IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 10, Issue 5 Ver. V (Sep-Oct. 2014), PP 10-12
www.iosrjournals.org

Combinatorial Theory of a Complete Graph K_5

Geetha N. K.
Assistant Professor (S.G),
Department of Mathematics, Saveetha School of Engineering Saveetha University, Chennai, India

Abstract : *For two given graphs G and H, the Ramsey number R(G,H) is the positive integer N such that for every graph F of order N, either F contains G as a subgraph. The Ramsey number $R(F_l , K_4)$ where F_l is the graph of every triangle. The aim of this paper is to prove that $R(F_l , K_n) = 2l(n-1) + 1$ for n=4 & l = 3 and $R(F_l , K_n) = 2l(n-1) + 1$ for $l \geq n \geq 5$.*

Keywords*: Fan, Graph, Ramsey number, Tree, Wheel.*

I. Introduction

A complete graph is a graph with an edge between every pair of vertices. A tree is a connected graph T that does not contain any cycles. The complete graph on n vertices is denoted by K_n . The graph □ is the compliment of G which is obtained from the complete graph on |V(G)| vertices by deleting the edges of G. A graph G is complete p- particle if its vertices can be partitioned into p non empty independent sets V_1 , V_2 V_3 V_4 ….., V_p such that its edge set E is formed by all edges that have one end vertex in V_i and the other one in V_j for $1 \leq i \leq j \leq p$. A complete 2-partite graph is called a complete m by n bipartite graph and denoted by $K_{m,n}$ if $|V_1| = m$ and $|V_2| = n$. A star S_n is a complete 2-partite graph with independent sets $V_1 = \{r\}$ and $V_2 = n$. the vertex r is called the root and the vertices in V_2 are called the leaves of $S_{n..}$ A wheel W_m is a graph on m+ 1 vertices obtained from a cycle on m vertices by adding a new vertex and edges joining it to all the vertices of the cycle (W_m is the join of K_1 and C_m). A kipas □$_m$ is a graph on m + 1 vertices obtained from the join of K_1 and P_m. A fan F_m is a graph on 2m+1 vertices obtained from m disjoint triangles (K_3s) by identifying precisely one vertex of every triangle (F_m is the join of K_1 and mK_2).

Let V(G) is the vertex set and E(G) is the edge set for the graph G=(V(G),E(G)) and □ is the compliment of G. G(S) denotes the subgraph for $S \leq V(G)$, induced by S in G and G-S = G[V(G) – S]. $N_s(V)$ is the set of neighbours of vertex V in S and $d_s(V) = | N_s(V) |$. when S = V(G) , then $N_v = N_G(V)$, $N_{[V]} = N_{(v)}$ U {V} and $d(V) = d_G(V)$. let the graph of order n is k_n and mk_n is the union of m vertex. F_ℓ is the fan of order $(2\ell + 1)$ and the join of k_1 and ℓk_2 , which is ℓ triangles sharing exactly one vertex. Where k_1 is the center of F_1. The Ramsey number R(F,H) is the simplest integer for two given graphs F and H such that for any graph G of order N , either the compliment of G contains H or G contains F. Burr[1] formulated a lower bound for a connected graph F of the order P , that is $R(F,H) \geq (P-1)(X(H) -1) + S(H)$, if $P \geq S(H)$, where X(H) is the chromatic number of H and the minimum count of vertices in class under vertex coloring is S(H) by X(H) colors. Noting that $X(K_n) = n$ and $S(K_n) = 1$ for the pair F_1 and K_n.

By Burr's lower bound, $R(F_\ell , K_n) \geq 2\ell(n-1) + 1$.For n=3, Gupta [2] showed that ,$R(F_\ell , K_3) = 4\ell+1$ for $\ell \geq 2$

For n=4 , Surahmat [3] showed that , $R(F_\ell , K_4) = 6\ell+1$ for $\ell \geq 3$

The conjecture for n=5 is to be confined in this paper.

G is a graph of order $8\ell+1$, $\ell \geq 5$, let us show that G contains an F_ℓ or □ contains K_5 . let us assume that G does not contain an F_ℓ or □ does not contains K_5. Let □ € V(G) , $d(v) \leq 2\ell - 1$ then G – N[v] is a graph of order at least $6\ell+1$ for $\ell \geq 3$, □ - N[v] contains a k_4 , it means that □ contains a k_5 which is a contradiction. If $d[v] \geq 2\ell + 3$, then the maximum matching M of G[N(v)] contains at least ℓ edges for otherwise □[N(v) – V(M)] will be a complete graph of order S, which means that G has an F_ℓ which is a contradiction. So , $2\ell \leq d(v) \leq 2\ell+2$.

Let us assume G contains a sub graph $H=K_{2\ell-1}$. Let V_0 € V(G) – V(H) such that $d_H(V_0) = \max\{ d_H(v)/V€ V(G) – V(H) \}$. Then G- (V(H) U $\{V_0\}$) is a graph of order $6\ell+1$.

As □ has no K_5 ,
$V(H) \cup \{V_0\} \leq \cup^4_{i=1} N(U_i)$
It means that $\max \{d_H (V_i) / 1 \leq i \leq 4 \} \geq [(2\ell -1) /4] \geq 3$.

If $d_H (V_0) \geq 4$, the V_i has two adjacent values in $N_H(V_0)$ U { V_0}, If $d_H(V_0) = 3$, then $d_H(V_i) \leq d_H(V_0) = 3$ for $1 \leq i \leq 4$, which means that $d_H(V_i) \geq 2$ and $N_H(V_i) \cap N_H(V_0) \neq \phi$. In either cases, $G[V(H) \cup \{V_0, V_i\}]$ contains an $F_\square$, a contradiction. So G does not contain $K_{2\square-1}$.

For $1 \leq i \leq 4$, set $X_i = \{ V/d_v(v) = I, V \in V(G)\}$

$\sum_{i=1}^{4} | X_i | = 8\square - 3$

$\sum_{i=1}^{4} i \, | X_i | = \sum_{i=1}^{4} d(U_i)$

$| X_i | \geq 8\square - 14 + | X_3 | + 2 | X_4 | \geq 8\square - 14.$

$X_{1i} = N_{X1}(V_i)$, $1 \leq i \leq 4$

As $\square$ has no K_5, $G[X_{1i} \cup \{ U_i \}]$ is a complete graph.
And as G has no $K_{2\square-1}$, $| X_{1i} \cup \{ U_i\} | \leq 2\square -2$ which means $|X_{1i}| \leq 2\square - 3$ for $1 \leq i \leq 4$.

Hence $|X_{1i}| = \sum_{i=1}^{4} | X_i | = 8\square - 12$ and $| X_3 | + 2 | X_4 | \leq 2$
So $| X_2 | \geq 7$.

As G contains no $K_{2\square-1}$, $U_i - N_{(y)} \neq \phi$. Then $G[U_i \cup U_j - N_{(y)}]$ is a complete graph.

G has no $F_\square$, V_i and U_j are complete graphs. And $d_{uj}(U) \leq 3$ for any $U \in V_i$, Such that $|V_j| \geq 2\square - 4$ and $\square \geq 5$. $dV_j(y) \geq | U_j |$, $| U_j - N_{(y)} | \geq (2\square - 4) - 3 \geq 3$.

Where U_j is pair wise vertex-disjoint.

If $| U_4 | = 2\square - 2$, then, $X_2 = U_1 \leq i \leq j \leq 4$ and $Y_{ij} = \phi$ which contradicts $| X_2 | \geq 7$.
So, $2\square - 4 \leq |U_4| \leq 2\square - 3$.

$| U_3 | + | U_4 | \geq 4\square - 6$ and $| U_4 | \leq 2\square - 3$

$\sum_{i=1}^{4} d(U_i) = 8\square + 8$, $d_{X2}(U_4) = 6$

Assume $N_{x2}(U_4) = \{ y_i / 1 \leq i \leq 6 \}$ and $\square$ contains no K_5, $G \{ N_{X2}(U_4) \}$ contains at least one edge.

$Y_1, Y_2 \in E(G)$

And as G has no $F_\square$, $G[\{ y_3, y_4, y_5, y_6 \}]$ contains no edge. As $\square$ has no K_5,

$| \{ y_3, y_4, y_5, y_6 \} \cap (N(U_1) \cup N(U_2)) | \geq 2$

Let us assume $\{ y_3, y_4 \} \leq N(U_1) \cup N(U_2)$

$dU_4(y_3) \geq 3$ and $dU_4(y_4) \geq 3$,

which means that $d_{X14}(y_3) \geq 2$ and $d_{X14}(y_4) \geq 2$

This shows that there exists U' and $U \in X_{14}$ such that $U' y_3$, $U'' y_4 \in E(G)$ and shows that $G[U_4 \cup \{y_1 y_2 y_3 y_4\}]$ has an $F_\square$ having U_4 as center, which is a contradiction.

This proves that $R(F_\square, K_5) = 8\square+1$ for $\square \geq 5$.

References

[1] S.A. Burr, Ramsey numbers involving graphs with long suspended paths, Journal of London Mathematical Society 24 (1981), 405–413.

[2] S.K. Gupta, L. Gupta and A. Sudan, On Ramsey numbers for fan-fan graphs, Journal of Combinatorics, Information & System Sciences 22 (1997), 85–93.

[3] Surahmat, E.T. Baskoro and H.J. Broersma, The Ramsey numbers of fans versus K4, Bulletin of the Institute of Combinatorics and its Applications 43 (2005), 96–102.

[4] S.P. Radziszowski, Small Ramsey numbers, The Electronic Journal of Combinatorics, (2011), DS1.13.
[5] Yanbo Zhang, Yaojun Chen, The Ramsey numbers of fans versus a complete graph of order five , Electronic journal of graph theory and Applications 2 (1) (2014), 66–69.
[6] D. Li, H. Lai, M. Zhan, Eulerian subgraphs and Hamilton-connected line graphs, Discrete Appl. Math. 145 (2005), 422–428.
[7] H. Fleischner, The square of every two-connected graph is hamiltonian, J. Combin. Theory Ser. B 16 (1974), 29–34.
[8] B. Li, H.J. Broersma and S. Zhang, Pairs of forbidden induced subgraphs for homogeneously traceable graphs, Discrete Math. 312 (2012), 2800-2818.
[9] M. Miller and J. Sir´an, Moore graphs and beyond: A survey of the degree/diameter problem, Electron. J. Combin. 20(2) (2013), DS14v2.
[10] P. Morillo, M.A. Fiol, J. F`abrega, The diameter of directed graphs associated to plane tesselations, Ars Combin. 20A(4) (1985), 17–27.
[11] J.L.A. Yebra, M.A. Fiol, P. Morillo, and I. Alegre, The diameter of undirected graphs associated to plane tesselations, Ars Combin. 20B(4) (1985), 159–171.