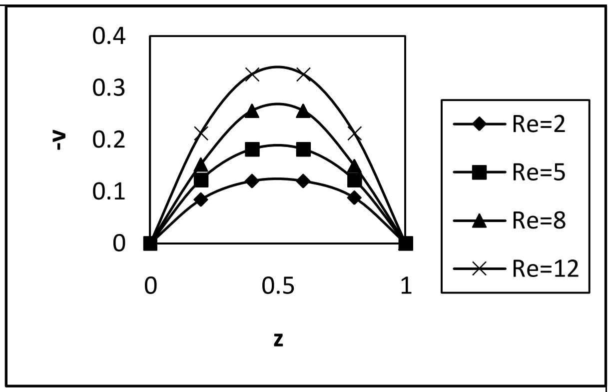

Fig. 3: The velocity profile for v on Reynolds number Re with $M=2,\ S=1,\ \alpha=-0.1, E=0.01, R=1.5, m=1$

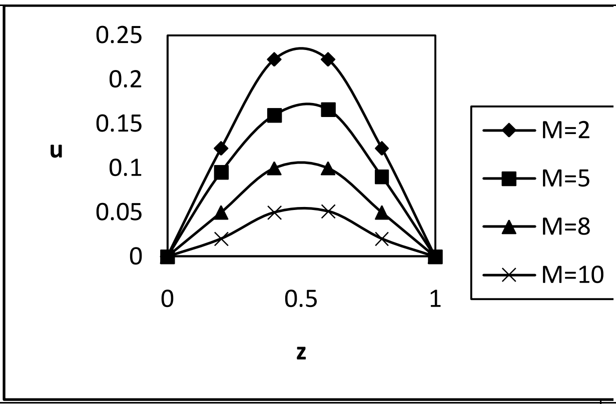

Fig. 4: The velocity profile for u on Hartmann number M with
$\mathrm{Re}=2,\ S=1,\ \alpha=-0.1, E=0.01, R=1.5, m=1$

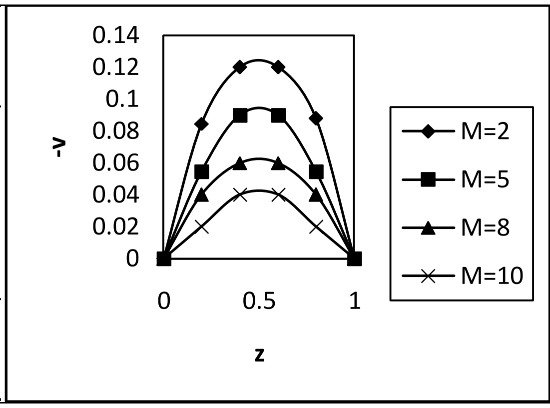

Fig. 5: The velocity profile for v on Hartmann number M with
$\mathrm{Re}=2,\ S=1,\ \alpha=-0.1, E=0.01, R=1.5, m=1$

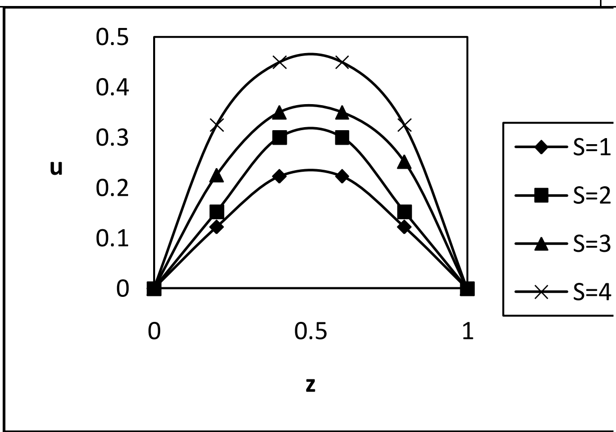

Fig. 6: The velocity profile for u on Porous parameter S with
$\mathrm{Re}=2,\ M=2,\ \alpha=-0.1, E=0.01, R=1.5, m=1$

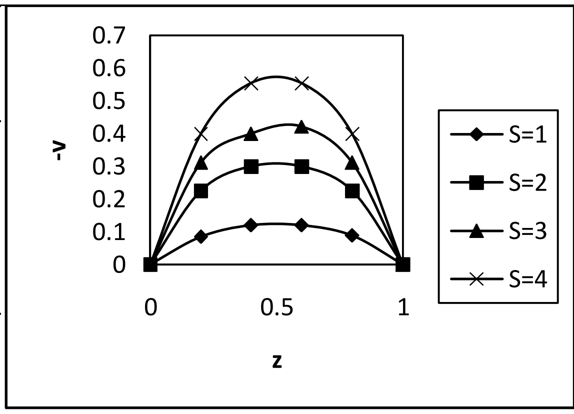

Fig. 7: The velocity profile for v on Porous parameter S with
$\mathrm{Re}=2,\ M=2,\ \alpha=-0.1, E=0.01, R=1.5, m$

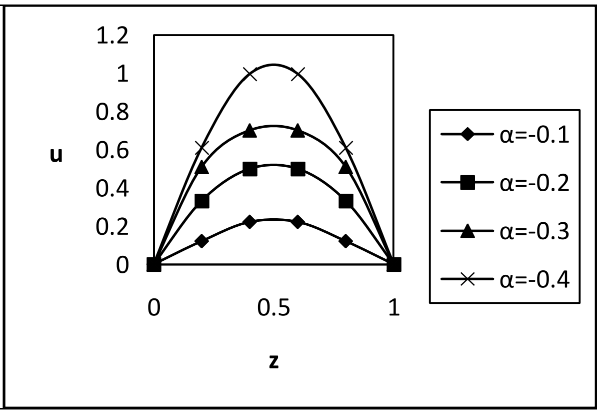

Fig. 8: The velocity profile for u on visco-elastic parameter α with
$\mathrm{Re}=2,\ M=2,\ S=1,\ E=0.01, R=1.5, m=1$

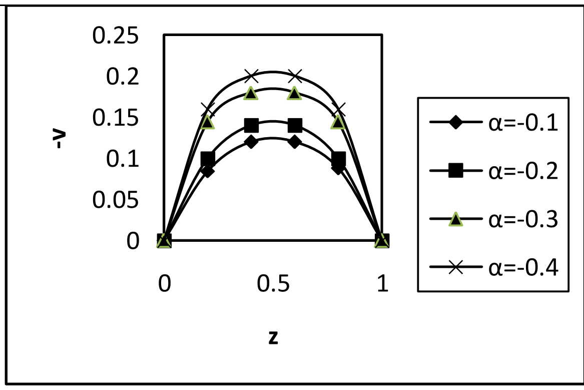

Fig. 9: The velocity profile for v on visco-elastic parameter α with
$\mathrm{Re}=2,\ M=2,\ S=1, E=0.01, R=1.5, m=1$

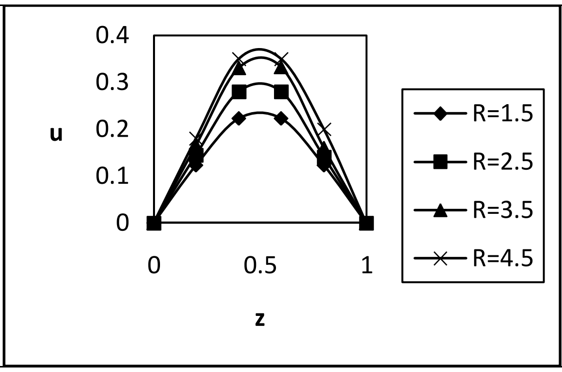

Fig. 10: The velocity profile for u on Radiation parameter R with
$\mathrm{Re}=2,\ M=2,\ S=1, E=0.01, \alpha=-0.1, m=$

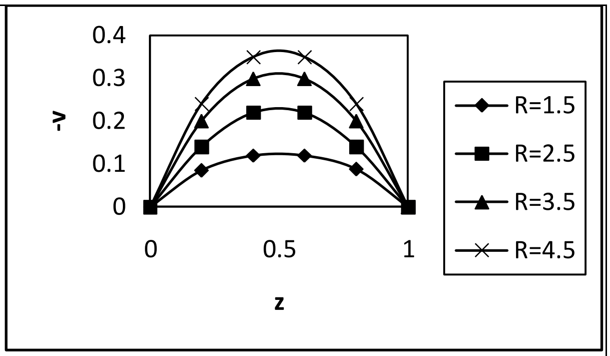

Fig. 11: The velocity profile for v on Radiation parameter R with
$\mathrm{Re}=2,\ M=2,\ S=1,\ \alpha=-0.1, E=0.01, m=1$

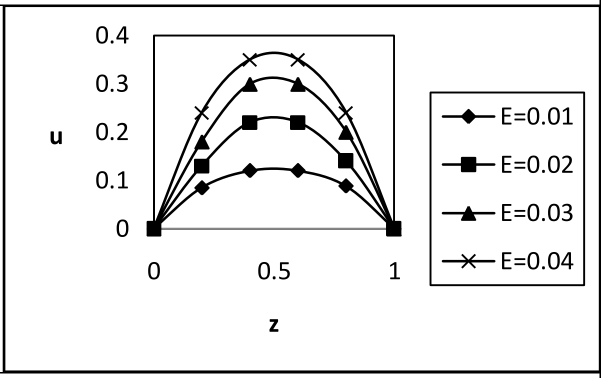

Fig. 12: The velocity profile for u on Eckmann number E with
$\mathrm{Re}=2,\ M=2,\ S=1,\ E=0.01, R=1.5,\ \alpha=$

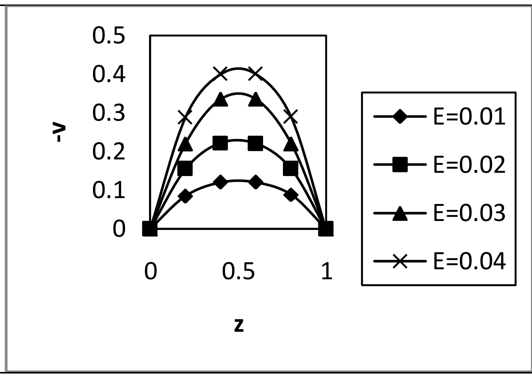

Fig. 13: The velocity profile for v on Eckmann number E with
$\mathrm{Re}=2,\ M=2,\ S=1, E=0.01, R=1.5,\ \alpha=-$

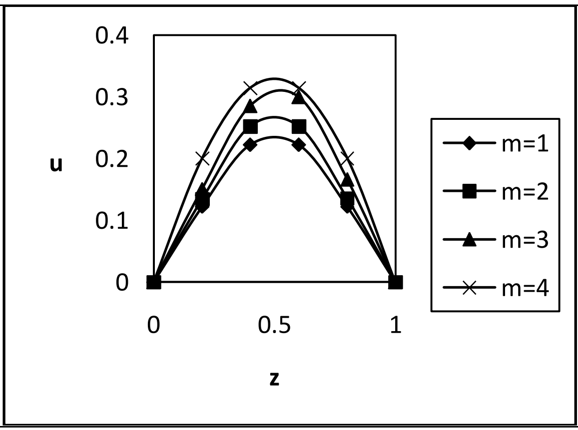

Fig. 14: The velocity profile for u on hall parameter m with
$\mathrm{Re}=2,\ M=2,\ S=1,\ E=0.01, R=1.5,\ \alpha=-0.1$

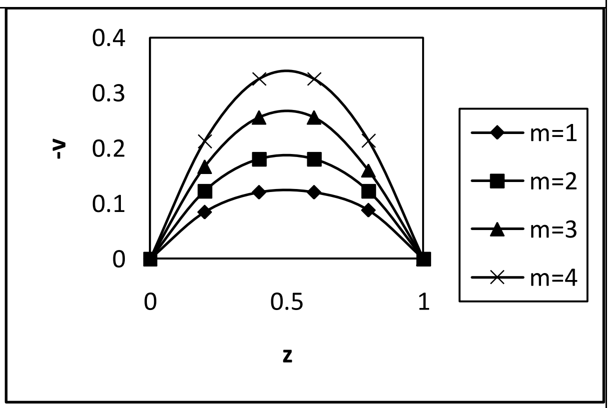

Fig. 15: The velocity profile for v on hall parameter m with
$\mathrm{Re}=2,\ M=2,\ S=1, E=0.01, R=1.5,\ \alpha=-0.1$

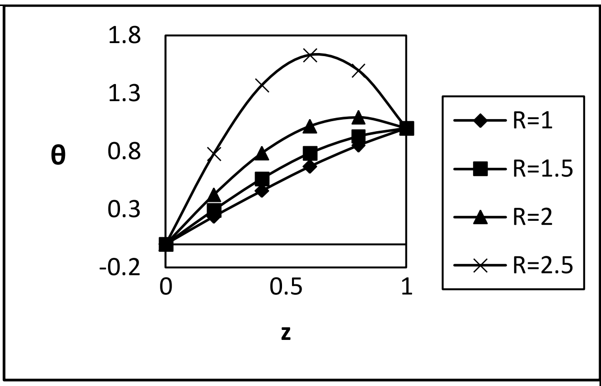

Fig. 16: The temperature profile for θ on Radiation parameter R with $Pe = 0.7, \omega = 1, t = 0.1$

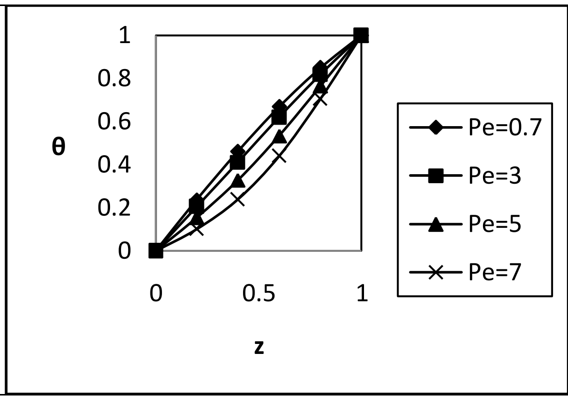

Fig. 17: The temperature profile for θ on Peclet number Pe with $R = 0.5, \omega = 1, t = 0.1$

Re	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	XIII
2	-0.50407	-0.41179	-0.23607	-0.58072	-0.46258	-0.44961	-0.39036	-0.23676	-0.28511	-0.50199	-0.49994	-0.37458	-0.31119
5	-0.35645	-0.37889	-0.23370	-0.43151	-0.40804	-0.33245	-0.30406	-0.23963	-0.22095	-0.35622	-0.35599	-0.29776	-0.27661
8	-0.33965	-0.34311	-0.22956	-0.36222	-0.35848	-0.34790	-0.36328	-0.25418	-0.21234	-0.33953	-0.33941	-0.33304	-0.33271
12	-0.30059	-0.30014	-0.22186	-0.30662	-0.30663	-0.31654	-0.34495	-0.25146	-0.20937	-0.30035	-0.30011	-0.29883	-0.29844

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	XIII
M	2	5	8	2	2	2	2	2	2	2	2	2	2
S	1	1	1	2	3	1	1	1	1	1	1	1	1
α	-0.1	-0.1	-0.1	-0.1	-0.1	-0.2	-0.3	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1
R	1.5	1.5	1.5	1.5	1.5	1.5	1.5	2.5	3.5	1.5	1.5	1.5	1.5
E	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.02	0.03	0.01	0.01
m	1	1	1	1	1	1	1	1	1	1	1	2	3

Table 1: The shear stresses (τ) at the wall $z = 0$

R	I	II	III	IV
1	0.626669	0.774768	1.05562	1.37396
1.5	0.095105	0.339636	0.742407	1.15424
2	-0.912429	-0.377591	0.312668	0.893296
2.5	-3.245460	-1.41121	-0.041387	0.745811

	I	II	III	IV
Pe	0.7	3	5	7

Table 2: Rate of heat transfer (Nusselt number) at the wall $z = 1$

IV. Conclusions

1. The magnitude of velocity u and v increase with the increasing values of the Reynolds number Re, Porous parameter S, the visco-elastic parameter α , Radiation parameter R hall parameter m and Eckmann number E.
2. The magnitude of the velocity component u experiences retardation and the behaviours of the velocity component v remains the same with the increasing values of the Hartmann number.
3. Lower the permeability of the porous medium lesser the fluid speed in the entire fluid region.
4. The resultant velocity q enhances with increasing the parameters Re, D, α , R, E and experiences retardation with increasing the intensity of the magnetic field.
5. The magnitude of the temperature increases with increasing R and experience retardation with in peclet number Pe
6. The shear stress enhances with increasing Re, M and α and reduces with increasing S. We also noted that it is increases firstly and then decreases with the increasing values of radiation parameter R.
7. The Nusselt number reduces with the increasing values of the Radiation parameter and the magnitude of the heat transfer enhances with increasing Pe.

References

[1]. McWhirter, J. D., Crawford, M. E. and Klein, D. E., "Magneto hydro dynamic Flows in Porous Media II: Experimental Results," Fusion Technol., Vol. 34, pp.187-197 (1998).
[2]. Geindreau, C. and Auriault, J. L., "Magneto hydro dynamic Flows in Porous Media," J Fluid Mech., Vol. 466, pp. 343-363 (2002).
[3]. Seth, G. S., Jana, R. N. and Maiti, M. K., "Unsteady hydro magnetic Couette Flow in a Rotating System," Int J Engng Sci., Vol. 20, pp. 989-999 (1982).
[4]. Singh, A. K., Sacheti, N. C. and Chandran, P., "Transient Effects in Magneto-Hydrodynamic Couette Flow with Rotation: Accelerated Motion," Int J Engng Sci., Vol. 32, pp. 133-139 (1994).
[5]. Chauhan, D. S. and Vyas, P., "Heat Transfer in hydro magnetic Couette Flow of Compressible Newtonian Fluid," ASCE J of Engng Mech., Vol. 121, pp. 57-61 (1995).
[6]. Attia, H. A., "Influence of Temperature-Dependent Viscosity in the MHD Couette Flow of Dusty Fluid with Heat Transfer," Diff Eq and Nonlinear Mechanics, pp. 1-14 (2006).
[7]. Attia, H. A., "Ion Slip Effect on Unsteady Couette Flow with Heat Transfer under Exponential Decaying Pressure Gradient," Tamkang J Sci Engng., Vol. 12, pp. 209-214 (2009).
[8]. Attia, H. A. and Ewis, K. M., "UnsteadyMHDCouette Flow with Heat Transfer of a Viscoelastic Fluid under Exponential Decaying Pressure Gradient," Tamkang J Sci Engng., Vol. 13, pp. 359-364 (2010).
[9]. Seth, G. S., Ansari, M. S. and Nandkeolyar, R., "Unsteady Hydromagnetic Couette Flow within a Porous Channel," Tamkang J Sci Engng., Vol. 14, pp. 7-14 (2011).
[10]. Attia, H. A., Al-kaisy, A. M. A. and Ewis, K. M., "MHD Couette Flow and Heat Transfer of a Dusty Fluid with Exponential Decaying Pressure Gradient," Tamkang J Sci Engng., Vol. 14, pp. 91-96 (2011).
[11]. Cramer, K. R. and Pai, S. I., Magneto fluid Dynamics, for Engineers and Applied Physicists, NY, USA: McGraw-Hill (1973).
[12]. Sato, H., "Hall Effect in the Viscous Flow of Ionized Gas between Parallel Plates under Transverse Magnetic Field," J Phys Soc Japan, Vol. 16, pp. 1427-1433 (1961).
[13]. Sutton, G.W. and Sherman, A., Engineering Magneto hydro dynamics, NY: McGraw-Hill (1965).
[14]. Ghosh, S. K. and Bhattacharjee, P. K., "Hall Effects on Steady Hydromagnetic Flow in a Rotating Channel in the Presence of an Inclined Magnetic Field," Chech J Phys., Vol. 50, pp. 759-767 (2000).
[15]. Ghosh, S. K., Bég, O. A. and Narahari, M., "Hall Effects on MHD Flow in a Rotating System with Heat Transfer Characteristics," Meccanica, Vol. 44, pp. 741-765 (2009).
[16]. Soundalgekar, V. M., Vighnesam, N. V. and Takhar, H. S., "Hall and Ion-Slip Effects in MHD Couette Flow with Heat Transfer," IEEE Trans Plasma Sci., Vol. 7, pp. 178-182 (1979).
[17]. Soundalgekar, V. M. and Uplekar, A. G., "Hall Effects in MHD Couette with Heat Transfer," IEEE Trans Plasma Sci., Vol. 14, pp. 579-583 (1986).
[18]. Attia, H. A., "MHD Couette Flow with Temperature Dependent Viscosity and Ion Slip," Tamkang J Sci Engng., Vol. 8, pp. 11-16 (2005).
[19]. Mandal, G. and Mandal, K. K., "Effect of Hall Current on MHD Couette Flow between Thick Arbitrarily Conducting Plates in a Rotating System," J Physical Soc Japan, Vol. 52, pp. 470-477 (1983).
[20]. Ghosh, S. K., "Effects of Hall Current on MHD Couette Flow in a Rotating System with Arbitrary Magnetic Field," Czech J Phys., Vol. 52, pp. 51-63 (2002).
[21]. Krishna, D. V., Prasada Rao, D. R. V. and Ramachandra Murthy, A. S., "Hydro magnetic Convection Flow through a Porous Medium in a Rotating Channel," J Eng Phys Thermophys, Vol. 75, pp. 281-291 (2002).
[22]. Bég, O. A., Zueco, J. and Takhar, H. S., "Unsteady Magneto hydro dynamic Hartmann-Couette Flow and Heat Transfer in a Darcian Channel with Hall Current, Ionslip, Viscous and Joule Heating Effects: Network Numerical Solutions," Commun in Nonlinear Sci and Numer Simulation, Vol. 14, pp. 1082-1097 (2009).
[23]. Ochoa-Tapia, J. A. and Whitaker, S., "Momentum Transfer at the Boundary between a Porous Medium and a Homogeneous Fluid-I. Theoretical Development," Int J Heat Mass Transfer, Vol. 38, pp. 2635-2646 (1995a).
[24]. Ochoa-Tapia, J. A. and Whitaker, S., "Momentum Transfer at the Boundary between a Porous Medium and a Homogeneous Fluid-I. Comparison with Experiment," Int J Heat Mass Transfer, Vol. 38, pp. 2647-2655 (1995b).
[25]. Kuznetsov, A. V., "Analytical Investigation of Couette Flow in a Composite Channel Partially Filled with a Porous Medium and Partially with a Clear Fluid," Int J Heat Mass Transfer, Vol. 41, pp. 2556-2560 (1998).
[26]. Chauhan, D. S. and Rastogi, P., "Hall Current and Heat Transfer Effects on MHD Flow in a Channel Partially Filled with a Porous Medium in a Rotating System," Turkish J Eng Env Sci., Vol. 33, pp. 167-184 (2009).
[27]. Chauhan, D. S. and Agrawal, R., "Effects of Hall Current on MHD Flow in a Rotating Channel Partially Filled with a Porous Medium," Chem Engng Comm., Vol. 197, pp. 830-845 (2010).

[28]. Chauhan, D. S. and Agrawal, R., “Effects of Hall Current on MHD Couette Flow in a Channel Partially Filled with a Porous Medium in a Rotating System,” Meccanica, DOI 10.1007/s11012-011-9446-9 (2011).

[29]. Cowling, T. G., Magnetohydrodynamics, New York: Interscience Pub. Inc (1957).

[30]. Nield, D. J. and Bejan, A., Convection in Porous Media, USA: Springer (2006).

[31]. Al-Hadhrami, A. K., Elliot, L. and Ingham, D. B., “A New Model for Viscous Dissipation in Porous Media across a Range of Permeability Values,” Transport in Porous Media, Vol. 53, pp. 117-122 (2003).

[32]. Dileep Singh Chauhan and Priyanka Rastogi, “Heat Transfer Effects on Rotating MHD Couette Flow in a Channel Partially Filled by a Porous Medium with Hall Current,” Journal of Applied Science and Engineering, Vol. 15, No. 3, pp. 281-290 (2012).

[33]. N. Ahmed and S. Talukdar, “ Transient magnetohydrodynamic (MHD) flow of a visco-elastic fluid past an infinite vertical porous plate embedded in a porous medium with Hall current and slip condition in a rotating system, International Journal of Physical Sciences Vol. 7(45), pp. 5942-5953 (2012).

[34]. Nayak, Anita Dash, G. C., “Oscillatory effect on magneto-hydrodynamic flow and heat transfer in a rotating horizontal porous channel” Annals of the Faculty of Engineering Hunedoara - International Journal, Vol. 11, Issue 1, p. 199 (2013).

[35]. Morteza Abbasi, Davood Domiri Ganji, Iman Rahimipetroudi, Mehran Khaki, “Comparative Analysis of MHD Boundary-Layer Flow of Viscoelastic Fluid in Permeable Channel with Slip Boundaries by using HAM, VIM, HPM,” Walailak Journal of Science and Technology (WJST), Vol. 11, No. 7 (2014).