IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 10, Issue 5 Ver. III (Sep-Oct. 2014), PP 83-85
www.iosrjournals.org

Ideal of Prime Γ-Rings with Right Reverse Derivations

Salah Mehdi Salih and Marwa Riyadh Salih
Al-Mustansirya University College of Education
Department of Mathematics

Abstract: *In this paper some results concerning to right reverse derivation on prime Γ-rings are presented if M be a prime Γ-ring with non-zero right reverse derivation d and U be the ideal of M, then M is commutative.*
Mathematics Subject Classification: *16A70, 16N60, 16W25*
Keywords: *prime Γ-ring, derivation,reverse derivation.*

I. Introduction:

The concepts of a Γ-ring was first by Nodusawa [5] in 1964. Now a day his Γ-ring is called a Γ-ring in the sense of Nobusawa this Γ-ring is generalized by W.E.Barnes [1] in a broad sense that served now- a day to call Γ-ring

Let M and Γ be additive abelian groups, if there exists a mapping M x Γx M→M: (x,α,y)→ xαy which satisfies the following conditions ,for all a,b,c∈M and α,β∈Γ:

1. (a + b) α c = aαc + bαc
   a(α + β) b = aαb + aβb
   aα(b + c) = aαb + aαc
2. (aαb)βc = aα(bβc)

Then M is called a Γ-ring. [1]

We writ $[x, y]_\alpha$ for xαy – yαx. Recall that a Γ-ring M is called prime if aΓMΓb = 0 implies a=0 or b=0and it is called semiprime if aΓMΓa = 0 implies a=0 ,a Γ-ring M is called commutative if $[x, y]_\alpha = 0$ for all x,y∈M and α∈Γ,Bresar and Vakman [2] have introduced the notion of a reverse derivation, the reverse derivation on semi prime rings have been studied by Samman and Alyamani [6] and K.KDey, A.IC.Paul, I.S.Rakhimov [3] have introduced the concepts of reverse derivation on Γ-ring as an additive mapping d from M in to M is called reverse derivation if d(xαy) = d(y)αx + yαd(x), for all x,y∈M ,α∈Γand we consider an assumption (*) by xαyβz = xαyβz for all x,y,z∈U,α,β∈Γ, where U is ideal of Γ-ring.

Taking the above as assumption (*) the basic commutate identities reduce to $[x\beta y, z]_\alpha = x\beta[y, z]_\alpha + [x, z]_\alpha\beta y$ and $[x, y\beta z]_\alpha = y\beta[x, z]_\alpha + [x, y]_\alpha\beta z$ for all x,y,z∈U and for all α,β∈Γ which are used extensively in our results

C.J.S.Reddy and K.Hemavathi [4] studied the right reverse derivation on prim ring and we extend in this paper the results mentioned above to prime Γ-rings case

II. The Main Results:

In this section we introduce the main results of this paper we begin with the following theorem:

Theorem(1): Let M be a prime Γ-ring,U a non zero ideal of M and d be a right reverse derivation of M ,if U is non-commutative such that (*) for all x, y, z ∈U And α,β∈Γ, then d=0.

Proof:

Since d is right reverse derivation and since (*) then

Let d (xαxβy) = d(y)βxαx + d(x)αxβy + d(x)αxβy
=d(y)βxαx + d(x)βxαy +d(x)αxβy ……(1)

On the other hand

d(xαxβy) =d(xα(xβy))
=d(xβy)αx + d(x)αxβy
=d(y)βxαx +d(x)βyαx + d(x)αxβy …….(2)

Compare (1) and (2) we get

d(x)βyαx = d(x)βxαy
⇒d(x)βyαx – d(x)βxαy = 0
⇒d(x)β(yαx-xαy) = 0
⇒d(x)β$[y, x]_\alpha$ = 0 for all x, y ∈U,α,β∈Γ……(3)

We replace y by yβz in equation (3) and using (3) we get :
d(x)β$[y\beta z, x]_\alpha$ = 0 for all x,y,z∈ U and α,β∈Γ
⇒d(x)βyβ$[z, x]_\alpha$ + d(x)β$[y, x]_\alpha$βz = 0
⇒d(x)βyβ$[z, x]_\alpha$ = 0 for all x,y,z ∈U and α,β∈Γ …..(4)
By writing y by yαm,m∈M in equation (4) we obtain
⇒d(x)βyαmβ$[z, x]_\alpha$ = 0 for all x,y,z∈U and α,β∈Γ ,m∈M
If we interchange m and y, then we get
⇒d(x)βmαyβ$[z, x]_\alpha$ = 0 for all x,y,z∈U , m∈M and α,β∈Γ
By primness property, either d(x) = 0 (or) $[z, x]_\alpha$ = 0
Since U is non-commutative, then d = 0.

Theorem(2): Let M be a prime Γ-ring, U is ideal of M and d be a non-zero right reverse derivation of M. if $[d(y), d(x)]_\alpha$ = $[y, x]_\alpha$ such that (*)for all x,y,z∈U and α,β∈Γ ,then$[x, d(x)]_\alpha$ = 0 and hence M is commutative.
Proof:
Gavin that $[d(y), d(x)]_\alpha$ = $[y, x]_\alpha$ for all x,y∈U ,α∈Γ
By taking yβ x instead of y in the hypothesis, then we get
$[y\beta x, x]_\alpha$ = $[d(y\beta x), d(x)]_\alpha$
⇒yβ$[x, x]_\alpha$ + $[y, x]_\alpha$x = $[d(x)\beta y + d(y)\beta x, d(x)]_\alpha$
⇒$[y, x]_\alpha$βx = (d(x)βy + d(y)βx)αd(x) – d(x)α(d(x)βy + d(y)βx)
⇒$[y, x]_\alpha$βx = d(x)βyαd(x) + d(y)βxαd(x) – d(x)αd(x)βy – d(x)αd(y)βx
Adding and subtracting d(y)βd(x)αx
⇒$[y, x]_\alpha\beta$x = d(x)βyαd(x) + d(y)βxαd(x) – d(x)αd(x)βy – d(x)αd(y)βx +d(y)βd(x)αx - d(y)βd(x)αx
⇒$[y, x]_\alpha\beta$x = d(x)βyαd(x) + d(y)βxαd(x) – d(x)βd(x)αy – d(x)αd(y)βx +d(y)αd(x)βx- d(y)βd(x)αx
⇒$[y, x]_\alpha$= d(x)βyαd(x) – d(x)βd(x)αy + d(y)βxαd(x) - d(y)βxαd(x) + d(y)αd(x)βx – d(x)αd(y)βx
⇒$[y, x]_\alpha\beta$x = d(x)β$[y, d(x)]_\alpha$ + d(y)β$[x, d(x)]_\alpha$ + $[d(y), d(x)]_\alpha\beta$x
⇒d(x)β$[y, d(x)]_\alpha$ + d(y)β$[x, d(x)]_\alpha$ = 0 …..(5)
Replace y by cαy where c∈Z(M) and using equation (5) we get,
⇒d(x)β$[y, x]_\alpha$ + d(cαy)β$[x, d(x)]_\alpha$ = 0
⇒d(x)β(cα$[y, d(x)]_\alpha$ + $[c, d(x)]_\alpha$αy) + (d(y)αc +d(c)αy)$\beta[x, d(x)]_\alpha$ = 0
⇒cαd(x)β$[y, d(x)]_\alpha$ +[d(x)β$c, d(x)]_\alpha$αy + cαd(y)β$[x, d(x)]_\alpha$
+ d(c)αyβ$[x, d(x)]_\alpha$ = 0
⇒ -cαd(y)β$[x, d(x)]_\alpha$ + d(x)β$c, d(x)]_\alpha$αy + cαd(y)β$[x, d(x)]_\alpha$+d(c)αyβ$[x, d(x)]_\alpha$ = 0
⇒d(c)αyβ$[x, d(x)]_\alpha$ = 0 for all x,y∈U and α,β∈Γ
Since 0≠ d(c) ∈Z(M) and U is ideal of M, then we have $[x, d(x)]_\alpha$ = 0 for all x∈U
By using the similar procedure as in theorem (1) then we get either d(x)=0 (or) $[z, x]_\alpha$ = 0
Since d is non-zero, then $[z, x]_\alpha$ = 0
Hence M is commutative.

Theorem (3): Let M be a prime Γ-ring, U is ideal of M and d be a non-zero right reverse derivation of M. if $[d(y), d(x)]_\alpha$ = 0 for all x,y∈U ,α,β∈Γ, then M is commutative.
Proof:
Given that $[d(y), d(x)]_\alpha$ = 0 for all x,y∈U and α∈Γ
By taking yβx instead of y in the hypothesis, then we get,
⇒$[d(y\beta x), d(x)]_\alpha$ = 0 for all x,y∈U and α,β∈Γ
⇒$[d(x)\beta y + d(y)\beta x, d(x)]_\alpha$ = 0
⇒$[d(x)\beta y, d(x)]_\alpha$ + $[d(y)\beta x, d(x)]_\alpha$ = 0
⇒$d(x)\beta[y, d(x)]_\alpha$ + $[d(x), d(x)]_\alpha\beta y$ + $d(y)\beta[x, d(x)]_\alpha$ + $[d(y), d(x)]_\alpha\beta x$ = 0
⇒d(x)β$[y, d(x)]_\alpha$ + d(y)β$[x, d(x)]_\alpha$ = 0 ……. (6)
The proof is now completed by using equation (5) of theorem (2).
Hence M is commutative.

References

[1] W.E.Barnes, 1966, "on the gamma rings of Nobusawa ",pacific .J. math 18, 411-422.
[2] M.Bresar and J.Vukman ,on some additive mapping in rings with involution, equation math, 38(1989), 178-185.
[3] K.K.Dey ,A.C.Paul, I.S.Rakhimove "Semiprim Gamma Rings With Orthogonal Reverse Derivations", International Journal Of Pure and Applied Mathematics, vol.83, No.2, 2013, 233-245.
[4] C.J.S.Reddy and K.Hemavathi, "right reverse derivation on prime rings", International Journal of Research in Engineering and Tec , vol.2, Issue 3, 2014, 141-144.
[5] N.Nobusawa, 1964, "on a generalization of the ring theory " Osaka J. Math.1, 81-89.
[6] M.Samman and N.Alyamani, "Derivation and reverse derivations in semiprim rings", International Mathematical forum2 , No.39, (2007), 1895-1902.