IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 10, Issue 5 Ver. VI (Sep-Oct. 2014), PP 52-57
www.iosrjournals.org

On Fuzzy Generalized b-connected Space in Fuzzy Topological Spaces on FuzzySet

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***Abstract*:** *In this paper we introduce and study fuzzy generalized b-connected spacein fuzzy Topological space on fuzzy sets and introduce some types of fuzzy (gp-connected,gs-connected,gα-connected and gsp–connected) space with some properties, relations andTheorems about this subject.*
Keywords:*fgb-separated sets, fgb-connected space.*

I. Introduction

The concept of fuzzy sets and fuzzy sets operation were first introduced by Zadeh [7] in 1965 The concepts of fuzzy topological space is study by Chang [1] in 1968.AndFuzzy generalized semi-Connected space is study by Fath Alla [3] in 2004 and fuzzyGeneralized semi pre-Connected space is study by Santhi, R.and D.Jayanthi[4] in 2012,in this paper we Introduce and study fuzzy generalized b–Connected space with some relations between them fuzzy (gp-connected,gs- connected,gα- connected and gsp – connected) space.

II. Basic Definitions

Definition (2.1): Let $(\tilde{A},\tilde{T})$ be a fuzzy topological space (f.t.s)and $\tilde{B},\tilde{C}$ are fuzzy sets in $\tilde{A}$, then $\tilde{B}$, and $\tilde{C}$ are said to be :

(1) Fuzzy gb – separated iff Min $\{\mu_{gbcl(\tilde{B})}(x), \mu_{\tilde{C}}(x)\} = 0$ and Min $\{\mu_{gbcl(\tilde{C})}(x), \mu_{\tilde{B}}(x)\} = 0$
(2) Fuzzy gα – separated iff Min $\{\mu_{g\alpha cl(\tilde{B})}(x), \mu_{\tilde{C}}(x)\} = 0$ and Min $\{\mu_{g\alpha cl(\tilde{C})}(x), \mu_{\tilde{B}}(x)\} = 0$,[6]
(3) Fuzzy gp – separated iff Min $\{\mu_{gpcl(\tilde{B})}(x), \mu_{\tilde{C}}(x)\} = 0$ and Min $\{\mu_{gpcl(\tilde{C})}(x), \mu_{\tilde{B}}(x)\} = 0$, [8]
(4) Fuzzy gs – separated iff Min $\{\mu_{gscl(\tilde{B})}(x), \mu_{\tilde{C}}(x)\} = 0$ and Min $\{\mu_{gscl(\tilde{C})}(x), \mu_{\tilde{B}}(x)\} = 0$, [3]
(5) Fuzzy gsp – separated iff Min $\{\mu_{gspcl(\tilde{B})}(x), \mu_{\tilde{C}}(x)\} = 0$ and Min $\{\mu_{gspcl(\tilde{C})}(x), \mu_{\tilde{B}}(x)\} = 0$

Definition (2.2)

1. A fuzzy topological space$(\tilde{A},\tilde{T})$is said to be fuzzy gα-connected if there is no proper non-empty maximal fuzzy gα-separated sets $\tilde{B}$ and $\tilde{C}$ in $\tilde{A}$ such that $\mu_{\tilde{A}}(x)$=Max $\{(\mu_{\tilde{C}}(x),(\mu_{\tilde{B}}(x)\}$. If $(\tilde{A},\tilde{T})$ is not fuzzy gα-connected then it is said to be fuzzy gα- disconnected space.[6]
2. A fuzzy topological space $(\tilde{A},\tilde{T})$ is said to be fuzzy gs-connected if there is no proper non-empty maximal fuzzy gs-separated sets $\tilde{B}$ and $\tilde{C}$ in $\tilde{A}$ such that $\mu_{\tilde{A}}(x)$=Max $\{(\mu_{\tilde{C}}(x),(\mu_{\tilde{B}}(x)\}$. If $(\tilde{A},\tilde{T})$ is not fuzzy gs-connected then it is said to be fuzzy
gs-disconnected space.[3]
3. A fuzzy topological space $(\tilde{A},\tilde{T})$ is said to be fuzzy gp-connected if there is no proper non-empty maximal fuzzy gp-separated sets $\tilde{B}$ and $\tilde{C}$ in $\tilde{A}$ such that $\mu_{\tilde{A}}(x)$=Max $\{(\mu_{\tilde{C}}(x),(\mu_{\tilde{B}}(x)\}$. If $(\tilde{A},\tilde{T})$ is not fuzzy gp-connected then it is said to be fuzzy gp- disconnected space.
4. A fuzzy topological space $(\tilde{A},\tilde{T})$is said to be fuzzy gsp-connected if there is no proper non-empty maximal fuzzy gsp-separated sets $\tilde{B}$ and $\tilde{C}$ in $\tilde{A}$ such that $\mu_{\tilde{A}}(x)$= Max $\{(\mu_{\tilde{C}}(x),(\mu_{\tilde{B}}(x)\}$. If $(\tilde{A},\tilde{T})$ is not fuzzy gsp-connected then it is said to be fuzzy gsp- disconnected space.[4]

Definition (2.3)[2]:A fuzzy set $\tilde{B}$ in $(\tilde{A},\tilde{T})$ is said to be fuzzy gb-clopen if and only if both fuzzy gb-open and fuzzy gb-closed set .

Definition (2.4)[3]:Let $\tilde{B}$ be a fuzzy set in$\tilde{A}$, then $\tilde{B}$ is said to be maximal fuzzy set in$\tilde{A}$
if for each $x \in X \mu_{\tilde{B}}(x) \neq 0$, then $\mu_{\tilde{B}}(x) = \mu_{\tilde{A}}(x)$

Proposition (2.5) [4]:Let $\tilde{B},\tilde{C}$ be any fuzzy sets in $(\tilde{A},\tilde{T})$, then

1. $\tilde{B}\, \tilde{\bar{q}}\, \tilde{C} \Rightarrow \mu_{\tilde{B}}(x) \leq \mu_{\tilde{C}^c}(x)$ or $\mu_{\tilde{C}}(x) \leq \mu_{\tilde{B}^c}(x)$.
2. $x_r\, \tilde{\bar{q}}\, \tilde{B} \Leftrightarrow \mu_{x_r}(x) \leq \mu_{\tilde{B}^c}(x)$.

3. $\tilde{B}\tilde{\bar{q}}\tilde{B}^c$, for any fuzzy set $\tilde{B}$ in $\tilde{A}$.
4. If Min $\{\mu_{\tilde{B}(x)}, \mu_{\tilde{C}(x)}\} = \mu_{\tilde{\emptyset}(x)}$, then $\mu_{\tilde{B}(x)} + \mu_{\tilde{C}(x)} \leq \mu_{\tilde{A}(x)}$.
5. Min $\{(\mu_{\tilde{C}}(x),(\mu_{\tilde{B}}(x)\} = \mu_{\tilde{\emptyset}}(x) \Rightarrow \mu_{\tilde{C}}(x) \leq \mu_{\tilde{B}}{}^c(x)$ or $\mu_{\tilde{B}}(x) \leq \mu_{\tilde{C}}{}^c(x)$.

Theorem (2.6)[6]: If $(\tilde{A},\tilde{T})$ is a f.t.s, $\tilde{B}$ and $\tilde{C}$ are fuzzy sets in $\tilde{A}$, then:

1. If $\tilde{B} \cap \tilde{C} = \tilde{\phi}$, $\tilde{B}$ and $\tilde{C}$ are fuzzy gb-closed sets in $\tilde{A}$, then $\tilde{B}$ and $\tilde{C}$ are fuzzy gb-separated in $\tilde{A}$.
2. If $\tilde{B} \cap \tilde{C} = \tilde{\phi}$, $\tilde{B}$ and $\tilde{C}$ are fuzzy gb-open sets in $\tilde{A}$, then $\tilde{B}$ and $\tilde{C}$ are fuzzy weak gb-separated in $\tilde{A}$.

Theorem (2.7)

1- If $(\tilde{A},\tilde{T})$ is a f.t.s and $\tilde{B},\tilde{C}$ are Fgα- separated in $\tilde{A}$ then $\tilde{B}$ and $\tilde{C}$ are Fgp– separated .
2- If $(\tilde{A},\tilde{T})$ is a f.t.s and $\tilde{B},\tilde{C}$ are Fgα- separated in $\tilde{A}$ then $\tilde{B}$ and $\tilde{C}$ are Fgs– separated .
3- If $(\tilde{A},\tilde{T})$ is a f.t.s and $\tilde{B},\tilde{C}$ are Fgα- separated in $\tilde{A}$ then $\tilde{B}$ and $\tilde{C}$ are Fgb– separated .
4- If $(\tilde{A},\tilde{T})$ is a f.t.s and $\tilde{B},\tilde{C}$ are Fgp- separated in $\tilde{A}$ then $\tilde{B}$ and $\tilde{C}$ are Fgb– separated .
5- If $(\tilde{A},\tilde{T})$ is a f.t.s and $\tilde{B},\tilde{C}$ are Fgs- separated in $\tilde{A}$ then $\tilde{B}$ and $\tilde{C}$ are Fgb– separated .
6- If $(\tilde{A},\tilde{T})$ is a f.t.s and $\tilde{B},\tilde{C}$ are Fgp- separated in $\tilde{A}$ then $\tilde{B}$ and $\tilde{C}$ are Fgsp– separated .
7- If $(\tilde{A},\tilde{T})$ is a f.t.s and $\tilde{B},\tilde{C}$ are Fgs- separated in $\tilde{A}$ then $\tilde{B}$and $\tilde{C}$ are Fgsp – separated .
8- If $(\tilde{A},\tilde{T})$ is a f.t.s and $\tilde{B},\tilde{C}$ are Fgb- separated in $\tilde{A}$ then $\tilde{B}$ and $\tilde{C}$ are Fgsp– separated .

Proof (1)

Since $\tilde{B}$ and $\tilde{C}$ are Fg α- separated in $\tilde{A}$, that implies, Min $\{\mu_{g\alpha cl(\tilde{B})}(x), \mu_{\tilde{C}}(x)\}=0$ and Min$\{\mu_{g\alpha cl(\tilde{C})}(x), \mu_{\tilde{B}}(x)\}=0$.
Since gαcl($\tilde{B}$) and gαcl($\tilde{C}$) are Fgα- closed sets in $\tilde{A}$. Then
Min $\{\mu_{gpcl(\tilde{B})}(x), \mu_{\tilde{C}}(x)\} = 0$ and Min $\{\mu_{gpcl(\tilde{C})}(x), \mu_{\tilde{B}}(x)\} = 0$
Hence, $\tilde{B}$ and $\tilde{C}$ are Fgp- separated in $\tilde{A}$. □

The proof(2,3,4,5,6,7,8) is similar to that of (1) theorem (2.7)

III. Some Properties of Fuzzy Generalized b-ConnectedSpace

Definition (3.1):

1.A fuzzy topological space $(\tilde{A},\tilde{T})$ is said to be fuzzy gb-connected if there is no proper non-empty maximal fuzzy gb-separated sets $\tilde{B}$and$\tilde{C}$in$\tilde{A}$suchthat $\mu_{\tilde{A}}(x)$=Max $\{(\mu_{\tilde{C}}(x),(\mu_{\tilde{B}}(x)\}$. If $(\tilde{A},\tilde{T})$ is not fuzzy gb-connected then it is said to be fuzzy gb-disconnected space.
2.A fuzzy topological space $(\tilde{A},\tilde{T})$ is said to be fuzzy weak gb-connected fuzzy if there is no proper non-empty maximal fuzzy weak gb-separated sets $\tilde{B}$ and $\tilde{C}$in $\tilde{A}$ such that $\mu_{\tilde{A}}(x)$=Max $\{(\mu_{\tilde{C}}(x),(\mu_{\tilde{B}}(x)\}$. If $(\tilde{A},\tilde{T})$ is not fuzzy weak gb-connected is said to be fuzzy weak gb-disconnected space.[3]

Definition (3.2) [4],[5]

A fuzzy topological space $(\tilde{A},\tilde{T})$is said to be fuzzy generalized b-connected(denoted by Fgb- connected) if and only if the fuzzy sets which are both Fgb-open and Fgb-closed.

Proposition (3.3): Every fuzzy gb-connected space $(\tilde{A},\tilde{T})$is fuzzy weak gb-connected space.

Proof:

Suppose that $\tilde{B}$ and $\tilde{C}$ be proper non-empty maximal fuzzy gb-separated sets in $\tilde{A}$
Implies that, $\tilde{B}$ and $\tilde{C}$are proper non-empty maximal fuzzy weak gb-separated sets in$\tilde{A}$
Since $(\tilde{A},\tilde{T})$is fuzzy gb-connected space
Then, $\mu_{\tilde{A}}(x) \neq$Max $\{(\mu_{\tilde{C}}(x),(\mu_{\tilde{B}}(x)\}$.
Therefore, $(\tilde{A},\tilde{T})$is a fuzzy weak gb-connected space. ■

Remark 3.4: The converse of proposition (3.3) is not true in general.

Example 3.5 : Let X = {a,b} and $\tilde{A}$, $\tilde{B}$,$\tilde{C}$, $\tilde{D}$, $\tilde{E}$, $\tilde{F}$ are fuzzy sets defined as follows:
$\tilde{A}$= {(a,0.3), (b,0.5)},$\tilde{B}$= {(a,0), (b,0.3)},$\tilde{C}$= {(a,0.3),(b,0)}, $\tilde{D}$ ={(a,0.3),(b,0.3)}, $\tilde{E}$ ={(a,0.2),(b,0.2)}, $\tilde{F}$ = {(a,0), (b,0.1)}.
Let $\tilde{T} = \{\tilde{\phi},\tilde{A},\tilde{B},\tilde{C},\tilde{D}\}$ be a fuzzy topology on $\tilde{A}$, then $\tilde{E}$ and $\tilde{F}$ are fuzzy weak gb-connected space but not fuzzy gb-connected space.

Theorem (3.6)

1- If $(\tilde{A},\tilde{T})$ a f.t.s, $\tilde{B}$and$\tilde{C}$ are fuzzy gb-connected in $\tilde{A}$, then $\tilde{B}$and$\tilde{C}$ are fuzzy gα-connected space.
2- If $(\tilde{A},\tilde{T})$ a f.t.s, $\tilde{B}$and$\tilde{C}$ are fuzzy gb-connected in $\tilde{A}$, then $\tilde{B}$and$\tilde{C}$ are fuzzy gs-connected space.

3- If (Ã,T̃) a f.t.s, B̃andC̃ are fuzzy gb-connected in Ã, then B̃andC̃ are fuzzy gp-connected space.
4- If (Ã,T̃) a f.t.s, B̃andC̃ are fuzzy gs-connected in Ã, then B̃andC̃ are fuzzy gα-connected space.
5- If (Ã,T̃) a f.t.s, B̃andC̃ are fuzzy gp-connected in Ã, then B̃andC̃ are fuzzy gα-connected space.
6- If (Ã,T̃) a f.t.s, B̃andC̃ are fuzzy gsp-connected in Ã, then B̃andC̃ are fuzzy gb-connected space.
7- If (Ã,T̃) a f.t.s, B̃andC̃ are fuzzy gsp-connected in Ã, then B̃andC̃ are fuzzy gs-connected space.
8- If (Ã,T̃) a f.t.s, B̃andC̃ are fuzzy gsp-connected in Ã, then B̃andC̃ are fuzzy gp-connected space.

Proof (1) : Let (Ã,T̃) is fuzzy gb-connected space
Suppose that (Ã,T̃) is fuzzy gα-disconnected space
Then this implies that there exist non-empty maximal fuzzy gα-separated sets B̃andC̃ in Ã such that Ã=B̃∪C̃.Then by theorem (2.7) there exist non-empty maximal fuzzy gb-separated sets B̃andC̃ in Ã such that Ã=B̃∪C̃. Implies that (Ã,T̃) is fuzzy gb-disconnected which is a contradiction
Hence (Ã,T̃) is fuzzy gα-connected space. ■
The Proof(2,3,4,5,6,7,8,9) is similar to that of (1) theorem (3.6).

Remark(3.7): The converse of (1) of theorem (3.6) is not true in general as shown by the following example.
Example (3.8): Let X = {a,b} and Ã,B̃,C̃,D̃,Ẽ are fuzzy sets defined as follows:
Ã={(a,0.8),(b,0.8)}, B̃={(a,0.6),(b,0.6)}, C̃={(a,0.6),(b,0.8)}, D̃={(a,0.8),(b,0.6)}Ẽ={(a,0.0),(b,0.8)}, F̃={(a,0.8) ,(b,0.0)}, let τ̃ ={ϕ̃ , Ã, B̃,C̃,D̃} be fuzzy topology on Ã and the FGαO(Ã) ={ϕ̃ ,Ã,B̃,C̃, D̃,Ẽ,F̃}, then (Ã, τ̃) is a fuzzy gα-connected space but not fuzzy gb-connected space.

Remark (3.9): The converse of (2) of theorem (3.6) is not true in general as shown by the following example.
Example (3.10): Let X={a,b}, and Ã,B̃,C̃,D̃,Ẽ, F̃ are fuzzy sets defined as follows:
Ã={(a,0.7),(b,0.7)}, B̃={(a,0.7),(b,0.3)}, C̃={(a,0.3),(b,0.7)},D̃={(a,0.3),(b,0.3)}, Ẽ ={(a,0.7),(b,0.0)}, F̃={(a,0.0) ,(b,0.7)} be a fuzzy sets in Ã,
τ̃ ={ϕ̃ ,Ã,B̃,C̃, D̃} be afuzzy topology on Ã and the FGSO(Ã)={ϕ̃ ,Ã,B̃,C̃,D̃,Ẽ,F̃}then (Ã, τ̃) is a fuzzy gs-connected space but not fuzzy gb-connected space.

Remark (3.11): The converse of (3) of theorem (3.6) is not true in general as shown by the following example.
Example (3.12): Let X={a,b}and Ã,$\tilde{B}_1$, $\tilde{B}_2$,…,$\tilde{B}_{17}$ are fuzzy sets defined as follows:
Ã={(a,0.7),(b,0.9)},$\tilde{B}_1$={(a,0.1),(b,0.9)},$\tilde{B}_2$={(a,0.7),(b,0.1)},$\tilde{B}_3$={(a,0.1),(b,0.1)}, $\tilde{B}_4$={(a,0.0),(b,0.8)}, $\tilde{B}_5$={(a,0.5),(b,0.0)},$\tilde{B}_6$={(a,0.5),(b,0.9)},$\tilde{B}_7$={(a,0.1),(b,0.7)},$\tilde{B}_8$={(a,0.0),(b,0.1)}, $\tilde{B}_9$={(a,0.1),(b,0.0)},$\tilde{B}_{10}$={(a,0.7),(b,0.8)}, $\tilde{B}_{11}$={(a,0.1),(b,0.8)}, $\tilde{B}_{12}$={(a,0.0),(b,0.7)}, $\tilde{B}_{13}$={(a,0.5),(b,0.1)}, $\tilde{B}_{14}$={(a,0.7),(b,0.7)}, $\tilde{B}_{15}$={(a,0.5),(b,0.8)}, $\tilde{B}_{16}$={(a,0.0),(b,0.9)}, $\tilde{B}_{17}$={(a,0.7),(b,0.0)},Be a fuzzy sets in Ã , τ̃ ={ϕ̃ ,Ã,$\tilde{B}_1$,$\tilde{B}_2$,$\tilde{B}_3$,$\tilde{B}_4$,$\tilde{B}_5$,$\tilde{B}_6$,$\tilde{B}_7$,$\tilde{B}_8$,$\tilde{B}_9$,$\tilde{B}_{10}$,$\tilde{B}_{11}$,$\tilde{B}_{12}$,$\tilde{B}_{13}$, $\tilde{B}_{14}$,$\tilde{B}_{15}$} be a fuzzy topology on Ã, the FGPO(Ã)={ϕ̃ ,Ã,$\tilde{B}_1$,$\tilde{B}_2$,$\tilde{B}_3$,$\tilde{B}_4$,$\tilde{B}_5$,$\tilde{B}_6$,$\tilde{B}_7$,$\tilde{B}_8$,$\tilde{B}_9$,$\tilde{B}_{10}$,$\tilde{B}_{11}$,$\tilde{B}_{12}$,$\tilde{B}_{13}$, $\tilde{B}_{15}$, $\tilde{B}_{16}$, $\tilde{B}_{17}$}, then (Ã, τ̃) is a fuzzy gp - connected space but not fuzzy gb-connected space.

Remark (3.13): The converse of (4) of theorem (3.6) is not true in general as shown by the following example.
Example (3.14): Let X = {a,b} and Ã,B̃,C̃,D̃,Ẽ,F̃ are fuzzy sets defined as follows:
Ã={(a,0.5),(b,0.5)}, B̃={(a,0.3),(b,0.5)}, C̃={(a,0.5),(b,0.3)}, D̃={(a,0.3),(b,0.3)}, Ẽ={(a,0.5),(b,0.0)}, F̃={(a,0.0),(b,0.5)}, let T̃={Ã,Ø̃,B̃,C̃,D̃} be fuzzy Topology on Ãand the FGαO(Ã)={ϕ̃ ,Ã,B̃,C̃,D̃,Ẽ,F̃} , then (Ã , τ̃) is a fuzzy gα-connected space but not fuzzy gs-connected space.

Remark (3.15): The converse of (5) of theorem (3.6) is not true in general as shown by the following example.
Example (3.16): Let X={a,b}, and Ã,B̃,C̃,D̃,Ẽ,F̃,are fuzzy sets defined as follows:
Ã{(a,0.6),(b,0.6)}, B̃={(a,0.3),(b,0.6)}, C̃={(a,0.6),(b,0.3)}, D̃={(a,0.3),(b,0.3)}, Ẽ={(a,0.6),(b,0.0)}, F̃={(a,0.0) ,(b,0.6)}be a fuzzy sets in Ã, τ̃ ={ϕ̃ ,Ã,B̃,C̃,D̃} be a fuzzy topology on Ã and the FGαO(Ã)={ϕ̃ ,Ã,B̃,C̃,D̃,Ẽ,F̃}, then (Ã, τ̃) is a fuzzy gα-connected space but not fuzzy gp-connected space.

Remark (3.17): The converse of (6) of theorem (3.6) is not true in general as shown by the following example.
Example (3.18): Let X={a,b} and Ã,B̃,C̃,D̃,Ẽ,F̃ are fuzzy sets defined as follows:
Ã={(a,0.7),(b,0.9)},$\tilde{B}_1$={(a,0.1),(b,0.9)},$\tilde{B}_2$={(a,0.7),(b,0.1)},
$\tilde{B}_3$={(a,0.1),(b,0.1)},$\tilde{B}_4$={(a,0.0),(b,0.8)},$\tilde{B}_5$={(a,0.7),(b,0.0)},$\tilde{B}_6$={(a,0.7),(b,0.7)},$\tilde{B}_7$={(a,0.1),(b,0.7)},$\tilde{B}_8$={(a,0

.0),(b,0.1)},B̃₉={(a,0.1),(b,0.0)},B̃₁₀={(a,0.0),(b,0.1)},B̃₁₁={(a,0.7),(b,0.8)},B̃₁₂={(a,0.1),(b,0.8)},B̃₁₃={(a,0.0),(b ,0.7)}, B̃₁₄={(a,0.0),(b,0.9)}, B̃₁₅={(a,0.7),(b,0.0)},

letbe $\tilde{\tau}$={$\tilde{\phi}$,Ã,B̃₁,B̃₂,B̃₃,B̃₄,B̃₅,B̃₆,B̃₇,B̃₈,B̃₉,B̃₁₀,B̃₁₁,B̃₁₂,B̃₁₃} be a fuzzy topology on Ãand the FGBO(Ã)={$\tilde{\phi}$,Ã,B̃₁,B̃₂,B̃₃,B̃₄,B̃₅,B̃₆,B̃₇,B̃₈,B̃₉,B̃₁₀,B̃₁₁,B̃₁₂,B̃₁₃,B̃₁₅} Then (Ã,$\tilde{\tau}$) is a fuzzy gb-connected space but not fuzzy gsp-connected space.

Remark (3.19): The converse of (7) of theorem (3.6) is not true in general as shown by the following example.
Example (3.20):Ã ={(a,0.7),(b,0.7)},B̃ ={(a,0.4),(b,0.7)}, C̃={(a,0.7),(b,0.4)}, D̃={(a,0.4),(b,0.4)}, Ẽ={(a,0.0),(b,0.2)}, F̃={(a,0.2) ,(b,0.0)},G̃={(a,0.2),(b,0.2)},H̃ ={(a,0.0),(b,0.7)},Ĩ={(a,0.7),(b,0.0)}, Let $\tilde{\tau}$={$\tilde{\phi}$,Ã, B̃, C̃, D̃, Ẽ, F̃,G̃} be fuzzy topology on Ã and the FGSO(Ã)={$\tilde{\phi}$,Ã,B̃,C̃,D̃,Ẽ,F̃,G̃,H̃,Ĩ}, then (Ã ,$\tilde{\tau}$) is a fuzzy gs-connected space but not fuzzy gsp-connected space.

Remark (3.21): The converse of (8) of theorem (3.6) is not true in general as shown by the following example.
Example (3.22): The space (Ã ,$\tilde{\tau}$) in the example 3.12 is a fuzzy gp-connected space but not fuzzy gsp-connected space.

We will explain the relationship between of some types of fuzzy generalized connected in fuzzy topological space on fuzzy set by fig. (1)

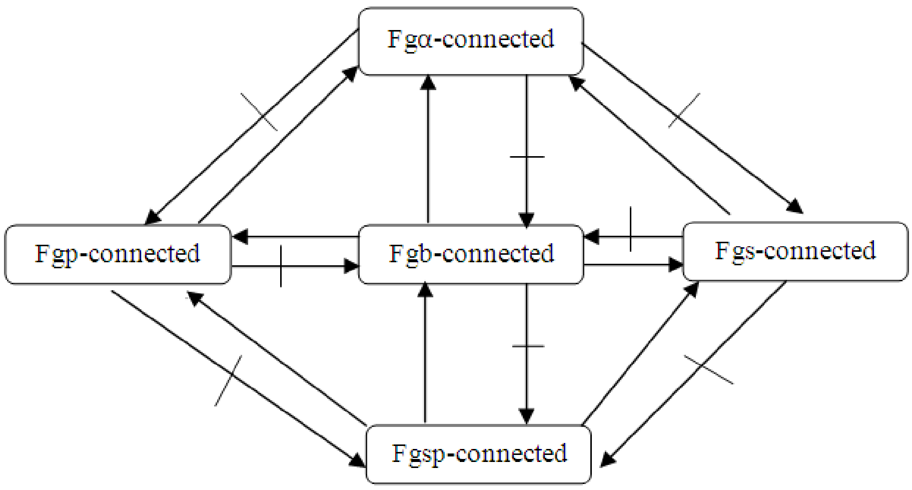

Fig.(1)

Theorem (3.23):A f.t.s(Ã,T̃)is fuzzy gb-connected if and only if there exist nonon-empty fuzzy gb-closedsets ẼandF̃inÃ, such that $\mu_{\tilde{A}}$(x)=Max{($\mu_{\tilde{E}}$(x),$\mu_{\tilde{F}}$(x)}and Min{($\mu_{\tilde{E}}$(x),$\mu_{\tilde{F}}$(x)}=0.
Proof:
(⇒) Suppose that (Ã,T̃)is fuzzy gb-connected space.
Suppose that there exists non-empty fuzzy gb-closed sets Ẽ andF̃inÃ, such that
$\mu_{\tilde{A}}$(x)=Max{($\mu_{\tilde{E}}$(x),$\mu_{\tilde{F}}$(x)}and Min{($\mu_{\tilde{E}}$(x),$\mu_{\tilde{F}}$(x)} =0.
Since ẼandF̃are fuzzy gb-closed sets in Ã and Min{($\mu_{\tilde{E}}$(x),$\mu_{\tilde{F}}$(x)} = 0 .
Implies that by theorem (2.6) ẼandF̃ are fuzzy gb-separated sets in Ã
Since, $\mu_{\tilde{A}}$(x) =Max{($\mu_{\tilde{E}}$(x),$\mu_{\tilde{F}}$(x)}
Then (Ã,T̃) is fuzzy gb-disconnected, which is a contradiction.
(⇐) Suppose that there exists no non-empty fuzzy s-closed sets Ẽ andF̃inÃ, such that $\mu_{\tilde{A}}$(x)=Max{($\mu_{\tilde{E}}$(x),$\mu_{\tilde{F}}$(x)}and Min{($\mu_{\tilde{E}}$(x),$\mu_{\tilde{F}}$(x)} =0 .
Suppose that (Ã,T̃) is fuzzy gb -disconnected space
Then this implies that there exist non-empty maximal fuzzy gb -separated sets B̃ and C̃ in Ã, such that$\mu_{\tilde{A}}$ (x) =Max {$\mu_{\tilde{B}}$(x) ,$\mu_{\tilde{C}}$(x)}
Since B̃ and C̃ are fuzzy gb-separated sets in Ã
Implies that, Min {$\mu_{gbcl(\tilde{B})}$(x) ,$\mu_{\tilde{C}}$(x)} = 0 and Min {$\mu_{gbcl(\tilde{C})}$(x) ,$\mu_{\tilde{B}}$(x)} = 0
Then, $\mu_{\tilde{C}}$(x)≤ [$\mu_{gbcl(\tilde{B})}$(x)]^{c}and $\mu_{\tilde{B}}$(x) ≤ [$\mu_{gbcl(\tilde{C})}$(x)]^c
Since $\mu_{\tilde{A}}$(x) =Max {$\mu_{\tilde{B}}$(x) ,$\mu_{\tilde{C}}$(x)} ≤Max{ [$\mu_{gbcl(\tilde{B})}$(x)]^c ,[$\mu_{gbcl(\tilde{C})}$(x)]^c
Implies that, $\mu_{\tilde{A}}$(x) =Max{ [$\mu_{gbcl(\tilde{B})}$(x)]^c , [$\mu_{gbcl(\tilde{C})}$(x)]^c

Then:
$\mu_{\tilde{A}}(x) = \text{Min}\{ [\mu_{gbcl(\tilde{B})}(x)]^c , [\mu_{gbcl(\tilde{C})}(x)]^c$
$\mu_{\tilde{A}}{}^c(x) = \text{Min}\ \{\mu_{gbcl(\tilde{B})}(x) , \mu_{gbcl(\tilde{C})}(x)\}$
$\mu_{\tilde{\emptyset}}(x) = \text{Min}\ \{\mu_{gbcl(\tilde{B})}(x) , \mu_{gbcl(\tilde{C})}(x)$
Let gb-cl($\tilde{C}$) =$\tilde{E}$ and gb-cl($\tilde{B}$) =$\tilde{F}$
Then, $\text{Min}\{(\mu_{\tilde{E}}(x),\mu_{\tilde{F}}(x)\} = \mu_{\tilde{\emptyset}}(x)$
$\text{Max}\{(\mu_{\tilde{E}}(x),\mu_{\tilde{F}}(x)\} = \text{Max}\{ \mu_{gbcl(\tilde{C})}(x),\mu_{gbcl(\tilde{B})}(x) \}$
$\leq \mu_{gbcl(\max\{\mu_{\tilde{B}}(x),\ \mu_{\tilde{C}}(x)\})}(x)$
$= \mu_{gbcl(\tilde{A})}(x) = \tilde{A}$
Implies that, $\text{Max}\{(\mu_{\tilde{E}}(x),\mu_{\tilde{F}}(x)\} = \mu_{\tilde{A}}(x)$, which is a contradiction.
Hence, ($\tilde{A}$,$\tilde{T}$) is fuzzy gb-connected space. ■

Corollary (3.24) :A f.t.s ($\tilde{A}$,$\tilde{T}$) is fuzzy gb-connected if and only if there exist no non-empty fuzzy gb-open sets $\tilde{G}$ and $\tilde{H}$ in $\tilde{A}$, such that$\mu_{\tilde{A}}(x)=\text{Max}\{(\mu_{\tilde{G}}(x),\mu_{\tilde{H}}(x)\}$ and$\text{Min}\{(\mu_{\tilde{G}}(x),\mu_{\tilde{H}}(x)\} = \mu_{\tilde{\emptyset}}(x)$
Proof:
($\Rightarrow$) Suppose that ($\tilde{A}$,$\tilde{T}$)is fuzzy gb-connected space.
Suppose that there exists non-empty fuzzy gb-open sets $\tilde{G}$ and $\tilde{H}$ in $\tilde{A}$, such that $\mu_{\tilde{A}}(x)=\text{Max}\{(\mu_{\tilde{G}}(x),\mu_{\tilde{H}}(x)\}$ and $\text{Min}\{(\mu_{\tilde{G}}(x),\mu_{\tilde{H}}(x)\} = \mu_{\tilde{\emptyset}}(x)$
Then, $\mu_{\tilde{A}}{}^c(x)=\text{Min}\{(\mu_{\tilde{G}}{}^c(x),\mu_{\tilde{H}}{}^c(x)\}$ and $\text{Max}\{(\mu_{\tilde{G}}{}^c(x),\mu_{\tilde{H}}{}^c(x)\} = \mu_{\tilde{\emptyset}}{}^c(x)$
Implies that, $\mu_{\tilde{\emptyset}}(x)=\text{Min}\{(\mu_{\tilde{G}}{}^c(x),\mu_{\tilde{H}}{}^c(x)\}$ and $\text{Max}\{(\mu_{\tilde{G}}{}^c(x),\mu_{\tilde{H}}{}^c(x)\} = \mu_{\tilde{A}}(x)$
Let$\tilde{G}^c$=$\tilde{E}$ and$\tilde{H}^c$=$\tilde{F}$,then there exist a non-empty fuzzy gb-closed sets $\tilde{E}$ and$\tilde{F}$in$\tilde{A}$, such that
$\mu_{\tilde{A}}(x) = \text{Max}\{(\mu_{\tilde{E}}(x),\mu_{\tilde{F}}(x)\}$and $\text{Min}\{(\mu_{\tilde{E}}(x),\mu_{\tilde{F}}(x)\} = \mu_{\tilde{\emptyset}}(x)$
Then by theorem (3.23) ($\tilde{A}$,$\tilde{T}$)is fuzzy gb-disconnected space, which is a contradiction
($\Leftarrow$) Suppose that there exist no non-empty fuzzy s-open sets $\tilde{G}$ and $\tilde{H}$ in $\tilde{A}$, such that $\mu_{\tilde{A}}(x)=\text{Max}\{(\mu_{\tilde{G}}(x),\mu_{\tilde{H}}(x)\}$ and $\text{Min}\{(\mu_{\tilde{G}}(x),\mu_{\tilde{H}}(x)\} = \mu_{\tilde{\emptyset}}(x)$.
Suppose that ($\tilde{A}$,$\tilde{T}$)is fuzzy gb-disconnected space then there exists non-empty maximal fuzzy gb-separated sets$\tilde{B}$ and $\tilde{C}$ in $\tilde{A}$, such that $\mu_{\tilde{A}}(x) = \text{Max}\ \{\mu_{\tilde{B}}(x) ,\mu_{\tilde{C}}(x)\}$
Since $\tilde{B}$ and $\tilde{C}$ are fuzzy gb-separated sets in $\tilde{A}$
Implies that $\text{Min}\ \{\mu_{gbcl(\tilde{B})}(x) ,\mu_{\tilde{C}}(x)\} = 0$ and $\text{Min}\ \{\mu_{gbcl(\tilde{C})}(x) ,\mu_{\tilde{B}}(x)\} = 0$
Then, $\mu_{\tilde{C}}(x) \leq [\mu_{gbcl(\tilde{B})}(x)]^c$and $\mu_{\tilde{B}}(x) \leq [\mu_{gbcl(\tilde{C})}(x)]^c$
Since $\mu_{\tilde{A}}(x) = \text{Max}\ \{\mu_{\tilde{B}}(x) ,\mu_{\tilde{C}}(x)\} \leq \text{Max}\{ [\mu_{gbcl(\tilde{B})}(x)]^c , [\mu_{gbcl(\tilde{C})}(x)]^c$
Then $\mu_{\tilde{A}}(x) = \text{Max}\{ [\mu_{gbcl(\tilde{B})}(x)]^c , [\mu_{gbcl(\tilde{C})}(x)]^c$
Let [gbcl($\tilde{B}$)]^c=$\tilde{G}$ and [gbclcl($\tilde{C}$)]^c=$\tilde{H}$
Implies that $\mu_{\tilde{A}}(x)=\text{Max}\{(\mu_{\tilde{G}}(x),\mu_{\tilde{H}}(x)\}$
$\text{Min}\{(\mu_{\tilde{G}}(x),\mu_{\tilde{H}}(x)\}=\text{Min}\{ [\mu_{gbcl(\tilde{B})}(x)]^c , [\mu_{gbcl(\tilde{C})}(x)]^c$
$= [\text{Max}\{ [\mu_{gbcl(\tilde{B})}(x)] , [\mu_{gbcl(\tilde{C})}(x)]\}]^c$
$\leq [\mu_{gbcl(\max\{\mu_{\tilde{B}}(x),\ \mu_{\tilde{C}}(x)\})}(x)]^c$
$=[\mu_{gbcl(\tilde{A})}(x)]^c = \mu_{\tilde{A}}{}^c(x) = \mu_{\tilde{\emptyset}}(x)$
Which is a contradiction.
Hence, ($\tilde{A}$,$\tilde{T}$)is fuzzy gb-connected space.
Corollary (3.25):A f.t.s ($\tilde{A}$,$\tilde{T}$)is fuzzy weak gb-connected if and only if there exist no non-empty fuzzy gb-closed sets $\tilde{E}$ and$\tilde{F}$in$\tilde{A}$, such that $\mu_{\tilde{A}}(x) = \text{Max}\{(\mu_{\tilde{E}}(x),\mu_{\tilde{F}}(x)\}$and $\text{Min}\{(\mu_{\tilde{E}}(x),\mu_{\tilde{F}}(x)\} = \mu_{\tilde{\emptyset}}(x)$
Proof :Obvious . ■
Corollary (3.26):A f.t.s ($\tilde{A}$,$\tilde{T}$)is fuzzy weak gb-connected if and only if there exist no non-empty fuzzy s-open sets $\tilde{G}$ and $\tilde{H}$ in $\tilde{A}$, such that $\mu_{\tilde{A}}(x)=\text{Max}\{(\mu_{\tilde{G}}(x),\mu_{\tilde{H}}(x)\}$ and $\text{Min}\{(\mu_{\tilde{G}}(x),\mu_{\tilde{H}}(x)\} = \mu_{\tilde{\emptyset}}(x)$
Proof:Obvious . ■
Theorem (3.27):A f.t.s ($\tilde{A}$,$\tilde{T}$) is fuzzy gb-connected if and only if has no proper non-empty maximal fuzzy gb-clopen set in $\tilde{A}$.
Proof:
($\Rightarrow$) Suppose that ($\tilde{A}$,$\tilde{T}$)is fuzzy gb-connected space. Suppose that ($\tilde{A}$,$\tilde{T}$) has a proper non-empty maximal fuzzy gb-clopen set $\tilde{B}$ in $\tilde{A}$
Since $\tilde{B}$is fuzzy gb-clopen set in $\tilde{A}$
Implies that $\tilde{B}$,$\tilde{B}^c$ are fuzzy gb-closed sets in $\tilde{A}$
Since $\tilde{B}$ is maximal fuzzy set
Then $\mu_{\tilde{A}}(x)=\text{Max}\ \{\mu_{\tilde{B}}(x) ,\mu_{\tilde{B}}{}^c(x)\}$, $\text{Min}\ \{\mu_{\tilde{B}}(x) ,\mu_{\tilde{B}}{}^c(x)\} = \mu_{\tilde{\emptyset}}(x)$
($\Rightarrow$) by theorem (3.23) ($\tilde{A}$,$\tilde{T}$) is fuzzy gb-disconnected space, which is a contradiction.

(⇐) Suppose that (Ã,T̃) has no proper non-empty maximal fuzzy gb-clopen set in Ã. Suppose that (Ã,T̃) is fuzzy gb-disconnected space. Implies that, by corollary (3.24) there exist non-empty fuzzy gb-open sets G̃ and H̃ in Ã, such that Min{($\mu_{\tilde{G}}(x)$,$\mu_{\tilde{H}}(x)$)}= $\mu_{\tilde{\emptyset}}(x)$. Since Min{($\mu_{\tilde{G}}(x)$,$\mu_{\tilde{H}}(x)$)}= $\mu_{\tilde{\emptyset}}(x)$.
Implies that by proposition (2.5) G̃$\tilde{\bar{q}}$H̃
Since G̃ and H̃ are maximal fuzzy sets in Ã
Implies that:

$$\mu_{\tilde{G}}(x)+\mu_{\tilde{H}}(x)=\mu_{\tilde{A}}(x)$$

$$\mu_{\tilde{G}}(x)=\mu_{\tilde{A}}(x)-\mu_{\tilde{H}}(x)$$

Hence, G̃=$\tilde{H}^c$
Implies that G̃ is fuzzy gb-clopen set in Ã, which is a contradiction. ■

Corollary (3.28):A f.t.s (Ã,T̃) is fuzzy weak gb-connected if and only if there is no proper non-empty maximal fuzzy gb-clopen set in Ã.
Proof: Obvious.

Theorem (3.29):A f.t.s (Ã,T̃) is fuzzy gb-connected if and only if it has no non-empty maximal fuzzy gb-open sets $\tilde{B}_1$ and$\tilde{B}_2$such that $\mu_{\tilde{A}}(x)$=Max{($\mu_{\tilde{G}}(x)$,$\mu_{\tilde{H}}(x)$)}and Min{($\mu_{\tilde{G}}(x)$,$\mu_{\tilde{H}}(x)$)}= $\mu_{\tilde{\emptyset}}(x)$
$\mu_{\tilde{B}_1}(x)+\mu_{\tilde{B}_2}(x)=\mu_{\tilde{A}}(x)$, for all x ∈ X

Proof:
Ã, B̃, C̃, D̃, Ẽ, F̃,G̃,H̃
(⇒) Suppose that (Ã,T̃)is fuzzy gb-connected space. Suppose that $\tilde{B}_1$ and $\tilde{B}_2$ exist
Since $\mu_{\tilde{B}_1}(x)+\mu_{\tilde{B}_2}(x)=\mu_{\tilde{A}}(x)$
Then, $\mu_{\tilde{B}_1}(x)=\mu_{\tilde{A}}(x)-\mu_{\tilde{B}_2}(x)=\mu_{\tilde{B}_2^c}(x)$
Hence, $\tilde{B}_1$is fuzzy gb-clopen set in Ã.
Implies that by theorem (3.27) (Ã,T̃)is fuzzy gb-disconnected, which is a contradiction.
(⇐) Suppose that (Ã,T̃)has no non-empty fuzzy gb-open sets $\tilde{B}_1$ and $\tilde{B}_2$, such that $\mu_{\tilde{B}_1}(x)+\mu_{\tilde{B}_2}(x)=\mu_{\tilde{A}}(x)$
Suppose that (Ã,T̃)is fuzzy gb-disconnected space
Then by theorem (3.27)(Ã,T̃) has a proper fuzzy set $\tilde{B}_1$which is both fuzzy gb-open set and fuzzy gb-closed set.
Let$\tilde{B}_2$=$\tilde{B}_1^c$
Implies that$\tilde{B}_2$isfuzzy gb-openset, such that $\tilde{B}_2 \neq \tilde{\phi}$,and $\mu_{\tilde{B}_2}(x)=\mu_{\tilde{A}}(x)-\mu_{\tilde{B}_1}(x)$ Then $\mu_{\tilde{B}_1}(x)+\mu_{\tilde{B}_2}(x)=\mu_{\tilde{A}}(x)$, which is a contradiction. ■

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