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On fuzzy generalized b- closed set in fuzzy topological spaces on fuzzy Sets

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Abstract: *In this paper we introduce and study fuzzy generalized b - closed set in fuzzy Topological spaces on fuzzy set with some properties on them and study fuzzy (gp-closed ,gs –closed ,ga-closed and gsp –closed) setes with some theorems and Some relations between them in fuzzy topological spaces on fuzzy sets .*
Keywords: *fuzzy b - closed set,fuzzy p - closed set,fuzzy α - closed set,fuzzy s- closed set,fuzzy sp- closed set,fuzzy generalized p - closed set,fuzzy generalized b - closed set .*

I. Introduction

The concepts of fuzzy sets was introduced by Zadeh in [10] in 1965.
The concepts of fuzzy topological space by chang in [2] in 1968.
The concepts of fuzzy generalized b – closed is study by Benchalli and Jenifer [1] and
The concepts of fuzzy generalized pre – closed sets **,**fuzzy generalized semi – closed sets **,** Were studied by Murugesan and Thangavelu [5] **.** The concepts generalied α -- closed Sets **,** fuzzy generalied semi – pre closed sets were studied by R.KSARAF [8] **,**Murugsan [5] respectively **.**

II. Basic Defintions

Defintion 2.1,[4],[9]

A collection $\tilde{T}$ of fuzzy subset of $\tilde{A}$, that is $\tilde{T} \subseteq P(\tilde{A})$ is said to be fuzzy topology on $\tilde{A}$ if satisfied the following conditions :

- $\tilde{\emptyset}, \tilde{A} \in \tilde{T}$.
- If $\tilde{G}, \tilde{U} \in \tilde{T}$,then $\min \{\mu_{\tilde{G}}(x), \mu_{\tilde{U}}(x)\} \in \tilde{T}$.
- If $\tilde{G}_\alpha \in \tilde{T}$, then $\sup \{\mu_{\tilde{G}_\alpha}(x) : \alpha \in \tilde{A}\} \in \tilde{T}$.

The pair $(\tilde{A},\tilde{T})$is said to be fuzzy topological space .
Every member of $\tilde{T}$ is called a fuzzy open set in $\tilde{A}$.
The complement of fuzzy open set is called a fuzzy closed set .

Defintion2.2

A Fuzzy set $\tilde{B}$ in $(\tilde{A},T)$ is said to be :
1.Fuzzy b- closed set (denoted by Fb- closed) if $\mu_{\tilde{B}}(x) \geq \min \{\mu_{int(cl(\tilde{B}))}(x), \mu_{cl(int(\tilde{B}))}(x)\}$.[4]
2.Fuzzy semi - closed set (denoted byFs- closed) if $\mu_{int(cl(\tilde{B}))}(x) \leq \mu_{\tilde{B}}(x)$.[9],[3]
3.Fuzzy pre- closed set (denoted byFp- closed) if $\mu_{cl(int(\tilde{B}))}(x) \leq \mu_{\tilde{B}}(x)$.[8],[5]
4.Fuzzy semi-pre closed set (denoted by Fsp –closed) if $\mu_{int\,(cl(int(\tilde{B})))}(x) \leq \mu_{\tilde{B}}(x)$.[8],[5]
5.Fuzzy α – closed set (denoted by Fα- closed) if $\mu_{cl(int(cl(\tilde{B})))}(x) \leq \mu_{\tilde{B}}(x)$.[8],[9]

Defintion 2.3

If $\tilde{B}$ is a fuzzy set in $(\tilde{A},\tilde{T})$ then :
1 . The b-closure of $\tilde{B}$ is denoted by $(bcl(\tilde{B}))$ and denfined by
$\mu_{bcl(\tilde{B})}(x) = \min\{ \mu_{\tilde{F}}(x) : \tilde{F}$ is a fuzzy b- closed set in $\tilde{A}$, $\mu_{\tilde{B}}(x) \leq \mu_{\tilde{F}}(x)\}$.[4]
2.The pre closure of is denoted by $(pcl(\tilde{B}))$ and denfined by
$\mu_{pcl(\tilde{B})}(x) = \min \{ \mu_{\tilde{F}}(x) : \tilde{F}$ is a fuzzy pre closed set in $\tilde{A}$, $\mu_{\tilde{B}}(x) \leq \mu_{\tilde{F}}(x)\}$.[9]
3. The αclosure of is denoted by $(\alpha cl(\tilde{B}))$ and defined by
$\mu_{\alpha cl(\tilde{B})}(x) = \min \{ \mu_{\tilde{F}}(x) : \tilde{F}$ is a fuzzy α closed set in $\tilde{A}, \mu_{\tilde{B}}(x) \leq \mu_{\tilde{F}}(x)\}$. [3],[9]
4. The semi pre closure of is denoted by $(spcl(\tilde{B}))$ and defined by. $\mu_{spcl(\tilde{B})}(x) = \min \{\mu_{\tilde{F}}(x) : \tilde{F}$ is a fuzzy semi pre closed set in $\tilde{A}, \mu_{\tilde{B}}(x) \leq \mu_{\tilde{F}}(x)\}$. [8]
5. The semi closure of is denoted by $(scl(\tilde{B}))$ and defined by $\mu_{scl(\tilde{B})}(x) = \min \{ \mu_{\tilde{F}}(x) : \tilde{F}$ is a fuzzy semi closed set in $\tilde{A}, \mu_{\tilde{B}}(x) \leq \mu_{\tilde{F}}(x)\}$.[3]

Remark 2.4,[5]

1.$\mu_{scl(\tilde{B})}(x) = \max\{\mu_{\tilde{B}}(x), \mu_{int(cl(\tilde{B}))}(x)\}$.
2. $\mu_{pcl(\tilde{B})}(x) = \max\{\mu_{\tilde{B}}(x), \mu_{cl(int(\tilde{B}))}(x)\}$.
3. . $\mu_{bcl(\tilde{B})}(x) = \max\{\mu_{\tilde{B}}(x), \min(\mu_{cl(int(\tilde{B}))}(x), \mu_{int(cl(\tilde{B}))}(x))\}$.
4. $\mu_{spcl(\tilde{B})}(x) = \max\{\mu_{\tilde{B}}(x), \mu_{int(cl(int(\tilde{B})))}(x)\}$.
5. $\mu_{\alpha cl(\tilde{B})}(x) = \max\{\mu_{\tilde{B}}(x), \mu_{cl(int(cl(\tilde{B})))}(x)\}$.

Defintions 2.5
A fuzzy set $\tilde{B}$in fuzzy topological $(\tilde{A}, \tilde{T})$ is called.
1.Fuzzy generalized pre-closed set if $\mu_{pcl(\tilde{B})(X)} \le \mu_{\tilde{U}(X)}$ whenever $\mu_{\tilde{B}(X)} \le \mu_{\tilde{U}(X)}$ and $\tilde{U}$ is fuzzy open set in $\tilde{A}$ (denoted by Fgp- closed set).**[8]**,**[7]**
2.Fuzzy generalized semi- closed set if $\mu_{scl(\tilde{B})(X)} \le \mu_{\tilde{U}(X)}$ whenever $\mu_{\tilde{B}(X)} \le \mu_{\tilde{U}(X)}$ and $\tilde{U}$ is fuzzy open set in $\tilde{A}$ (denoted by Fgs- closed set).**[7]**
3.Fuzzy generalized semi pre - closed set if $\mu_{spcl(\tilde{B})(X)} \le \mu_{\tilde{U}(X)}$ whenever $\mu_{\tilde{B}(X)} \le \mu_{\tilde{U}(X)}$ and $\tilde{U}$ is fuzzy open set in $\tilde{A}$ (denoted by Fgsp- closed set).**[8]**

4.Fuzzy generalizedα - closed set if $\mu_{\alpha cl(\tilde{B})(X)} \le \mu_{\tilde{U}(X)}$ whenever $\mu_{\tilde{B}(X)} \le \mu_{\tilde{U}(X)}$ and $\tilde{U}$ is fuzzy α - open set in $\tilde{A}$(denoted by Fgα- closed set).**[8],[7]**
5.Fuzzy generalized closed set if $\mu_{cl(\tilde{B})}(x) \le \mu_{\tilde{u}}(x)$ Whenever $\mu_{\tilde{B}}(x) \le \mu_{\tilde{U}}(x)$ and $\tilde{U}$ is fuzzy open set in $\tilde{A}$ (denoted by Fg - closed set) . **[7],[8]**
Defintion 2.6 ,[6],[7]

A fuzzy point $\tilde{p}$ in a set X is also a fuzzy set with membership function:

$$\mu_{\tilde{p}}(x) = \begin{cases} r, & \text{for } x = y \\ 0, & \text{for } x \neq y \end{cases}$$

where $x \in X$ and $0 < r \le 1$, y is called the support of $\tilde{p}$ and r the value of $\tilde{p}$.

We denote this fuzzy point by x_r or $\tilde{p}$. Two fuzzy points x_r and y_s are said to be distinct if and only if $x \neq y$. A fuzzy point x_r is said to be belonged to a fuzzy subset $\tilde{A}$ in X, denoted by $x_r \in \tilde{A}$ if and only if $r \le \mu_{\tilde{A}}(x)$.(where $\mu_{xr(x)} = r$)

Defintion2.7
Afuzzy topological space$(\tilde{A}, \tilde{T})$ is said to be :
FT_{gs}if every Fgs- closed subset of $(\tilde{A},\tilde{T})$ is a Fsg closed set.
Remark 2.8
The definition (2.7) is equivalent e to be :
every singleton is either Fp- open set or Fuzzy closed set .
Lemma 2.9
In any fuzzy topological space $(\tilde{A},\tilde{T})$ the following are equivalent :
(1) $(\tilde{A},\tilde{T})$ is FT_{gs} .
(2) Every Fgp – closed subset of $(\tilde{A},\tilde{T})$ is a Fp – closed set.
Lemma 2.10,[4],[5]
Let $\tilde{B}$ be a fuzzy set in $(\tilde{A},\tilde{T})$, then :
(i).$\mu_{spcl(\tilde{B})}(x) \le \mu_{scl(\tilde{B})}(x) \le \mu_{\alpha cl(\tilde{B})}(x) \le \mu_{cl(\tilde{B})}(x)$.
(ii).$\mu_{spcl(\tilde{B})}(x) \le \mu_{pcl(\tilde{B})}(x) \le \mu_{\alpha cl(\tilde{B})}(x)$.
Proposition 2.11,[4]
If $\tilde{B}$, $\tilde{C}$ are a fuzzy sets in $(\tilde{A},\tilde{T})$,then :
1. $\mu_{bcl(\tilde{\emptyset})}(x) = \mu_{\tilde{\emptyset}}(x)$ and $\mu_{bcl(\tilde{A})}(x) = \mu_{\tilde{A}}(x)$, **2.**$\mu_{\tilde{B}}(x) \le \mu_{\tilde{C}}(x)$, then $\mu_{bcl(\tilde{B})}(x) \le \mu_{bcl(\tilde{C})}(x)$
3.$\mu_{\tilde{B}}(x) \le \mu_{bcl(\tilde{B})}(x)$,**4.**$\mu_{bcl(bcl(\tilde{B}))}(x) = \mu_{bcl(\tilde{B})}(x)$,**5.**$\mu_{bcl(\min\{\mu_{\tilde{B}(x)}, \mu_{\tilde{c}(x)}\})}(x) \le \min\{\mu_{bcl(\tilde{B})}(x), \mu_{bcl(\tilde{c})}(x)\}$,**6.**$\mu_{bcl(\max\{\mu_{\tilde{B}(X)}, \mu_{\tilde{c}(x)}\})}(x) = \max\{\mu_{bcl(\tilde{B})}(x), \mu_{bcl(\tilde{c})}(x)\}$,**7.**$\tilde{B}$ is a fuzzy b-closed set iff $\mu_{\tilde{B}}(x) = \mu_{bcl(\tilde{B})}(x)$.
Remark 2.12
Let $(\tilde{A},\tilde{T})$ be fuzzy a topological space , $\tilde{B}$ is closed subset of $\tilde{A}$ Then :
1. $\mu_{bcl(\tilde{B})}(x)) \le \mu_{scl(\tilde{B})}(x) \le \mu_{cl(\tilde{B})}(x)$.
2. $\mu_{bcl(\tilde{B})}(x)) \le \mu_{pcl(\tilde{B})}(x) \le \mu_{cl(\tilde{B})}(x)$.
3. $\mu_{bcl(\tilde{B})}(x)) \le \mu_{\alpha cl(\tilde{B})}(x) \le \mu_{cl(\tilde{B})}(x)$.
4. $\mu_{\tilde{B}}(x) \le \mu_{gbcl(\tilde{B})}(x) \le \mu_{bcl(\tilde{B})}(x) \le \mu_{cl(\tilde{B})}(x)$.

Proof: Obvious .

Proposition 2.13

If (Ã,T̃) a fuzzy topological space , then :

1. Every Fb– closedset isFgb- closed.
2. Every Fs closed set is Fgb- closed set.
3. Every Fp closed set isFgb- closed set.
4. Every Fg- closed set isFgb- closed set.
5. EveryFα - closed set isFgb- closed set.
6. Every Fsg- closed set isFg s-closed set.
7. Every F b – closed set isFgsp –closed set.
8. Every Fα – closedset isbothFs - closed set and Fp -closed set .

Proof (1)

Let B̃ be Fb –closed set in(Ã,T̃) such that$\mu_{B}(x) \leq \mu_{U}(x)$, where Ũ is fuzzy open set in Ã,sinceB̃ is Fb –closed set,$\mu_{bcl(B)}(x) = \mu_{B}(x)$ .Therefore $\mu_{bcl(B)}(x) \leq \mu_{U}(x)$.

Hence B̃ isFgb- closed set. □

The proof(2,3,4,5,6,7,8) is similar to that of (1) proposition (2.13)

Lemma 2.14,[3],[4]

If B̃ is a fuzzy set in (Ã,T̃) ,then;

•$\mu_{gbint(\tilde{B})}(x) = \mu_{(gbcl(\tilde{B}^c))^c}(x)$.

•$\mu_{gbcl(\tilde{B})}(x) = \mu_{(gbint(\tilde{B}^c))^c}$

III. Properties of fuzzy generalized b- closed set

In this section we introduce the concept of Fgb-closed sets in fuzzy topological spaces and study some properties and theorem of the subject .

Definition 3.1,[1]

A fuzzy set B̃ in fuzzy topological space (Ã,T̃) is called .

Fuzzy generalized b-closed set(denoted by Fgb-closed set) if $\mu_{bcl(\tilde{B})}(x) \leq \mu_{\tilde{U}}(x)$ whenever $\mu_{\tilde{B}}(x) \leq \mu_{\tilde{U}}(x)$ and Ũ is fuzzy open set .

Theorem 3.2

If (Ã,T̃)afuzzy topological space , then :

1. Every Fgα-closed set is Fgs-closed set.
2. Every Fgα-closed set is Fgp-closed set.
3. Every Fgp–closed set is Fgsp – closed set.
4. Every Fgs–closed set is Fgsp – closed set.
5. Every Fgs–closed set is Fgb – closed set.
6. Every Fgp–closed set is Fgb – closed set.
7. Every Fgb-closed set is Fgsp– closed set.
8. Every Fgα-closed set is Fgb-closed set .

Proof (1)

Let B̃ be a fuzzy gα –closed set in(Ã,T̃) such that$\mu_{\alpha cl(\tilde{B})}(x) \leq \mu_{\tilde{U}}(x)$,$\mu_{B}(x) \leq \mu_{U}(x)$ where Ũ is fuzzy open set in Ã,since$\mu_{scl(\tilde{B})}(x) \leq \mu_{\alpha cl(\tilde{B})}(x) \leq \mu_{\tilde{U}}(x)$

Hence B̃ is Fgs-closed set . □

Remark 3.3

The converse of (1) of theorem (3.2) is not true in general as shown by the following example

Example 3.4

Let $X = \{a, b\}$ and Ã,B̃,C̃,D̃,**Ẽ**are fuzzy sets defined as follows:

Ã ={(a,0.5),(b,0.3)},B̃ ={(a,0.1),(b,0.3)},C̃ ={(a,0.3),(b,0.0)},D̃={(a,0.3),(b,0.3)},Ẽ ={(a,0.1),(b,0.0)} ,F̃ ={(a,0,0),(b,0.2)},Let $\tilde{T} = \{\tilde{\phi}, \tilde{A}, \tilde{B}, \tilde{C}$, D̃, Ẽ} be fuzzy topology on $\tilde{A}$ then F̃ is Fgs-closed set in $\tilde{A}$ .But not Fgα– closed set .

The proof(2,3,4,5,6,7,8) is similar to that of (1) theorem (3.2)

Remrk 3.5

The converse of (2) of theorem (3.2) is not true in general as shown by the following example.

Example 3.6

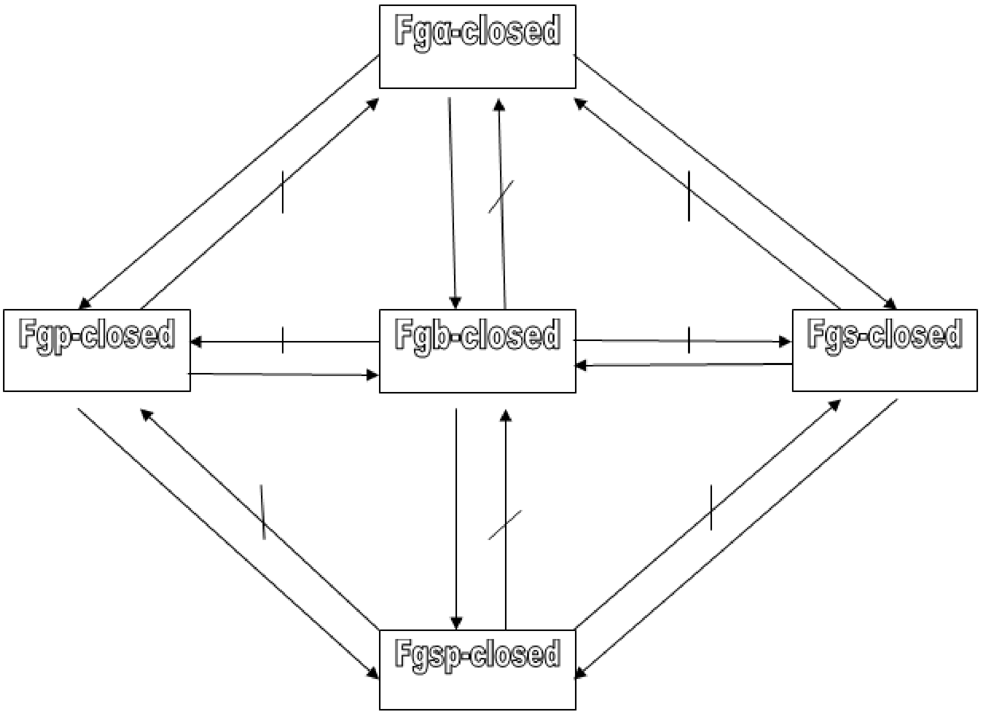
Let $X = \{a, b\}$ and are Ã,B̃,C̃,D̃,**Ẽ**fuzzy sets defined as follows:

Ã ={(a,0.6),(b,0.4)},B̃ ={(a,0.0),(b,0.4)},C̃ ={(a,0.4),(b,0.0)},)},D̃={(a,0.4),(b,0.4)}, Ẽ ={(a,0.0),(b,0.3)} ,Let T̃ ={Ø̃,Ã,B̃,D̃,C̃} be a fuzzy topology on $\tilde{A}$ then C̃ is Fgp-closed set but not Fgα-closed set.

Remark 3.7
The converse of (3) of theorem (3.2) is not true in general as shown by the following example.
Example 3.8
Ã ={(a,0.8),(b,0.8)},B̃ ={(a,0.3),(b,0.3)},C̃ ={(a,0.1),(b,0.1)},D̃ = {(a,0.2),(b,0.2)}
Let $\tilde{T} = \{\tilde{\phi}, \tilde{A}, \tilde{B}, \tilde{C}\}$ be a fuzzy topology on $\tilde{A}$ then D̃ is Fgsp-closed set but not Fgp- closed set.
Remark 3.9
The converse of (4) of theorem (3.2) is not true in general as shown by the following example.
Example 3.10
Let X={(a,b)} and Ã,B̃,C̃,D̃are fuzzy set defined as follows :
Ã = {(a,0.6),(b,0.6)},B̃ = {(a,0.2),(b,0.2)},,C̃ = {(a,0.5),(b,0.5)},D̃ ={(a,0.1),(b,0.5)},Let $\tilde{T} = \{\tilde{\phi}, \tilde{A}, \tilde{B}, \tilde{C}\}$ be a fuzzy topology on $\tilde{A}$ then D̃ is Fgsp-closed set but not Fgs– closed set.
Remark 3.11
The converse of (5) of theorem (3.2) is not true in general as shown by the following example
Example 3.12
Let $X = \{a, b\}$ and Ã,$\tilde{B}_1$,$\tilde{B}_2$,$\tilde{B}_3$are fuzzy set :
Ã ={(a,0.7),(b,0.6)},$\tilde{B}_1$ ={(a,0.4),(b,0.5)},$\tilde{B}_2$ ={(a,0.0),(b,0.5)},$\tilde{B}_3$ ={(a,0.4),(b,0.3)},Let T̃={Ø̃,Ã,$\tilde{B}_1$,$\tilde{B}_2$} be a fuzzy topology on ($\tilde{A}$, T̃) then $\tilde{B}_3$ is Fgb-closed set but not Fgs-closed set.

Remark 3.13
The converse (6) of the theorm (3.2) is not true in general as shown by the following example.
Example 3.14
Let x={a,b}and Ã,B̃,C̃,D̃ are fuzzy set
Ã ={(a,0.7),(b,0.6)},B̃ ={(a,0.4),(b,0.5)},C̃ ={(a,0.0),(b,0.2)},D̃ ={(a,0.0),(b,0.4)},Let T̃={Ø̃,Ã,B̃,C̃} be a fuzzy topology on $\tilde{A}$ then D̃ is Fgb-closed set but not is Fgp-closed set .
Remark 3.15
The converse (7) of the theorm (3.2) is not true in general as shown by the following example.
Example 3.16
Let X={a,b} and $\tilde{A}, \tilde{B}, \tilde{C}$, D̃ are fuzzy set :
Ã = {(a,0.8),(b,0.8)},B̃ = {(a,0.7),(b,0.7)},C̃ ={(a,0.1),(b,0.1)} ,D̃ ={(a,0.3),(b,0.3)},
Ẽ ={(a,0.4),(b,0.4)} ,F̃ ={(a,2),(b,0.2)} ,Let T̃={Ø̃,Ã,B̃,C̃,D̃, Ẽ} be a fuzzy topology on $\tilde{A}$ thenF̃ is Fgsp but not is Fgb-closed set.
Remark 3.17
The converse (8) of the theorm (3.2) is not true in general as shown by the following
Example 3.18
Let x={a,b}andÃ,B̃,C̃,D̃,**Ẽ**are fuzzy sets defined as follows:
Ã ={(a,0.5),(b,0.3)},B̃ ={(a,0.1),(b,0.3)},C̃ ={(a,0.3),(b,0.0)},D̃={(a,0.3),(b,0.3)}, Ẽ ={(a,0.1),(b,0.0)} ,F̃ ={(a,0),(b,0.2)},Let $\tilde{T} = \{\tilde{\phi}, \tilde{A}, \tilde{B}, \tilde{C}$, D̃, Ẽ} be fuzzy topology on $\tilde{A}$ then F̃is Fgb-closed set but not is Fgα-closed set.
Remark 3.19:The intersection of two Fgb-closed set is not necessary Fgb-closed set.
Example 3.20: Let $X = \{a, b, c\}$ and are fuzzy:
Ã = {(a,0.6),(b,0.5),(c,0.3)},B̃ ={(a,0.6),(b,0.0),(c,0.0)},C̃ ={(a,0.6),(b,0.5),(c,0.0)},
D̃ ={(a,0.6),(b,0.0),(c,0.3)}, *the collection* $\tilde{T} = \{\tilde{\phi}, \tilde{A}, \tilde{B}\}$ is fuzzy topology on $\tilde{A}$ then $\tilde{C}$ and $\tilde{D}$ are Fgb closed set. but - min{$\mu_{\tilde{C}}(x)$,$\mu_{\tilde{D}}(x)$ is not Fgb-closed set .
Theorem 3.21 : Let $(\tilde{A}, \tilde{T})$ be fuzzy topological space then the union of two Fgb-closed set is Fgb-closed set.
Proof :Let B̃ and C̃ are Fgb- closed set in (Ã,T̃) and Ũ be Fuzzy open set containing B̃ and C̃ Therefore $\mu_{bcl(\tilde{B})}(x) \leq \mu_{\tilde{U}}(x)$, $\mu_{bcl(\tilde{C})}(x) \leq \mu_{\tilde{U}}(x)$. Since$\mu_{\tilde{B}}(x) \leq \mu_{\tilde{U}}(x)$, $\mu_{\tilde{C}}(x) \leq \mu_{\tilde{U}}(x)$ then
max {$\mu_{\tilde{B}}(x)$, $\mu_{\tilde{C}}(x)$ }≤ $\mu_{\tilde{U}}(x)$.
Hence $\mu_{bcl(\max\{\mu_{\tilde{B}(X)}, \mu_{\tilde{C}(x)}\})}(x)$ = max { $\mu_{bcl(\tilde{B})}(x)$,$\mu_{bcl(\tilde{C})}(x)$ }≤ $\mu_{\tilde{U}}(x)$.
Therefore max {$\mu_{\tilde{B}}(x)$, $\mu_{\tilde{C}}(x)$ } is Fgb-closed set. □

We will explain the relationship between some types of fuzzy genrelalized closed sets in fuzzy topological space on fuzzy sets by fig



Theorem 3.22

If $\tilde{B}$is F - open and Fgb- closed then $\tilde{B}$isFb-closed .

Proof:

Since$\tilde{B}$fuzzy open and fuzzy gb- closedset

Then Ũ any fuzzy open in Ã,$\mu_{\tilde{B}}(x) \leq \mu_{\tilde{U}}(x)$ .Then$\mu_{bcl(\tilde{B})}(x) \leq \mu_{\tilde{U}}(x)$.

Let Ũ=B̃,we have $\mu_{\tilde{B}}(x) \leq \mu_{\tilde{B}}(x)$

$\Longrightarrow \mu_{bcl(\tilde{B})}(x) \leq \mu_{\tilde{B}}(x)$…………………………..(1) But:

$\mu_{\tilde{B}}(x) \leq \mu_{bcl(\tilde{B})}(x)$………………………….(2)

From (1) and (2) $\mu_{\tilde{B}}(x) = \mu_{bcl(\tilde{B})}(x)$.

Hence $\tilde{B}$ is Fb – closed . □

Theorem 3.23

If $\tilde{B}$isFgb- closed and $\tilde{B} \leq \tilde{C} \leq bcl(\tilde{B})$ then C̃ isFgb – closed set .

Proof:

Let $\tilde{U}$ be a fuzzy open set of $\tilde{A}$ such that$\mu_{\tilde{C}}(x) \leq \mu_{\tilde{U}}(x)$ then $\mu_{\tilde{B}}(x) \leq \mu_{\tilde{U}}(x)$ since $\tilde{B}$ is Fuzzy gb- closed.Then$\mu_{bcl(\tilde{B})}(x) \leq \mu_{\tilde{U}}(x)$

Now$\mu_{bcl(\tilde{C})}(x) \leq \mu_{bcl(bcl(\tilde{B}))}(x) = \mu_{bcl(\tilde{B})}(x) \leq \mu_{\tilde{U}}(x)$

Therefor$\mu_{bcl(\tilde{C})}(x) \leq \mu_{\tilde{U}}(x)$. Hence$\tilde{C}$ is is fuzzy gb – closed .□

Theorem 3.24

If B̃ isFgb- closed set in (Ã,T̃) then $bcl(\tilde{B}) - \tilde{B}$ does not contain any non-empty fuzzy closed set .

Proof:

Let F̃ be fuzzy closed set in Ã such that $\mu_{\tilde{F}}(x) \leq \mu_{bcl(\tilde{B})}(x) - \mu_{\tilde{B}}(x)$,since $(\tilde{A} - \tilde{F})$ is open,so$\mu_{\tilde{F}}(x) \leq \mu_{(\tilde{A} - \tilde{F})}(x)$, since B̃ is Fgb- closed set and Ã- F̃ is open ,then $\mu_{bcl(\tilde{B})}(x) \leq \mu_{(\tilde{A} - \tilde{F})}(x)$ This $\Longrightarrow \mu_{\tilde{F}}(x) \leq \mu_{(A)}(x) - \mu_{bcl(\tilde{B})}(x)$ .So $\mu_{\tilde{F}}(x) \leq \min\{\mu_{(\tilde{A})}(x) - \mu_{bcl(\tilde{B})}(x) , \mu_{(\tilde{A})}(x) - \mu_{bcl(\tilde{B})}(x)\} = \tilde{\emptyset}$

therefore $\tilde{F} = \tilde{\emptyset}$ □

Theorem 3.25

If B̃ be Fgb- closed set then B̃ is Fb- closed set iffbcl$(\tilde{B}) - \tilde{B} = \emptyset$

is closed set .

Proof:

$\Longrightarrow$ Let B̃ be fuzzy gb- closed set.ifB̃is closed set .$\mu_{bcl(\tilde{B})}(x) = \mu_{\tilde{B}}(x)$

Hencebcl$(\tilde{B}) - \tilde{B} = \tilde{\emptyset}$ is closed set .

$\Longleftarrow$Let bcl$(\tilde{B}) - \tilde{B}$ be fuzzy closed . since $\tilde{B}$ is fuzzy gb-closed, So by theorem (3.25)

$bcl(\tilde{B}) - \tilde{B}$ does not contain any non-empty fuzzy closed set .
$bcl(\tilde{B}) - \tilde{B}$ is closed set , than $bcl(\tilde{B}) - \tilde{B} = \emptyset$
This implies that$\mu_{bcl(\tilde{B})}(x) = \mu_{\tilde{B}}(x)\tilde{B}$ and $\tilde{B}$ is Fb-closed set □

Theorem 3.26:Let $(\tilde{A},\tilde{T})$ be any fuzzy topological space ,then the following are equivalent
(1) Every Fgb- closed set is Fgp- closed set .
(2) Every Fb- closed set is Fgp- closed set .

Proof .
(1) $\Rightarrow$ (2) : is obvious .
(2) $\Rightarrow$ (1) : let $\tilde{B}$ be a Fgb- closed set ,such that $\mu_{\tilde{B}}(x) \leq \mu_{\tilde{U}}(x)$ where $\tilde{U}$ is open . By definition(3.1) .Then $\mu_{bcl(\tilde{B})}(x) \leq \mu_{\tilde{U}}(x)$,sincebcl($\tilde{B}$) isFb-closed set,then by (2)
bcl($\tilde{B}$) is Fgp- closed set .So $\mu_{pcl(\tilde{B})}(x) \leq \mu_{pcl(bcl)}(x) \leq \mu_{\tilde{U}}(x)$.
Therefore $\tilde{B}$ is Fgp- closed set. □

Proposition 3.27
If $\tilde{G}$ is a fuzzy closed set and $\tilde{U}$ is a Fg b-closed set infuzzy topological space $(\tilde{A},\tilde{T})$then min $\{\mu_{\tilde{G}}(x) ,\mu_{\tilde{U}}(x) \}$ is a Fgb-closed set in$(\tilde{A},,\tilde{T})$.

Proof
To prove min $\{ \mu_{\tilde{G}}(x) , \mu_{\tilde{U}}(x) \} = \mu_{gbcl(\min\{\mu_{\tilde{G}(X)},\mu_{\tilde{U}(X)}\})}(x)$.
Since $\mu_{\tilde{G}}(x)=\mu_{cl(\tilde{G})}(x)$ and $\mu_{\tilde{u}}(x) = \mu_{gbcl(\tilde{u})}(x)$ then
Min$\{\mu_{gbcl(\tilde{G})}(x) ,\mu_{gbcl(\tilde{U})}(x) \}\leq \min\{\mu_{cl(\tilde{G})}(x) ,\mu_{gbcl(\tilde{U})}(x)\}$
Hence$\mu_{gbcl(\min\{\mu_{\tilde{G}(x)},\mu_{\tilde{u}(x)}\})}(x) \leq \min\{\mu_{\tilde{G}}(x),\mu_{\tilde{u}}(x)\}$ ……………(*)
And since min $\{\mu_{G\mu}(x) ,\mu_{\tilde{u}}(x)\} \leq \mu_{gbcl(\min\{\mu_{\tilde{G}(x)},\mu_{\tilde{u}(X)}\})}(x)$..................(**)
Then from (*)and (**) we get min $\{ \mu_{\tilde{G}}(x) ,(x) , \mu_{\tilde{U}}(x) \} = \mu_{gbcl(\min\{\mu_{\tilde{G}(X)},\mu_{\tilde{U}(X)}\})}(x)$.
Thus min $\{ \mu_{\tilde{G}}(x) , \mu_{\tilde{U}}(x) \}$is a Fgb- closed set in$(\tilde{A},\tilde{T})$. □

Lemma 3.28
Let$(\tilde{A},\tilde{T})$ be any fuzzy topological space $(\tilde{A},\tilde{T})$, every singleton is either fuzzy pre open or nowhere dense .**Proof**: Obvious .

Theorem 3.29:Let Let$(\tilde{A},\tilde{T})$ be any fuzzy topological space $(\tilde{A},\tilde{T})$,then:
Every Fgb- closed set is a Fb- closed set if and only if $(\tilde{A},\tilde{T})$ is every singleton is either
Fp- open or Fuzzy closed set .

Proof .
$\Longrightarrow$ suppose that every Fgb – closed set of $(\tilde{A},\tilde{T})$ is a Fb – closed set .Let $\mu_{xr}(x) < \mu_{A}(x)$. Then by lemma (3.28) , $\{x_r\}$ is either Fuzzy preopen or nowhere dense , if $\{x_r\}$ is Fuzzy preopen , Then $(\tilde{A},\tilde{T})$ is FT_{gs} , Now suppose that $\{x_r\}$is nowhere dense ,thus $\mu_{int(cl(\{Xr\}))}(x) = \emptyset$ and not fuzzy closed . Then $\tilde{A}$ - $\{x_r\}$ is not fuzzy open . Thus $\tilde{A}$ is the only open set which contains $\tilde{A}$ - $\{x_r\}$andbcl($\tilde{A}$ - $\{x_r\}$) .That is $\tilde{A}$ - $\{x_r\}$ is Fgb – closed set . By assumption , every Fgb- closed set is Fb- closed set . So $\tilde{A}$ - $\{x_r\}$ isFb – closed set , thus $\{x_{r\}}$ Fb- open, and so$\mu_{\{xr\}}(x) \leq \max\{\mu_{int(cl(\{xr\}))}(x),\mu_{cl(int(\{xr\}))}(x) \}$, a contradiction to the fact that $\{x_r\}$*is nowhere dense* .Then $\{x_r\}$is closed set . Thus $(\tilde{A},\tilde{T})$isFT_{gs} by Remark (2.8) .
$\Longleftarrow$suppose that $(\tilde{A},\tilde{T})$ is a $FT_{gs}$and let $\tilde{B}$ be Fgb – closed subset of $\tilde{A}$. we want to show that $\mu_{bcl(\tilde{B})}(x)\leq \mu_{(\tilde{B})}(x)$. Let $\mu_{xr}(x) < \mu_{bcl(\tilde{B})}(x)$ and suppose that $\mu_{xr}(x) > \mu_{(\tilde{B})}(x)$ .So $\mu_{(\tilde{B})}(x) \leq \mu_{(\tilde{A}-\{xr\})}(x)$. If$\{x_r\}$ is Fp- open , then $\tilde{A}$- $\{x_r\}$ is Fp- closed , thus $\mu_{pcl(\tilde{B})}(x) \leq \mu_{(\tilde{A}-\{xr\})}(x)$. Since every Fp- closed set is a Fb- closed set .Then $\mu_{bcl(\tilde{B})}(x) \leq\mu_{pcl(\tilde{B})}(x) \leq \mu_{(\tilde{A}-\{xr\})}(x)$.$\mu_{xr}(x) > \mu_{bcl(\tilde{B})}(x)$, a contradiction . If $\{x_r\}$ is F-closed set then $\tilde{A}$- $\{x_r\}$ is F- open set .Since $\tilde{B}$ is Fgb – closed , then $\mu_{bcl(\tilde{B})}(x) \leq\mu_{(\tilde{A}-\{xr\})}(x)$. Thus $\mu_{xr}(x) > \mu_{bcl(\tilde{B})}(x)$, a contradiction .Thus is $\mu_{bcl(\tilde{B})}(x) \leq\mu_{(\tilde{B})}(x)$ .Since $\mu_{(\tilde{B})}(x) \leq\mu_{bcl(\tilde{B})}(x)$ then $\mu_{(\tilde{B})}(x) = \mu_{bcl(\tilde{B})}(x)$. Hence $\tilde{B}$ is F b- closed set. □

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