IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 10, Issue 5 Ver. III (Sep-Oct. 2014), PP 69-82
www.iosrjournals.org

Jordan (σ,τ)-Higher Homomorphisms of a Γ-Ring M into a Γ -Ring M'

Dr. Salah Mehdi Salih and Fawaz Ra'ad Jarallah
Mathematical Department, Education college, Al-Mustansiriya University

Abstract: *Let M and M' be two prime Γ-rings and σ^n, τ^n be two higher homomorphism of a Γ-ring M, for all n ∈ N in the present paper we show that under certain conditions of M, every Jordan (σ, τ)-higher homomorphism of a Γ-Ring M into a prime Γ'-Ring M 'is either (σ, τ)-higher homomorphism or (σ, τ)-anti-higher homomorphism.*
Key Words: *prime Γ-ring , homomorphism,Jordan homomorphism.*
Mathematics Subject Classication : *16N60*; 16U80

I. Introduction:

Let M and Γ be two additive abelian groups, suppose that there is a mapping from M×Γ×M ⟶ M (the image of (a,α,b) being denoted by $a\alpha b$, a, $b \in$ M and $\alpha \in \Gamma$). Satisfying for all a, b, $c \in$ M and $\alpha, \beta \in \Gamma$:

(i) $(a + b)\alpha c = a\alpha c + b\alpha c$
$a(\alpha + \beta)c = a\alpha c + a\beta c$
$a\alpha(b + c) = a\alpha b + a\alpha c$

(ii) $(a\alpha b)\beta c = a\alpha(b\beta c)$

Then M is called a Γ-ring. This definition is due to Barnes [1], [9].

A Γ-ring M is called a prime if $a\Gamma M\Gamma b = (0)$ implies $a = 0$ or $b = 0$, where $a, b \in M$, this definition is due to [5].

A Γ-ring M is called semiprime if $a\Gamma M\Gamma a = (0)$ implies $a = 0$, such that $a \in M$, this definition is due to [7].

Let M be a 2-torsion free semiprime Γ-ring and suppose that $a, b \in M$ if $a\Gamma m\Gamma b + b\Gamma m\Gamma a = 0$ for all m ∈ M, then $a\Gamma m\Gamma b = b\Gamma m\Gamma a = 0$ this definition is due to [11].

Let M be Γ-ring then M is called 2-torsion free if $2a = 0$ implies $a = 0$, for every $a \in M$, this definition is due to [6].

Let σ^i, τ^i be two higher homomorphism of a Γ-ring M then σ^i, τ^i are called commutative if $\sigma^i\tau^i = \tau^i\sigma^i$, for all i∈N, this definition is due to Barnes [1].

Let M be a Γ-ring and d: M ⟶ M be an additive mapping (that is $d(a + b) = d(a) + d(b)$) of a Γ-ring M into itself then d is called a derivation on M if :

$d(a\alpha b) = d(a)\alpha b + a\alpha d(b)$, for all $a, b \in M$ and $\alpha \in \Gamma$.

d is called a Jordan derivation on M if $d(a\alpha a) = d(a)\alpha a + a\alpha d(a)$, for all $a \in M$ and $\alpha \in \Gamma$, [4], [10].

Let M be a Γ-ring and f: M ⟶ M be an additive map (that is $f(a + b) = f(a) + f(b)$), Then f is called a generalized derivation if there exists a derivation d: M ⟶ M such that

$f(a\alpha b) = f(a)\alpha b + a\alpha d(b)$, for all $a, b \in M$ and $\alpha \in \Gamma$.

And f is called a generalized Jordan derivation if there exists a Jordan derivation d: M ⟶ M such that $f(a\alpha a) = f(a)\alpha a + a\alpha\, d(a)$, for all $a \in M$ and $\alpha \in \Gamma$, [2], [3].

Let θ be an additive mapping of Γ-ring M into a Γ'-ring M', θ is called a homomorphism if for all $a, b \in M$ and $\alpha \in \Gamma$

$\theta(a\alpha b) = \theta(a)\alpha\theta(b)$ [1] .

And θ is called a Jordan homomorphism if for all $a, b \in M$ and $\alpha \in \Gamma$

$\theta(a\alpha b + b\alpha a) = \theta(a)\alpha\theta(b) + \theta(b)\alpha\,\theta(a)$, [8].

Let F be an additive mapping of a Γ-ring M into a Γ'-ring M', F is called a generalized homomorphism if there exists a homomorphism θ from a Γ-ring M into a Γ'-ring M', such that

$F(a\alpha b) = F(a)\alpha\theta(b)$, for all $a, b \in M$ and $\alpha \in \Gamma$, where θ is called a relating homomorphism, and F is called a generalized Jordan homomorphism if there exists a Jordan homomorphism θ from a Γ-ring M into a Γ'-ring M', such that

$F(a\alpha b + b\alpha a) = F(a)\alpha\theta(b) + F(b)\alpha\,\theta(a)$, for all $a, b \in M$ and $\alpha \in \Gamma$, where θ is called a relating Jordan homomorphism, [8].

Let $\theta = (\phi_i)_{i\in N}$ be a family of additive mappings, ϕ_n: M ⟶ M' then θ is said to be a higher homomorphism (resp. Jordan higher homomorphism) on a Γ-ring M into a Γ'-ring M' if $\phi_0 = I_{M'}$ (the identity mapping on M') and $\phi_n(a\alpha b)=\sum_{i=1}^{n}\phi_i(a)\alpha\phi_i(b)$ (resp. $\phi_n(a\alpha b+b\alpha a)=\sum_{i=1}^{n}\phi_i(a)\alpha\phi_i(b)+\sum_{i=1}^{n}\phi_i(b)\alpha\phi_i(a)$), for all $a, b \in$ M and $\alpha \in \Gamma$), [8].

Now, the main purpose of this paper is that every Jordan (σ,τ)-higher homomorphism of a Γ-ring M into a prime Γ'-ring M' is either (σ,τ)-higher homomorphism or (σ,τ)-antihigher homomorphism and every Jordan (σ,τ)-higher homomorphism from a Γ-ring M into a 2-torsion free Γ'-ring M' is a Jordan triple (σ,τ)-higher homomorphism.

II. 2-Jordan (σ,τ)-Higher Homomorphisms of a Γ-Rings

Definition (2.1):

Let $\theta = (\phi_i)_{i\in N}$ be a family of additive mappings of a Γ-ring M into a Γ-ring M' and σ,τ be two homomorphism of a Γ-ring M, θ is called a (σ,τ)-higher homomorphism if for all $a, b \in$ M, $\alpha \in \Gamma$ and $n \in N$

$$\phi_n(a\alpha b)=\sum_{i=1}^{n}\phi_i(\sigma^i(a))\alpha\phi_i(\tau^i(b)).$$

Definition (2.2):

Let $\theta = (\phi_i)_{i\in N}$ be a family of additive mappings of a Γ-ring M into a Γ-ring M' and σ,τ be two homomorphism of a Γ-ring M, θ is called Jordan (σ,τ)-higher homomorphism if for all $a, b \in$ M, $\alpha \in \Gamma$ and for $n \in N$

$$\phi_n(a\alpha b+b\alpha a)=\sum_{i=1}^{n}\phi_i(\sigma^i(a))\alpha\phi_i(\tau^i(b))+\sum_{i=1}^{n}\phi_i(\sigma^i(b))\alpha\phi_i(\tau^i(a)).$$

Definition (2.3):

Let $\theta = (\phi_i)_{i\in N}$ be a family of additive mappings of a Γ-ring M into a Γ-ring M' and σ,τ be two homomorphism of a Γ-ring M, θ is called a Jordan triple (σ,τ)-higher homomorphism if for all $a, b \in$ M, $\alpha, \beta \in \Gamma$ and $n \in N$

$$\phi_n(a\alpha b\beta a)=\sum_{i=1}^{n}\phi_i(\sigma^i(a))\alpha\phi_i(\sigma^i\tau^{n-i}(b))\beta\phi_i(\tau^i(a))$$

Definition (2.4):

Let $\theta = (\phi_i)_{i\in N}$ be a family of additive mappings of a Γ-ring M into a Γ-ring M' and σ,τ be two homomorphism of a Γ-ring M, θ is called a(σ,τ)-anti-higher homomorphism if for all $a, b \in$ M, $\alpha \in \Gamma$ and $n \in N$

$$\phi_n(a\alpha b)=\sum_{i=1}^{n}\phi_i(\sigma^i(b))\alpha\phi_i(\tau^i(a))$$

Now, we present below an example of (σ,τ)-higher homomorphism and it is clearly is a Jordan (σ,τ)-higher homomorphism.

Example (2.5):

Let S_1, S_2 be two rings and $\theta = (\theta_i)_{i\in N}$ be a (σ,τ)-higher homomorphism of a ring S_1 into a ring S_2, let M = {(a,b): $a, b \in S_1$}, M' = {(a',b'): $a', b' \in S_2$} and Γ={(n,m): n,m∈Z},we define $\phi = (\phi_i)_{i\in N}$ be a family of additive mappings from a Γ-ring M into a Γ-ring M', by $\phi_n((a,b))=(\theta_n(a),\theta_n(b))$ for all a,b $\in S_1$,let σ_1^n, τ_1^n be two homomorphisms of a Γ-ring M such that

$\sigma_1^n((a,b))=(\sigma^n(a),\sigma^n(b))$, $\tau_1^n((a,b))=(\tau^n(a),\tau^n(b))$ then ϕ_n is a (σ,τ)-higher homomorphism and Jordan (σ,τ)-higher homomorphism of a Γ-ring M into a Γ-ring M'.

Lemma (2.6):

Let $\theta = (\phi_i)_{i\in N}$ be a Jordan (σ,τ)-higher homomorphism of a Γ-ring M into a Γ-ring M', then for all $a, b, c \in$ M, $\alpha, \beta \in \Gamma$ and $n \in N$ if $\sigma^{i^2}=\sigma^i$, $\tau^{i^2}=\tau^i$, $\sigma^i\tau^i=\sigma^i\tau^{n-i}$ and $\sigma^i\tau^i=\tau^i\sigma^i$

(i) $$\phi_n(a\alpha b\beta a + a\beta b\alpha a) = \sum_{i=1}^{n} \phi_i(\sigma^i(a))\alpha\phi_i(\sigma^i\tau^{n-i}(b))\beta\phi_i(\tau^i(a)) + \sum_{i=1}^{n}\phi_i(\sigma^i(a))\beta\phi_i(\sigma^i\tau^{n-i}(b))\alpha\phi_i(\tau^i(a))$$

(ii) $$\phi_n(a\alpha b\beta c + c\alpha b\beta a) = \sum_{i=1}^{n} \phi_i(\sigma^i(a))\alpha\phi_i(\sigma^i\tau^{n-i}(b))\beta\phi_i(\tau^i(c)) + \sum_{i=1}^{n} \phi_i(\sigma^i(c))\alpha\phi_i(\sigma^i\tau^{n-i}(b))\beta\phi_i(\tau^i(a))$$

In particular, if M' is a 2-torsion free commutative Γ-ring.

(iii) $$\phi_n(a\alpha b\beta c) = \sum_{i=1}^{n} \phi_i(\sigma^i(a))\alpha\phi_i(\sigma^i\tau^{n-i}(b))\beta\phi_i(\tau^i(c))$$

(iv) $$\phi_n(a\alpha b\alpha c + c\alpha b\alpha a) = \sum_{i=1}^{n} \phi_i(\sigma^i(a))\alpha\phi_i(\sigma^i\tau^{n-i}(b))\alpha\phi_i(\tau^i(c)) + \sum_{i=1}^{n} \phi_i(\sigma^i(c))\alpha\phi_i(\sigma^i\tau^{n-i}(b))\alpha\phi_i(\tau^i(a))$$

Proof:

(i) Replac $a\beta b + b\beta a$ for b in the definition (2.2), we get:

$$\phi_n(a\alpha(a\beta b + b\beta a) + (a\beta b + b\beta a)\alpha a) = \sum_{i=1}^{n} \phi_i(\sigma^i(a))\alpha\phi_i(\tau^i(a\beta b + b\beta a)) + \sum_{i=1}^{n} \phi_i(\sigma^i(a\beta b + b\beta a))\alpha\phi_i(\tau^i(a))$$

$$= \sum_{i=1}^{n} \phi_i(\sigma^i(a))\alpha\phi_i(\tau^i(a)\beta\tau^i(b) + \tau^i(b)\beta\tau^i(a)) + \sum_{i=1}^{n} \phi_i(\sigma^i(a)\beta\sigma^i(b) + \sigma^i(b)\beta\sigma^i(a))\alpha\phi_i(\tau^i(a))$$

$$= \sum_{i=1}^{n} \phi_i(\sigma^i(a))\alpha\left(\sum_{j=1}^{i}\phi_j(\sigma^j\tau^j(a)\beta\phi_j(\tau^{j^2}(b)) + \sum_{j=1}^{i}\phi_j(\sigma^j\tau^j(b)\beta\phi_j(\tau^{j^2}(a))\right) + \sum_{i=1}^{n}\left(\sum_{j=1}^{i} \phi_j(\sigma^{j^2}(a)\beta\phi_j(\tau^j\sigma^j(b)) + \sum_{j=1}^{i} \phi_j(\sigma^{j^2}(b)\beta\phi_j(\tau^j\sigma^j(a)\right)\alpha\phi_i(\tau^i(a))$$

$$= \sum_{i=1}^{n} \phi_i(\sigma^i(a))\alpha\phi_i(\sigma^i\tau^i(a))\beta\phi_i(\tau^{i^2}(b)) + \sum_{i=1}^{n}\phi_i(\sigma^i(a))\alpha\phi_i(\sigma^i\tau^i(b)\beta\phi_i(\tau^{i^2}(a)) + \sum_{i=1}^{n} \phi_i(\sigma^{i^2}(a))\beta\phi_i(\tau^i\sigma^i(b))\alpha\phi_i(\tau^i(a)) + \sum_{i=1}^{n}\phi_i(\sigma^{i^2}(b))\beta\phi_i(\tau^i\sigma^i(a)\alpha\phi_i(\tau^i(a))$$

Since $\sigma^{i^2} = \sigma^i$, $\tau^{i^2} = \tau^i$, $\sigma^i\tau^i = \sigma^i\tau^{n-i}$ and $\sigma^i\tau^i = \tau^i\sigma^i$

$$=\sum_{i=1}^{n}\phi_i(\sigma^i(a))\alpha\phi_i(\sigma^i\tau^{n-i}(a))\beta\phi_i(\tau^i(b))+\sum_{i=1}^{n}\phi_i(\sigma^i(a))\alpha\phi_i(\sigma^i\tau^{n-i}(b))\beta\phi_i(\tau^i(a))+$$
$$\sum_{i=1}^{n}\phi_i(\sigma^i(a))\beta\phi_i(\sigma^i\tau^{n-i}(b))\alpha\phi_i(\tau^i(a))+\sum_{i=1}^{n}\phi_i(\sigma^i(b))\beta\phi_i(\sigma^i\tau^{n-i}(a))\alpha\phi_i(\tau^i(a)) \quad \ldots(1)$$

On the other hand:

$$\phi_n(a\alpha(a\beta b+b\beta a)+(a\beta b+b\beta a)\alpha a)=\phi_n(a\alpha a\beta b+a\alpha b\beta a+a\beta b\alpha a+b\beta a\alpha a)$$
$$=\sum_{i=1}^{n}\phi_i(\sigma^i(a))\alpha\phi_i(\sigma^i\tau^{n-i}(a))\beta\,\phi_i(\tau^i(b))+\sum_{i=1}^{n}\phi_i(\sigma^i(b))\beta\,\phi_i(\sigma^i\tau^{n-i}(a))\alpha\phi_i(\tau^i(a))+$$
$$\phi_n(a\alpha b\beta a+a\beta b\alpha a) \quad \ldots(2)$$

Comparing (1) and (2), we get:

$$\phi_n(a\alpha b\beta a+a\beta b\alpha a)=\sum_{i=1}^{n}\phi_i(\sigma^i(a))\alpha\phi_i(\sigma^i\tau^{n-i}(b))\beta\phi_i(\tau^i(a))+$$
$$\sum_{i=1}^{n}\phi_i(\sigma^i(a))\beta\phi_i(\sigma^i\tau^{n-i}(b))\alpha\,\phi_i(\tau^i(a))$$

(ii) Replace $a+c$ for a in the definition (2.3), we get:

$$\phi_n((a+c)\alpha b\beta(a+c))=\sum_{i=1}^{n}\phi_i(\sigma^i(a+c))\alpha\phi_i(\sigma^i\tau^{n-i}(b))\beta\phi_i(\tau^i(a+c))$$
$$=\sum_{i=1}^{n}\phi_i(\sigma^i(a)+\sigma^i(c))\alpha\phi_i(\sigma^i\tau^{n-i}(b))\beta\phi_i(\tau^i(a)+\tau^i(c))$$
$$=\sum_{i=1}^{n}\phi_i(\sigma^i(a)\alpha\phi_i(\sigma^i\tau^{n-i}(b))\beta\phi_i(\tau^i(a))+\sum_{i=1}^{n}\phi_i(\sigma^i(a)\alpha\phi_i(\sigma^i\tau^{n-i}(b))\beta\phi_i(\tau^i(c))+$$
$$\sum_{i=1}^{n}\phi_i(\sigma^i(c)\alpha\phi_i(\sigma^i\tau^{n-i}(b))\beta\phi_i(\tau^i(a))+\sum_{i=1}^{n}\phi_i(\sigma^i(c)\alpha\phi_i(\sigma^i\tau^{n-i}(b))\beta\phi_i(\tau^i(c)) \ldots(1)$$

On the other hand:

$$\phi_n((a+c)\alpha b\,\beta(a+c))=\phi_n(a\alpha b\beta a+a\alpha b\beta c+c\alpha b\beta a+c\alpha b\beta a)$$
$$=\sum_{i=1}^{n}\phi_i(\sigma^i(a))\alpha\phi_i(\sigma^i\tau^{n-i}(b))\beta\,\phi_i(\tau^i(a))+\sum_{i=1}^{n}\phi_i(\sigma^i(c))\alpha\,\phi_i(\sigma^i\tau^{n-i}(b))\beta\phi_i(\tau^i(c))+$$
$$\phi_n(a\alpha b\beta c+c\alpha b\beta a)$$
$$\ldots(2)$$

Comparing (1) and (2), we get:

$$\phi_n(a\alpha b\beta c+c\alpha b\beta a)=\sum_{i=1}^{n}\phi_i(\sigma^i(a))\alpha\phi_i(\sigma^i\tau^{n-i}(b))\beta\phi_i(\tau^i(c))+$$
$$\sum_{i=1}^{n}\phi_i(\sigma^i(c))\alpha\,\phi_i(\sigma^i\tau^{n-i}(b))\beta\phi_i(\tau^i(a))$$

(iii) By (ii) since M' is a 2-torsion free commutative Γ-ring

$$\phi_n(a\alpha b\,\beta c+c\,\alpha b\,\beta a)=2\phi_n(a\alpha b\,\beta c)$$
$$=\sum_{i=1}^{n}\phi_i(\sigma^i(a))\alpha\,\phi_i(\sigma^i\tau^{n-i}(b))\beta\phi_i(\tau^i(c))$$

(iv) Replace β for α in (ii), we get:

$$\phi_n(a\alpha b\alpha c + c\alpha b\alpha a) = \sum_{i=1}^{n} \phi_i(\sigma^i(a))\alpha\phi_i(\sigma^i\tau^{n-i}(b))\alpha\phi_i(\tau^i(c)) + \sum_{i=1}^{n} \phi_i(\sigma^i(c))\alpha\phi_i(\sigma^i\tau^{n-i}(b))\alpha\phi_i(\tau^i(a))$$

Definition (2.7):
Let $\theta = (\phi_i)_{i\in N}$ be a Jordan (σ,τ)-higher homomorphism from a Γ-ring M into a Γ'-ring M', then for all a, $b \in$ M, $\alpha \in \Gamma$ and $n \in N$, we define $G_n(a,b)_\alpha$: M×Γ×M ⟶ M' by:

$$G_n(a,b)_\alpha = \phi_n(a\alpha b) - \sum_{i=1}^{n} \phi_i(\sigma^i(a))\alpha\phi_i(\tau^i(b))$$

Now, we present the properties of $G_n(a\,,b)_\alpha$

Lemma (2.8):
Let $\theta = (\phi_i)_{i\in N}$ be a Jordan (σ,τ)-higher homomorphism from a Γ-ring M into a Γ'-ring M', then for all a, b, $c \in$ M, α, β ∈ Γ and n ∈ N:

(i) $G_n(a+b,c)_\alpha = G_n(a,c)_\alpha + G_n(b,c)_\alpha$
(ii) $G_n(a,b+c)_\alpha = G_n(a,b)_\alpha + G_n(a,c)_\alpha$
(iii) $G_n(a,b)_{\alpha+\beta} = G_n(a,b)_\alpha + G_n(a,b)_\beta$

Proof:

(i) $$G_n(a+b,c)_\alpha = \phi_n((a+b)\alpha c) - \sum_{i=1}^{n} \phi_i(\sigma^i(a+b))\alpha\phi_i(\tau^i(c))$$

$$= \phi_n(a\alpha c + b\alpha c) - \sum_{i=1}^{n} \phi_i(\sigma^i(a))\alpha\phi_i(\tau^i(c)) - \sum_{i=1}^{n} \phi_i(\sigma^i(b))\alpha\phi_i(\tau^i(c))$$

$$= \phi_n(a\alpha c) - \sum_{i=1}^{n} \phi_i(\sigma^i(a))\alpha\phi_i(\tau^i(c)) + \phi_n(b\alpha c) - \sum_{i=1}^{n} \phi_i(\sigma^i(b))\alpha\phi_i(\tau^i(c))$$

$= G_n(a,c)_\alpha + G_n(b,c)_\alpha$

(ii) $$G_n(a,b+c)_\alpha = \phi_n(a\alpha(b+c)) - \sum_{i=1}^{n} \phi_i(\sigma^i(a))\alpha\phi_i(\tau^i(b+c))$$

$$= \phi_n(a\alpha b + a\alpha c)) - \sum_{i=1}^{n} \phi_i(\sigma^i(a))\alpha\phi_i(\tau^i(b)) - \sum_{i=1}^{n} \phi_i(\sigma^i(a))\alpha\phi_i(\tau^i(c))$$

$$= \phi_n(a\alpha b) - \sum_{i=1}^{n} \phi_i(\sigma^i(a))\alpha\phi_i(\tau^i(b)) + \phi_n(a\alpha c)) - \sum_{i=1}^{n} \phi_i(\sigma^i(a))\alpha\phi_i(\tau^i(c))$$

$= G_n(a,b)_\alpha + G_n(a,c)_\alpha$

(iii) $$G_n(a,b)_{\alpha+\beta} = \phi_n(a(\alpha+\beta)b) - \sum_{i=1}^{n} \phi_i(\sigma^i(a))(\alpha+\beta)\phi_i(\tau^i(b))$$

$$= \phi_n(a\alpha b) - \sum_{i=1}^{n} \phi_i(\sigma^i(a))\alpha\phi_i(\tau^i(b)) + \phi_n(a\beta b) - \sum_{i=1}^{n} \phi_i(\sigma^i(a))\beta\phi_i(\tau^i(b))$$

$= G_n(a,b)_\alpha + G_n(a,b)_\beta$

Remark (2.9):
Note that $\theta = (\phi_i)_{i\in N}$ is a (σ,τ)-higher homomorphism from a Γ-ring M into a Γ-ring M' if and only if $G_n(a,b)_\alpha = 0$ for all a, $b \in$ M, $\alpha \in \Gamma$ and n ∈ N.

Lemma (2.10):

Let $\theta = (\phi_i)_{i\in N}$ be a Jordan (σ,τ)-higher homomorphism of a 2-torsion free Γ-ring M into a Γ-ring M', such that $\sigma^{n^2} = \sigma^n$, $\tau^n\sigma^n = \sigma^n$, $\sigma^i\tau^{n-i} = \tau^i\sigma^i$ and $\sigma^i\tau^i = \tau^i\sigma^i$, then for all a, b, m $\in$ M, α, $\beta \in \Gamma$ and n $\in$ N

(i) $G_n(\sigma^n(a),\sigma^n(b))_\alpha\beta\phi_n(\sigma^n(m))\beta G_n(\tau^n(b),\tau^n(a))_\alpha +$
$G_n(\sigma^n(b),\sigma^n(a))_\alpha\beta\phi_n(\sigma^n(m))\beta G_n(\tau^n(a),\tau^n(b))_\alpha = 0$

(ii) $G_n(\sigma^n(a),\sigma^n(b))_\alpha\alpha\phi_n(\sigma^n(m))\alpha G_n(\tau^n(b),\tau^n(a))_\alpha +$
$G_n(\sigma^n(b),\sigma^n(a))_\alpha\alpha\phi_n(\sigma^n(m))\alpha G_n(\tau^n(a),\tau^n(b))_\alpha = 0$

(iii) $G_n(\sigma^n(a),\sigma^n(b))_\beta\alpha\phi_n(\sigma^n(m))\alpha G_n(\tau^n(b),\tau^n(a))_\beta +$
$G_n(\sigma^n(b),\sigma^n(a))_\beta\alpha\phi_n(\sigma^n(m))\alpha G_n(\tau^n(a),\tau^n(b))_\beta = 0$.

Proof:

(i) We prove by using the induction, we can assume that:

$G_s(\sigma^s(a),\sigma^s(b))_\alpha\beta\phi_s(\sigma^s(m))\beta G_s(\tau^s(b),\tau^s(a))_\alpha +$
$G_s(\sigma^s(b),\sigma^s(a))_\alpha\beta\phi_s(\sigma^s(m))\beta G_s(\tau^s(a),\tau^s(b))_\alpha = 0$ for all a, b, m$\in$R, and s, n $\in$ N, s < n.

Let w = $a\alpha b\beta m\beta b\alpha a + b\alpha a\beta m\beta a\alpha b$, since θ is a Jordan (σ,τ)-higher homomorphism

$\phi_n(w) = \phi_n(a\alpha(b\beta m\beta b)\alpha a + b\alpha(a\beta m\beta a)\alpha b)$

$$= \sum_{i=1}^{n} \phi_i(\sigma^i(a))\alpha\phi_i(\sigma^i\tau^{n-i}(b\beta m\beta b))\alpha\phi_i(\tau^i(a)) +$$

$$\sum_{i=1}^{n} \phi_i(\sigma^i(b))\alpha\phi_i(\sigma^i\tau^{n-i}(a\beta m\beta a))\alpha\phi_i(\tau^i(b))$$

$$= \sum_{i=1}^{n} \phi_i(\sigma^i(a))\alpha\left(\sum_{j=1}^{i}\phi_j(\sigma^j(\sigma^j\tau^{n-j}(b)))\beta\phi_j(\sigma^j\tau^{n-j}(\sigma^j\tau^{n-j}(m)))\beta\phi_j(\tau^j(\sigma^j\tau^{n-j}(b)))\right)\alpha\phi_i(\tau^i(a)) +$$

$$\sum_{i=1}^{n} \phi_i(\sigma^i(b))\alpha\left(\sum_{j=1}^{i}\phi_j(\sigma^j(\sigma^j\tau^{n-j}(m)))\beta\phi_j(\sigma^j\tau^{n-j}(\sigma^j\tau^{n-j}(m)))\beta\phi_j(\tau^j(\sigma^j\tau^{n-j}(a)))\right)\alpha\phi_i(\tau^i(b))$$

$$= \sum_{i=1}^{n} \phi_i(\sigma^i(a))\alpha\phi_i(\sigma^i(\sigma^i\tau^{n-i}(b)))\beta\phi_i(\sigma^i\tau^{n-i}(\sigma^i\tau^{n-i}(m))\beta\phi_i(\tau^i(\sigma^i\tau^{n-i}(b))\alpha\phi_i(\tau^i(a)) +$$

$$\sum_{i=1}^{n} \phi_i(\sigma^i(b))\alpha\phi_i(\sigma^i(\sigma^i\tau^{n-i}(a)))\beta\phi_i(\sigma^i\tau^{n-i}(\sigma^i\tau^{n-i}(m))\beta\phi_i(\tau^i(\sigma^i\tau^{n-i}(a))\alpha\phi_i(\tau^i(b))$$

$$= \sum_{i=1}^{n} \phi_i(\sigma^i(a))\alpha\phi_i(\sigma^i(\sigma^i\tau^{n-i}(b)))\beta\phi_i(\sigma^i\tau^{n-i}(\sigma^i\tau^{n-i}(m))\beta\sum_{j=1}^{i}\phi_j(\tau^j(\sigma^j\tau^{n-j}(b)))\alpha\phi_j(\tau^j(a)) +$$

$$\sum_{i=1}^{n} \phi_i(\sigma^i(b))\alpha\phi_i(\sigma^i(\sigma^i\tau^{n-i}(a)))\beta\phi_i(\sigma^i\tau^{n-i}(\sigma^i\tau^{n-i}(m))\beta\sum_{j=1}^{i}\phi_j(\tau^j(\sigma^j\tau^{n-j}(a)))\alpha\phi_j(\tau^j(b))$$

$$= \phi_n(\sigma^n(a))\alpha\phi_n(\sigma^n(\sigma^n(b)))\beta\phi_n(\sigma^n(\sigma^n(\mathrm{m})))\beta\sum_{j=1}^{i}\phi_j(\tau^j(\sigma^j\tau^{n-j}(b)))\alpha\phi_j(\tau^j(a)) +$$

$$\sum_{i=1}^{n-1}\phi_i(\sigma^i(a))\alpha\phi_i(\sigma^i(\sigma^i\tau^{n-i}(b))\beta\phi_i(\sigma^i\tau^{n-i}(\sigma^i\tau^{n-i}(\mathrm{m}))\beta\sum_{j=1}^{i}\phi_j(\tau^j(\sigma^j\tau^{n-j}(b)))\alpha\phi_j(\tau^j(a)) + \quad \ldots(1)$$

$$\phi_n(\sigma^n(b))\alpha\phi_n(\sigma^n(\sigma^n(a)))\beta\phi_n(\sigma^n(\sigma^n(\mathrm{m})))\beta\sum_{j=1}^{i}\phi_j(\tau^j(\sigma^j\tau^{n-j}(a)))\alpha\phi_j(\tau^j(b)) +$$

$$\sum_{i=1}^{n-1}\phi_i(\sigma^i(b))\alpha\phi_i(\sigma^i(\sigma^i\tau^{n-i}(a))\beta\phi_i(\sigma^i\tau^{n-i}(\sigma^i\tau^{n-i}(\mathrm{m}))\beta\sum_{j=1}^{i}\phi_j(\tau^j(\sigma^j\tau^{n-j}(a)))\alpha\phi_j(\tau^j(b))$$

On the other hand:

$\phi_n(\mathrm{w}) = \phi_n((a\alpha b)\beta \mathrm{m}\beta(b\alpha a) + (b\alpha a)\beta \mathrm{m}\beta(a\alpha b))$

$$= \sum_{i=1}^{n}\phi_i(\sigma^i(a\alpha b))\beta\phi_i(\sigma^i\tau^{n-i}(\mathrm{m}))\beta\phi_i(\tau^i(b\alpha a)) +$$

$$\sum_{i=1}^{n}\phi_i(\sigma^i(b\alpha a))\beta\phi_i(\sigma^i\tau^{n-i}(\mathrm{m}))\beta\phi_i(\tau^i(a\alpha b))$$

$$= \sum_{i=1}^{n}\phi_i(\sigma^i(a\alpha b))\beta\phi_i((\sigma^j\tau^{n-j}(\mathrm{m}))\beta\left(\sum_{j=1}^{i}\phi_j(\sigma^j\tau^j(a))\alpha\phi_j(\tau^{j^2}(b)) + \sum_{j=1}^{i}\phi_j(\sigma^j\tau^j(b))\alpha\phi_j(\tau^{j^2}(a)) - \phi_i(\tau^i(a\alpha b))\right) + \sum_{i=1}^{n}\left(\sum_{j=1}^{i}\phi_j(\sigma^{j^2}(a))\alpha\phi_j(\tau^j\sigma^j(b)) + \sum_{j=1}^{i}\phi_j(\sigma^{j^2}(b))\alpha\phi_j(\tau^j\sigma^j(a)) - \phi_i(\sigma^i(a\alpha b))\right)\beta\phi_i(\sigma^j\tau^{n-j}(\mathrm{m}))\beta\phi_i(\tau^i(a\alpha b))$$

$$= \sum_{i=1}^{n}\phi_i(\sigma^i(a\alpha b))\beta\phi_i(\sigma^j\tau^{n-j}(\mathrm{m})\beta\sum_{j=1}^{i}\phi_j(\sigma^j\tau^j(a))\alpha\phi_j(\tau^{j^2}(b)) + \sum_{i=1}^{n}\phi_i(\sigma^i(a\alpha b))\beta\phi_i((\sigma^j\tau^{n-j}(\mathrm{m}))\beta\cdot$$

$$\sum_{j=1}^{i}\phi_j(\sigma^j\tau^j(b))\alpha\phi_j(\tau^{j^2}(a)) - \sum_{i=1}^{n}\phi_i(\sigma^i(a\alpha b)\beta\phi_i(\sigma^i\tau^{n-i}(\mathrm{m}))\beta\phi_i(\tau^i(a\alpha b)) +$$

$$\sum_{i=1}^{n}\phi_i(\sigma^{i^2}(a))\alpha\phi_i(\tau^i\sigma^i(b))\beta\phi_i(\sigma^i\tau^{n-i}(\mathrm{m}))\beta\phi_i(\tau^i(a\alpha b)) + \sum_{i=1}^{n}\phi_i(\sigma^{i^2}(b))\alpha\phi_i(\tau^i\sigma^i(a))\beta\phi_i(\sigma^i\tau^{n-i}(\mathrm{m}))\beta\cdot$$

$$\phi_i(\tau^i(a\alpha b)) - \phi_i(\sigma^i(a\alpha b)\beta\phi_i(\sigma^i\tau^{n-i}(\mathrm{m}))\beta\phi_i(\tau^i(a\alpha b))$$

$$=-\sum_{i=1}^{n}\phi_i(\sigma^i(a\alpha b))\beta\phi_i(\sigma^j\tau^{n-j}(m))\beta(\phi_i(\tau^i(a\alpha b))-\sum_{j=1}^{i}\phi_j(\sigma^j\tau^j(a))\alpha\phi_j(\tau^{j^2}(b))-$$

$$\sum_{i=1}^{n}\phi_i(\sigma^i(a\alpha b))\beta\phi_i(\sigma^i\tau^{n-i}(m))\beta(\phi_i(\tau^i(a\alpha b))-\sum_{j=1}^{i}\phi_j(\sigma^j\tau^j(b))\alpha\phi_j(\tau^{j^2}(a))+$$

$$\sum_{i=1}^{n}\phi_i(\sigma^{i^2}(a))\alpha\phi_i(\tau^i\sigma^i(b))\beta\phi_i(\sigma^i\tau^{n-i}(m))\beta\phi_i(\tau^i(a\alpha b))+$$

$$\sum_{i=1}^{n}\phi_i(\sigma^{i^2}(b))\alpha\phi_i(\tau^i\sigma^i(a))\beta\phi_i(\sigma^i\tau^{n-i}(m))\beta\phi_i(\tau^i(a\alpha b))$$

$$=-\phi_n(\sigma^n(a\alpha b)\beta\phi_n((\sigma^n(m))\beta G_n(\tau^n(a),\tau^n(b))_\alpha-$$

$$\sum_{i=1}^{n-1}\phi_i(\sigma^i(a\alpha b))\beta\phi_i((\sigma^i\tau^{n-i}(m))\beta G_i(\tau^i(a),\tau^i(b))_\alpha-$$

$$\phi_n(\sigma^n(a\alpha b))\beta\phi_n(\sigma^n(m))\beta G_n(\tau^n(b),\tau^n(a))_\alpha-$$

$$\sum_{i=1}^{n-1}\phi_i(\sigma^i(a\alpha b))\beta\phi_i(\sigma^i\tau^{n-i}(m))\beta G_i(\tau^i(a),\tau^i(b))_\alpha+$$

$$\phi_n(\sigma^{n^2}(a))\alpha\phi_n(\tau^n\sigma^n(b))\beta\phi_n((\sigma^n(m))\beta\phi_n(\tau^n(a\alpha b))+$$

$$\sum_{i=1}^{n-1}\phi_i(\sigma^{i^2}(a)\alpha\phi_i(\tau^i\sigma^i(b))\beta\phi_i(\sigma^i\tau^{n-i}(m))\beta\phi_i(\tau^i(a\alpha b))+$$

$$\phi_n(\sigma^{n^2}(b))\alpha\phi_n(\tau^n\sigma^n(a))\beta\phi_n((\sigma^n(m))\beta\phi_n(\tau^n(a\alpha b))+$$

$$\sum_{i=1}^{n-1}\phi_i(\sigma^{i^2}(b)\alpha\phi_i(\tau^i\sigma^i(a))\beta\phi_i(\sigma^i\tau^{n-i}(m))\beta\phi_i(\tau^i(a\alpha b))$$

…(2)

Compare (1), (2) and since $\sigma^{n^2}=\sigma^n$, $\tau^n\sigma^n=\sigma^n$, $\sigma^i\tau^{n-i}=\tau^i\sigma^i$ and $\sigma^i\tau^i=\tau^i\sigma^i$

$$0=-\phi_n(\sigma^n(a\alpha b))\beta\phi_n((\sigma^n(m))\beta G_n(\tau^n(a),\tau^n(b))_\alpha-$$

$$\phi_n(\sigma^n(a\alpha b))\beta\phi_n((\sigma^n(m))\beta G_n(\tau^n(b),\tau^n(a))_\alpha+$$

$$\phi_n(\sigma^n(a))\alpha\phi_n(\sigma^n(b))\beta\phi_n((\sigma^n(m))\beta(\phi_n(\tau^n(a\alpha b))-$$

$$\sum_{i=1}^{n}\phi_i(\tau^i(\sigma^i\tau^{n-i}(a)))\alpha\phi_i(\tau^i(b))+\phi_n(b)\alpha\phi_n(\sigma^n(a))\beta(\phi_n((\sigma^n(m))\beta(\phi_n(\tau^n(a\alpha b))-$$

$$\sum_{i=1}^{n}\phi_i(\tau^i(\sigma^i\tau^{n-i}(a)))\alpha\phi_i(\tau^i(b))-\sum_{i=1}^{n-1}\phi_i(\sigma^i(a\alpha b))\beta\phi_i((\sigma^i\tau^{n-i}(m))\beta G_i(\tau^i(a),\tau^i(b))_\alpha-$$

$$\sum_{i=1}^{n-1}\phi_i(\sigma^i(a\alpha b))\beta\phi_i(\sigma^i\tau^{n-i}(m))\beta G_i(\tau^i(b),\tau^i(a))_\alpha+$$

$$\sum_{i=1}^{n-1}\phi_i(\sigma^i(a)\alpha\phi_i(\tau^i\sigma^i(b))\beta\phi_i(\sigma^i\tau^{n-i}(m))\beta(\phi_i(\tau^i(a\alpha b))-$$

$$\sum_{j=1}^{i}\phi_j(\tau^j(\sigma^j\tau^{n-j}(b)))\alpha\phi_j(\tau^i(a)))+\sum_{i=1}^{n-1}\phi_i(\sigma^i(b)\alpha\phi_i(\tau^i\sigma^i(a))\beta\phi_i(\sigma^i\tau^{n-i}(m))\beta(\phi_i(\tau^i(a\alpha b))-$$

$$\sum_{j=1}^{i}\phi_j(\tau^j(\sigma^j\tau^{n-j}(a)))\alpha\phi_j(\tau^i(b)))$$

$$=-\phi_n(\sigma^n(a\alpha b))\beta\phi_n((\sigma^n(m))\beta G_n(\tau^n(a),\tau^n(b))_\alpha -$$
$$\phi_n(\sigma^n(a\alpha b))\beta\phi_n((\sigma^n(m))\beta G_n(\tau^n(b),\tau^n(a))_\alpha +$$
$$\phi_n(\sigma^n(a))\alpha\phi_n(\sigma^n(b))\beta\phi_n((\sigma^n(m))\beta G_n(\tau^n(b),\tau^n(a))_\alpha +$$
$$\phi_n(\sigma^n(b))\alpha\phi_n(\sigma^n(a))\beta\phi_n((\sigma^n(m))\beta G_n(\tau^n(a),\tau^n(b))_\alpha -$$
$$\sum_{i=1}^{n-1}\phi_i(\sigma^i(a\alpha b))\beta\phi_i((\sigma^i\tau^{n-i}(m))\beta G_i(\tau^i(a),\tau^i(b))_\alpha -$$
$$\sum_{i=1}^{n-1}\phi_i(\sigma^i(a\alpha b))\beta\phi_i(\sigma^i\tau^{n-i}(m))\beta G_i(\tau^i(b),\tau^i(a))_\alpha +$$
$$\sum_{i=1}^{n-1}\phi_i(\sigma^i(a)\alpha\phi_i(\tau^i\sigma^i(b))\beta\phi_i(\sigma^i\tau^{n-i}(m))\beta G_i(\tau^i(b),\tau^i(a))_\alpha +$$
$$\sum_{i=1}^{n-1}\phi_i(\sigma^i(b)\alpha\phi_i(\tau^i\sigma^i(a))\beta\phi_i(\sigma^i\tau^{n-i}(m))\beta G_i(\tau^i(a),\tau^i(b))_\alpha$$
$$=-G_n(\sigma^n(b),\sigma^n(a))_\alpha\beta\phi_n((\sigma^n(m))\beta G_n(\tau^n(a),\tau^n(b))_\alpha -$$
$$G_n(\sigma^n(a),\sigma^n(b))_\alpha\beta\phi_n((\sigma^n(m))\beta G_n(\tau^n(b),\tau^n(a))_\alpha -$$
$$\sum_{i=1}^{n-1}G_i(\sigma^i(b),\sigma^i(a))_\alpha\beta\phi_i((\sigma^i\tau^{n-i}(m))\beta G_i(\tau^i(a),\tau^i(b))_\alpha -$$
$$\sum_{i=1}^{n-1}G_i(\sigma^i(a),\sigma^i(b))_\alpha\beta\phi_i(\sigma^i\tau^{n-i}(m))\beta G_i(\tau^i(b),\tau^i(a))_\alpha$$
$$=-(G_n(\sigma^n(b),\sigma^n(a))_\alpha\beta\phi_n(\sigma^n(m))\beta G_n(\tau^n(a),\tau^n(b)_\alpha)+$$
$$G_n(\sigma^n(a),\sigma^n(b))_\alpha\beta\phi_n(\sigma^n(m))\beta G_n(\tau^n(b),\tau^n(a)_\alpha)-$$
$$(\sum_{i=1}^{n-1}\phi_i(\sigma^i(b),\sigma^i(a))_\alpha\beta\phi_i(\sigma^i\tau^{n-i}(m))\beta G_i(\tau^i(a),\tau^i(b))_\alpha +$$
$$\sum_{i=1}^{n-1}\phi_i(\sigma^i(a),\sigma^i(b))_\alpha\beta\phi_i(\sigma^i\tau^{n-i}(m))\beta G_i(\tau^i(b),\tau^i(a))_\alpha)$$

By our hypothesis, we have:

$$G_n(\sigma^n(a),\sigma^n(b))_\alpha\beta\phi_n(\sigma^n(m))\beta G_n(\tau^n(b),\tau^n(a))_\alpha +$$
$$G_n(\sigma^n(b),\sigma^n(a))_\alpha\beta\phi_n(\sigma^n(m))\beta G_n(\tau^n(a),\tau^n(b))_\alpha = 0$$

(ii) Replace β by α in (i) proceeding in the same way as in the proof of (i) by the similar arguments, we get (ii).

(iii) Interchanging α and β in (i), we get (iii).

Lemma (2.11):

Let $\theta = (\phi_i)_{i\in N}$ be a Jordan (σ,τ)-higher homomorphism from a Γ-ring M into Γ-ring M', then for all *a*, *b*, m ∈ M, α, β∈Γ and n ∈ N

(i) $$G_n(\sigma^n(a),\sigma^n(b))_\alpha\beta\phi_n(\sigma^n(m))\beta G_n(\tau^n(b),\tau^n(a))_\alpha =$$
$$G_n(\sigma^n(b),\sigma^n(a))_\alpha\beta\phi_n(\sigma^n(m))\beta G_n(\tau^n(a),\tau^n(b))_\alpha = 0$$

(ii) $$G_n(\sigma^n(a),\sigma^n(b))_\alpha\alpha\phi_n(\sigma^n(m))\alpha G_n(\tau^n(b),\tau^n(a))_\alpha =$$
$$G_n(\sigma^n(b),\sigma^n(a))_\alpha\alpha\phi_n(\sigma^n(m))\alpha G_n(\tau^n(a),\tau^n(b))_\alpha = 0$$

(iii) $G_n(\sigma^n(a),\sigma^n(b))_\beta \alpha \phi_n(\sigma^n(m))\alpha G_n(\tau^n(b),\tau^n(a))_\beta =$

$$G_n(\sigma^n(b),\sigma^n(a))_\beta \alpha \phi_n(\sigma^n(m))\alpha G_n(\tau^n(a),\tau^n(b))_\beta = 0$$

Proof:

(i) By lemma (2.10) (i), we have:

$$G_n(\sigma^n(a),\sigma^n(b))_\alpha \beta \phi_n(\sigma^n(m))\beta G_n(\tau^n(b),\tau^n(a))_\alpha +$$

$$G_n(\sigma^n(b),\sigma^n(a))_\alpha \beta \phi_n(\sigma^n(m))\beta G_n(\tau^n(a),\tau^n(b))_\alpha = 0$$

And by lemma (let M be a 2-torsion free semiprime Γ-ring and suppose that $a, b \in$ M if $a\Gamma m\Gamma b + b\Gamma m\Gamma a = 0$ for all m ∈ M, then $a\Gamma m\Gamma b = b\Gamma m\Gamma a = 0$), we get:

$$G_n(\sigma^n(a),\sigma^n(b))_\alpha \beta \phi_n(\sigma^n(m))\beta G_n(\tau^n(b),\tau^n(a))_\alpha =$$

$$G_n(\sigma^n(b),\sigma^n(a))_\alpha \beta \phi_n(\sigma^n(m))\beta G_n(\tau^n(a),\tau^n(b))_\alpha = 0$$

(ii) Replace β for α in (i), we get (ii).

(iii) Interchanging α and β in (i), we get (iii).

Theorem (2.12):

Let $\theta = (\phi_i)_{i\in N}$ be a Jordan (σ,τ)-higher homomorphism from a Γ-ring M into a prime Γ-ring M', then for all a, b, c, d, m ∈ M, α, β∈Γ and n ∈ N

(i) $G_n(\sigma^n(a),\sigma^n(b))_\alpha \beta \phi_n(\sigma^n(m))\beta G_n(\tau^n(d),\tau^n(c))_\alpha = 0$

(ii) $G_n(\sigma^n(a),\sigma^n(b))_\alpha \alpha \phi_n(\sigma^n(m))\alpha G_n(\tau^n(d),\tau^n(c))_\alpha = 0$

(iii) $G_n(\sigma^n(a),\sigma^n(b))_\alpha \alpha \phi_n(\sigma^n(m))\alpha G_n(\tau^n(d),\tau^n(c))_\beta = 0$

Proof:

(i) Replacing $a+c$ for a in lemma (2.11) (i), we get:

$$G_n(\sigma^n(a+c),\sigma^n(b))_\alpha \beta \phi_n(\sigma^n(m))\beta G_n(\tau^n(b),\tau^n(a+c))_\alpha = 0$$

$$G_n(\sigma^n(a),\sigma^n(b))_\alpha \beta \phi_n(\sigma^n(m))\beta G_n(\tau^n(b),\tau^n(a))_\alpha +$$

$$G_n(\sigma^n(a),\sigma^n(b))_\alpha \beta \phi_n(\sigma^n(m))\beta G_n(\tau^n(b),\tau^n(c))_\alpha +$$

$$G_n(\sigma^n(c),\sigma^n(b))_\alpha \beta \phi_n(\sigma^n(m))\beta G_n(\tau^n(b),\tau^n(a))_\alpha +$$

$$G_n(\sigma^n(c),\sigma^n(b))_\alpha \beta \phi_n(\sigma^n(m))\beta G_n(\tau^n(b),\tau^n(c))_\alpha = 0$$

By lemma (2.10)(i), we get:

$$G_n(\sigma^n(a),\sigma^n(b))_\alpha \beta \phi_n(\sigma^n(m))\beta G_n(\tau^n(b),\tau^n(c))_\alpha +$$

$$G_n(\sigma^n(c),\sigma^n(b))_\alpha \beta \phi_n(\sigma^n(m))\beta G_n(\tau^n(b),\tau^n(a))_\alpha = 0$$

Therefore, we get:

$$G_n(\sigma^n(a),\sigma^n(b))_\alpha \beta \phi_n(\sigma^n(m))\beta G_n(\tau^n(b),\tau^n(c))_\alpha \beta \phi_n(\sigma^n(m))\beta$$

$$G_n(\sigma^n(a),\sigma^n(b))_\alpha \beta \phi_n(\sigma^n(m))\beta G_n(\tau^n(b),\tau^n(c))_\alpha = 0$$

$$= -G_n(\sigma^n(a),\sigma^n(b))_\alpha \beta \phi_n(\sigma^n(m))\beta G_n(\tau^n(b),\tau^n(c))_\alpha \beta \phi_n(\sigma^n(m))\beta$$

$$G_n(\sigma^n(c),\sigma^n(b))_\alpha \beta \phi_n(\sigma^n(m))\beta G_n(\tau^n(b),\tau^n(a))_\alpha = 0$$

Hence, by the primness of M':

$$G_n(\sigma^n(a),\sigma^n(b))_\alpha \beta \phi_n(\sigma^n(m))\beta G_n(\tau^n(b),\tau^n(c))_\alpha = 0 \qquad \ldots(1)$$

Now, replacing $b+d$ for b in lemma (2.12) (i), we get:

$$G_n(\sigma^n(a),\sigma^n(b+d))_\alpha \beta \phi_n(\sigma^n(m))\beta G_n(\tau^n(b+d),\tau^n(a))_\alpha = 0$$

$$G_n(\sigma^n(a),\sigma^n(b))_\alpha\beta\phi_n(\sigma^n(m))\beta G_n(\tau^n(b),\tau^n(a))_\alpha +$$
$$G_n(\sigma^n(a),\sigma^n(b))_\alpha\beta\phi_n(\sigma^n(m))\beta G_n(\tau^n(d),\tau^n(a))_\alpha +$$
$$G_n(\sigma^n(a),\sigma^n(d))_\alpha\beta\phi_n(\sigma^n(m))\beta G_n(\tau^n(b),\tau^n(a))_\alpha +$$
$$G_n(\sigma^n(a),\sigma^n(d))_\alpha\beta\phi_n(\sigma^n(m))\beta G_n(\tau^n(d),\tau^n(a))_\alpha = 0$$

By lemma (2.12) (i), we get:

$$G_n(\sigma^n(a),\sigma^n(b))_\alpha\beta\phi_n(\sigma^n(m))\beta G_n(\tau^n(d),\tau^n(a))_\alpha +$$
$$G_n(\sigma^n(a),\sigma^n(d))_\alpha\beta\phi_n(\sigma^n(m))\beta G_n(\tau^n(b),\tau^n(a))_\alpha = 0$$

Therefore, we get:

$$G_n(\sigma^n(a),\sigma^n(b))_\alpha\beta\phi_n(\sigma^n(m))\beta G_n(\tau^n(d),\tau^n(a))_\alpha\beta\phi_n(\sigma^n(m))\beta$$
$$G_n(\sigma^n(a),\sigma^n(b))_\alpha\beta\phi_n(\sigma^n(m))\beta G_n(\tau^n(d),\tau^n(a))_\alpha = 0$$
$$= -G_n(\sigma^n(a),\sigma^n(b))_\alpha\beta\phi_n(\sigma^n(m))\beta G_n(\tau^n(d),\tau^n(a))_\alpha\beta\phi_n(\sigma^n(m))\beta$$
$$G_n(\sigma^n(a),\sigma^n(d))_\alpha\beta\phi_n(\sigma^n(m))\beta G_n(\tau^n(b),\tau^n(a))_\alpha = 0$$

Since M' is a prime Γ-ring, then:

$$G_n(\sigma^n(a),\sigma^n(b))_\alpha\beta\phi_n(\sigma^n(m))\beta G_n(\tau^n(d),\tau^n(a))_\alpha = 0 \qquad \ldots(2)$$

Thus, $G_n(\sigma^n(a),\sigma^n(b))_\alpha\beta\phi_n(\sigma^n(m))\beta G_n(\tau^n(b+d),\tau^n(a+c))_\alpha = 0$

$$G_n(\sigma^n(a),\sigma^n(b))_\alpha\beta\phi_n(\sigma^n(m))\beta G_n(\tau^n(b),\tau^n(a))_\alpha +$$
$$G_n(\sigma^n(a),\sigma^n(b))_\alpha\beta\phi_n(\sigma^n(m))\beta G_n(\tau^n(b),\tau^n(c))_\alpha +$$
$$G_n(\sigma^n(a),\sigma^n(b))_\alpha\beta\phi_n(\sigma^n(m))\beta G_n(\tau^n(d),\tau^n(a))_\alpha +$$
$$G_n(\sigma^n(a),\sigma^n(b))_\alpha\beta\phi_n(\sigma^n(m))\beta G_n(\tau^n(d),\tau^n(c))_\alpha = 0$$

By (1), (2) and lemma (2.12), we get:

$$G_n(\sigma^n(a),\sigma^n(b))_\alpha\beta\phi_n(\sigma^n(m))\beta G_n(\tau^n(d),\tau^n(c))_\alpha = 0.$$

(ii) Replace β for α in (1), we get (ii).

(iii) Replacing $\alpha+\beta$ for α in (ii), we get:

$$G_n(\sigma^n(a),\sigma^n(b))_{\alpha+\beta}\alpha\phi_n(\sigma^n(m))\alpha G_n(\tau^n(d),\tau^n(c))_{\alpha+\beta} = 0$$
$$G_n(\sigma^n(a),\sigma^n(b))_\alpha\alpha\phi_n(\sigma^n(m))\alpha G_n(\tau^n(d),\tau^n(c))_\alpha +$$
$$G_n(\sigma^n(a),\sigma^n(b))_\alpha\alpha\phi_n(\sigma^n(m))\alpha G_n(\tau^n(d),\tau^n(c))_\beta +$$
$$G_n(\sigma^n(a),\sigma^n(b))_\beta\alpha\phi_n(\sigma^n(m))\alpha G_n(\tau^n(d),\tau^n(c))_\alpha +$$
$$G_n(\sigma^n(a),\sigma^n(b))_\beta\alpha\phi_n(\sigma^n(m))\alpha G_n(\tau^n(d),\tau^n(c))_\beta = 0$$

By (i) and (ii), we get:

$$G_n(\sigma^n(a),\sigma^n(b))_{\alpha+\beta}\alpha\phi_n(\sigma^n(m))\alpha G_n(\tau^n(d),\tau^n(c))_{\alpha+\beta} = 0$$

$$G_n(\sigma^n(a),\sigma^n(b))_\alpha \alpha\phi_n(\sigma^n(m))\alpha G_n(\tau^n(d),\tau^n(c))_\alpha +$$
$$G_n(\sigma^n(a),\sigma^n(b))_\alpha \alpha\phi_n(\sigma^n(m))\alpha G_n(\tau^n(d),\tau^n(c))_\beta +$$
$$G_n(\sigma^n(a),\sigma^n(b))_\beta \alpha\phi_n(\sigma^n(m))\alpha G_n(\tau^n(d),\tau^n(c))_\alpha +$$
$$G_n(\sigma^n(a),\sigma^n(b))_\beta \alpha\phi_n(\sigma^n(m))\alpha G_n(\tau^n(d),\tau^n(c))_\beta = 0$$

By (i) and (ii), we get:

$$G_n(\sigma^n(a),\sigma^n(b))_\alpha \alpha\phi_n(\sigma^n(m))\alpha G_n(\tau^n(d),\tau^n(c))_\beta +$$
$$G_n(\sigma^n(a),\sigma^n(b))_\beta \alpha\phi_n(\sigma^n(m))\alpha G_n(\tau^n(d),\tau^n(c))_\alpha = 0$$

Therefore, we have:

$$G_n(\sigma^n(a),\sigma^n(b))_\alpha \alpha\phi_n(\sigma^n(m))\alpha G_n(\tau^n(d),\tau^n(c))_\beta \alpha\phi_n(\sigma^n(m))\alpha$$
$$G_n(\sigma^n(a),\sigma^n(b))_\alpha \alpha\phi_n(\sigma^n(m))\alpha G_n(\tau^n(d),\tau^n(c))_\beta = 0$$
$$= -G_n(\sigma^n(a),\sigma^n(b))_\alpha \alpha\phi_n(\sigma^n(m))\alpha G_n(\tau^n(d),\tau^n(c))_\beta \alpha\phi_n(\sigma^n(m))\alpha$$
$$G_n(\sigma^n(a),\sigma^n(b))_\beta \alpha\phi_n(\sigma^n(m))\alpha G_n(\tau^n(d),\tau^n(c))_\beta = 0$$

Since M' is a prime Γ'-ring, then:

$$G_n(\sigma^n(a),\sigma^n(b))_\alpha \alpha\phi_n(\sigma^n(m))\alpha G_n(\tau^n(d),\tau^n(c))_\beta = 0.$$

III. The Main Results

Theorem (3.1):

Every Jordan (σ,τ)-higher homomorphism from a Γ-ring M into a prime Γ-ring M' is either (σ,τ)-higher homomorphism or (σ,τ)-anti-higher homomorphism.

Proof:

Let $\theta = (\phi_i)_{i\in N}$ be a Jordan (σ,τ)-higher homomorphism of a Γ-ring M into a prime Γ'-ring M'.
Since M' is a prime Γ'-ring, we get from theorem (2.12) (i)

$G_n(\sigma^n(a),\sigma^n(b))_\alpha = 0$ or $G_n(\tau^n(d),\tau^n(c))_\alpha = 0$, for all *a*, *b*, *c*, *d* ∈ M, α, β∈Γ and n ∈ N.

If $G_n(\tau^n(d),\tau^n(c))_\alpha \neq 0$ for all *c*, *d* ∈ M, α∈Γ and n ∈ N then $G_n(\sigma^n(a),\sigma^n(b))_\alpha = 0$ for all *a*, *b* ∈ M, α∈Γ and n ∈ N, hence, we get θ is a (σ,τ)-higher homomorphism from a Γ-ring M into a prime Γ'-ring M'. But if $G_n(\tau^n(d),\tau^n(c))_\alpha = 0$ for all *c*, *d* ∈ M, α∈Γ and n ∈ N then θ is a (σ,τ)-anti-higher homomorphism from a Γ-ring M into a prime Γ'-ring M'

Proposition (3.2):

Let $\theta = (\phi_i)_{i\in N}$ be a Jordan (σ,τ)-higher homomorphism from a Γ-ring M into 2-torsion free Γ-ring M', such that *a*α*b*β*c* = *a*β*b*α*c*, for all *a*, *b*, *c* ∈ M and α, β∈Γ, *a*α'*b*β'*c* = *a*β'*b*α'*c*, for all *a*, *b*, *c* ∈ M' and α', β' ∈ Γ', $\sigma^{i^2} = \sigma^i$, $\tau^{i^2} = \tau^i$, $\sigma^i\tau^i = \sigma^i\tau^{n-i}$ and $\sigma^i\tau^i = \tau^i\sigma^i$ then θ is a Jordan triple (σ,τ)-higher homomorphism.

Proof:

Replace *b* by *a*β*b* + *b*β*a* in the definition (2.2), we get:

$$\phi_n(a\alpha(a\beta b + b\beta a) + (a\beta b + b\beta a)\alpha a)$$
$$= \sum_{i=1}^{n} \phi_i(\sigma^i(a))\alpha'\phi_i(\tau^i(a\beta' b + b\beta' a)) + \sum_{i=1}^{n} \phi_i(\sigma^i(a\beta' b + b\beta' a))\alpha'\phi_i(\tau^i(a))$$

$$=\sum_{i=1}^{n}\phi_i(\sigma^i(a))\alpha'\phi_i(\tau^i(a)\beta'\tau^i(b)+\tau^i(b)\beta'\tau^i(a))+$$

$$\sum_{i=1}^{n}\phi_i(\sigma^i(a)\beta'\sigma^i(b)+\sigma^i(b)\beta'\sigma^i(a))\alpha'\phi_i(\tau^i(a))$$

$$=\sum_{i=1}^{n}\phi_i(\sigma^i(a))\alpha'\left(\sum_{j=1}^{i}\phi_j(\sigma^j\tau^j(a)\beta'\phi_j(\tau^{j^2}(b))+\sum_{j=1}^{i}\phi_j(\sigma^j\tau^j(b)\beta'\phi_j(\tau^{j^2}(a))\right)+$$

$$\sum_{i=1}^{n}\left(\sum_{j=1}^{i}\phi_j(\sigma^{j^2}(a)\beta'\phi_j(\tau^j\sigma^j(b))+\sum_{j=1}^{i}\phi_j(\sigma^{j^2}(b)\beta'\phi_j(\tau^j\sigma^j(a))\right)\alpha'\phi_i\tau^i(a))$$

$$=\sum_{i=1}^{n}\phi_i(\sigma^i(a))\alpha'\phi_i(\sigma^i\tau^i(a)\beta'\phi_i(\tau^{i^2}(b))+\sum_{j=1}^{i}\phi_i(\sigma^i(a))\alpha'\phi_i(\sigma^i\tau^i(b)\beta'\phi_i(\tau^{i^2}(a))+$$

$$\sum_{i=1}^{n}\phi_i(\sigma^{i^2}(a))\beta'\phi_i(\tau^i\sigma^i(b)\alpha'\phi_i(\tau^i(a))+\sum_{j=1}^{i}\phi_i(\sigma^{i^2}(b))\beta'\phi_i(\sigma^i\tau^i(a)\alpha'\phi_i(\tau^i(a))$$

Since $a\alpha b\beta c = a\beta b\alpha c$, for all a, b, $c \in$ M' and α, $\beta \in \Gamma$, $\sigma^{i^2}=\sigma^i$, $\tau^{i^2}=\tau^i$, $\sigma^i\tau^i=\sigma^i\tau^{n-i}$ and $\sigma^i\tau^i=\tau^i\sigma^i$, we get

$$=\sum_{i=1}^{n}\phi_i(\sigma^i(a))\alpha'\phi_i(\sigma^i\tau^{n-i}(a)\beta'\phi_i(\tau^i(b))+2\sum_{i=1}^{n}\phi_i(\sigma^i(a))\alpha'\phi_i(\sigma^i\tau^{n-i}(b)\beta'\phi_i(\tau^i(a))+$$

$$\sum_{i=1}^{n}\phi_i(\sigma^i(b))\beta'\phi_i(\sigma^i\tau^{n-i}(a)\alpha'\phi_i(\tau^i(a) \qquad \ldots(1)$$

On the other hand:

$$\phi_n(a\alpha(a\beta b+b\beta a)+(a\beta b+b\beta a)\alpha a)=\phi_n(a\alpha a\beta b+a\alpha b\beta a+a\beta b\,\alpha a+b\beta a\alpha a)$$

Since $a\alpha b\beta a = a\beta b\alpha a$, for all a, $b \in$ M and α, $\beta\in\Gamma$

$$=\phi_n(a\alpha a\beta b+b\beta a\,\alpha a)+2\phi_n(a\alpha b\beta a)$$

$$=\sum_{i=1}^{n}\phi_i(\sigma^i(a))\alpha'\phi_i(\sigma^i\tau^{n-i}(a)\beta'\phi_i(\tau^i(b))+\sum_{i=1}^{n}\phi_i(\sigma^i(b))\beta'\phi_i(\sigma^i\tau^{n-i}(a)\alpha'\phi_i(\tau^i(a))+ \qquad \ldots(2)$$

$$2\phi_n(a\alpha b\beta a)$$

Compare (1) and (2), we get:

$$2\phi_n(a\alpha b\beta a)=2\sum_{i=1}^{n}\phi_i(\sigma^i(a))\alpha'\phi_i(\sigma^i\tau^{n-i}(b)\beta'\phi_i(\tau^i(a))$$

Since M' is a 2-torsion free Γ'-ring, we obtain that θ is a Jordan triple (σ,τ)-higher homomorphism from a Γ-ring M into a Γ'-ring M'.

References:

[1] W.E.Barnes, "On The Γ-Rings of Nobusawa", Pacific J.Math., Vol.18 (1966), pp.411-422.

[2] Y.Ceven and M.A.Ozturk, "On Jordan Generalized Derivations in Gamma Rings", Hocet type J.of Mathematics and Statistics, Vol.33 (2004), pp.11-14.

[3] K.K.Dey, A.C.Paul and I.S.Rakhimov, " Generalized Derivations in Semiprime Gama Rings", International Journal of Mathematics and Mathematical Sciences, Vol.(2012), 14 pages.

[4] A.K.Halder and A.C.Paul, "On Two Torsion Free Prime Gamma Ring with Jordan Left Derivations", Journal of Math., Vol.46(1) (2014), pp.59-66.

[5] M.F.Hoque and A.C.Paul, "On Generalized Derivations on Semiprime Gamma Ring with Involution", Palestine Journal of Math., Vol.3(2) (2014),pp. 235-239.

[6] A.K.Jordan and A.C.Paul, "On Jordan Triple Derivations of Prime Γ-Rings", International Journal of Engineering and Computer Science, Vol.3(3) (2014), pp.5196-5199.
[7] A.C.Paul and A.Nazneen, "On Jordan K-Derivations on Lie Ideals of Prime Γ-Rings", Universal Journal of Computational Mathematics, Vol.2(1) (2014), pp.17-21.
[8] S.M.Salih, "On Prime Γ-Rings with Derivations", Ph.D.Thesis University of Al-Mustansirya,2010
[9] S.M.Salih and B.M.Hammad, "On Jordan Higher Centralizer on Γ-Rings", International Mathematical Forum, Vol.8(11)(2013), pp.517-526.
[10] S.M.Salih, A.M.Kamal and B.M.Hammad, "On Jordan Higher K-Centralizer on Γ-Rings", Iosr Journal of Mathematics, Vol.7(1) (2013), pp.06-14.
[11] N.N.Suliman and A.H.Majeed, "On Orthogonal Generalized Derivations of Semiprime Gamma Ring", International Journal of Computational Science and Mathematics, Vol.4(2) (2012), pp.113-122.