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# On Intuitionistic Fuzzy $T_2$ −Spaces

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***Abstract:****The purpose of this paper is to introduce and study the intuitionistic fuzzy $T_2$–spaces. We investigate some relations amongthem. We also investigate the relationship between intuitionistic fuzzy topological spaces and intuitionistic topological spaces.*
***Keywords:****Intuitionistic set,Intuitionistic fuzzy set,Intuitionistic topological space, Intuitionistic fuzzytopological space, Intuitionistic fuzzy $T_2$–spaces*

## I. Introduction

The fundamental concepts of a fuzzy set was introduced by L. A. Zadeh[15] in1965. In 1968, Chang [10] introduced the concepts of the fuzzy topological spaces by using the fuzzy sets. As a generalization of fuzzy sets, the concept of intuitionistic fuzzy sets was introduced by Atanassov [3, 7]. Coker [1, 2, 4, 6, 11] and his colleagues introduced intuitionistic fuzzy topological spaces by using intuitionistic fuzzy sets. In this paper, we investigate the properties of $T_2$ –spaces.

**Definition 1.1**[1, 2, 8]An intuitionistic set $A$ is an object having the form$A = (x, A_1, A_2)$ where $A_1$and $A_2$ are subsets of$X$satisfying$A_1 \cap A_2 = \phi$. The set $A_1$ is called the set of member of $A$ while $A_2$ is called the set of non-member of A.
Throughout this paper, we use the simpler notation $A = (A_1, A_2)$ for an intuitionistic set.

**Remark 1.2**[1, 2, 8] Every subset$A$ on a nonempty set $X$ may obviously be regarded as an intuitionistic set having the form $A' = (A, A^C)$, where $A^C = X \setminus A$ is the complement of $A$ in$X$.

**Definition 1.3**[1, 2, 8] Let the intuitionistic sets $A$ and $B$ on $X$ be of the form $A = (A_1, A_2)$ and$B = (B_1, B_2)$ respectively. Furthermore, let $\{A_j : j \in J\}$ be an arbitrary family of intuitionistic sets in$X$ where $A_j = (A_j^{(1)}, A_j^{(2)})$Then

(a) $A \subseteq B$if and only if $A_1 \subseteq B_1$and$A_2 \supseteq B_2$.
(b) $A = B$if and only if $A \subseteq B$and$B \subseteq A$.
(c) $\overline{A} = (A_2, A_1)$, denotes the complement of$A$.
(d) $\cap A_j = (\cap A_j^{(1)}, \cup A_j^{(2)})$.
(e) $\cup A_j = (\cup A_j^{(1)}, \cap A_j^{(2)})$.
(f) $\phi_\sim = (\phi, X)$and$X_\sim = (X, \phi)$.

**Definition: 1.4[**2, 8**]**An intuitionistic topology on a set$X$ is a family $\tau$ of intuitionistic sets in $X$ satisfying the following axioms:

(1) $\phi_\sim, X_\sim \in \tau$.
(2) $G_1 \cap G_2 \in \tau$for any $G_1, G_2 \in \tau$.
(3) $\cup G_i \in \tau$for any arbitrary family $G_i \in \tau$.

In this case, the pair $(X, \tau)$ is called an intuitionistic topological space (ITS, in short) and any intuitionistic set in $\tau$ is known as an intuitionistic open set (IOS, in short) in X.

**Definition 1.5**[3, 4, 6, 9] Let $X$ be a non empty set and $I$ be the unit interval $[0, 1]$. An intuitionistic fuzzy set A (IFS, in short) in $X$ is an object having the form $A=\{(x, \mu_A(x), \nu_A(x)), x \in X\}$, where $\mu_A: X \to I$and $\nu_A: X \to I$ denote the degree of membership and the degree of non- membership respectively, and $\mu_A(x) + \nu_A(x) \leq 1$.
Let I(X) denote the set of all intuitionistic fuzzy sets in$X$. Obviously every fuzzy set $\mu_A$ in $X$ is an intuitionistic fuzzy set of the form $(\mu_A, 1 - \mu_A)$.

Throughout this paper, we use the simpler notation $A=(\mu_A, \nu_A)$ instead of $A=\{(x, \mu_A(x), \nu_A(x)), x \in X\}$.

**Definition 1.6**[3, 4, 6, 9] Let A=$(\mu_A, \nu_A)$ andB= $(\mu_B, \nu_B)$ be intuitionistic fuzzy sets inX. Then
(1) $A \subseteq B$if and only if $\mu_A \leq \mu_B$and $\nu_A \geq \nu_B$.
(2)$A = B$if and only if $A \subseteq B$ and $B \subseteq A$.
(3) $A^c$= $(\nu_A, \mu_A)$.
(4)$A \cap B$= $(\mu_A \cap \mu_B;\ \nu_A \cup \nu_B)$.
(5) $A \cup B$= $(\mu_A \cup \mu_B;\ \nu_A \cap \nu_B)$.
(6) $0_\sim = (0^\sim, 1^\sim)$ and $1_\sim = (1^\sim, 0^\sim)$.

**Definition 1.7**[4, 6, 9]An intuitionistic fuzzy topology (IFT, in short) on X is a family t of IFSs in X which satisfies the following properties:
(1)$0_\sim, 1_\sim \in t$.
(2) if$A_1, A_2 \in t$, then $A_1 \cap A_2 \in t$.
(3) if$A_i \in t$ for each i, then $\cup A_i \in t$.

The pair $(X, t)$ is called an intuitionistic fuzzy topological space (IFTS, in short). Let $(X, t)$ be an IFTS. Then any member oft is called an intuitionistic fuzzy open set (IFOS, in short) inX.The complement of an IFOS inX is called an intuitionistic fuzzy closed set (IFCS, in short) in X.

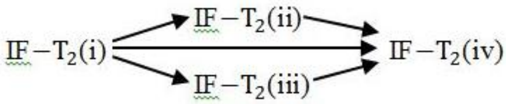
## II. Intuitionistic fuzzy $T_2$ −spaces

**Definition2.1** An intuitionistic fuzzy topological space $(X, t)$ is called
(1) $IF-T_2(i)$ if for allx, $y \in X$, $x \neq y$ there exists A= $(\mu_A, \nu_A)$, B=$(\mu_B, \nu_B) \in t$ such that$\mu_A(x)$ =1, $\nu_A(x)$ =0; $\mu_B(y)$ =1, $\nu_B(y)$ =0 and $A \cap B = 0_\sim$.
(2) $IF-T_2(ii)$ if for all x, $y \in X$, $x \neq y$ there exists A= $(\mu_A, \nu_A)$, B=$(\mu_B, \nu_B) \in t$ such that $\mu_A(x)$ =1, $\nu_A(x)$ =0; $\mu_B(y) > 0$, $\nu_B(y)$ =0 and$A \cap B = (0^\sim, \gamma^\sim)$ where $\gamma \in (0, 1]$.
(3) $IF-T_2(iii)$ if for allx, $y \in X$, $x \neq y$ there exists A= $(\mu_A, \nu_A)$, B=$(\mu_B, \nu_B) \in t$ such that $\mu_A(x) > 0$, $\nu_A(x)$ =0; $\mu_B(y)$ =1, $\nu_B(y)$ =0 and$A \cap B = (0^\sim, \gamma^\sim)$ where $\gamma \in (0, 1]$.
(4) $IF-T_2(iv)$ if for allx, $y \in X$, $x \neq y$ there exists A= $(\mu_A, \nu_A)$, B= $(\mu_B, \nu_B) \in t$ such that $\mu_A(x) > 0$, $\nu_A(x)$ =0; $\mu_B(y) > 0$, $\nu_B(y)$ =0 and$A \cap B = (0^\sim, \gamma^\sim)$ where $\gamma \in (0, 1]$.

**Definition2.2**Let $\alpha \in (0, 1)$. An intuitionistic fuzzy topological space $(X, t)$ is called

(a) $\alpha-IF-T_2(i)$ iffor all x, $y \in X$, $x \neq y$ there exists $A = (\mu_A, \nu_A)$, B=$(\mu_B, \nu_B) \in t$ such that$\mu_A(x) = 1$, $\nu_A(x) = 0$; $\mu_B(y) \geq \alpha$, $\nu_B(y) = 0$ and $A \cap B = 0_\sim$.
(b) $\alpha-IF-T_2(ii)$ if for all x, $y \in X$, $x \neq y$ there exists$A = (\mu_A, \nu_A)$, B=$(\mu_B, \nu_B) \in t$ such that$\mu_A(x) \geq \alpha$, $\nu_A(x) = 0$; $\mu_B(y) \geq \alpha$, $\nu_B(y) = 0$ and$A \cap B = (0^\sim, \gamma^\sim)$ where $\gamma \in (0, 1]$.
(c) $\alpha-IF-T_2(iii)$ if for all x, $y \in X$, $x \neq y$there exists $A = (\mu_A, \nu_A)$, B=$(\mu_B, \nu_B) \in t$ such that$\mu_A(x) > 0$, $\nu_A(x) = 0$; $\mu_B(y) \geq \alpha$, $\nu_B(y) = 0$ and $A \cap B = (0^\sim, \gamma^\sim)$ where $\gamma \in (0, 1]$.

**Theorem 2.3** Let $(X, t)$ be an intuitionistic fuzzy topological space. Then we have the following implications:




**Proof:** Suppose $(X, t)$ is $IF-T_2(i)$ space. We shall prove that$(X, t)$ is $IF-T_2(ii)$.Since $(X, t)$ is $IF-T_2(i)$, then for allx, $y \in X$, $x \neq y$ there exists $A = (\mu_A, \nu_A)$, $B = (\mu_B, \nu_B) \in t$such that $\mu_A(x) = 1$, $\nu_A(x) = 0$; $\mu_B(y)$=1, $\nu_B(y)$=0and $A \cap B = 0_\sim$ $\Longrightarrow \mu_A(x)$ =1, $\nu_A(x)$ =0; $\mu_B(y) > 0$, $\nu_B(y) = 0$ and $A \cap B = (0^\sim, \gamma^\sim)$ where $\gamma \in (0, 1]$. Which is$IF-T_2(ii)$space.Hence $IF-T_2(i) \Longrightarrow IF-T_2(ii)$.

Again, suppose$(X, t)$ is $IF-T_2(i)$ space. We shall prove that $(X, t)$ is$IF-T_2(iii)$. Since $(X, t)$ is $IF-T_2(i)$, then for all x, $y \in X$, $x \neq y$ there exists $A = (\mu_A, \nu_A)$,$B = (\mu_B, \nu_B) \in t$such that $\mu_A(x) = 1$,$\nu_A(x) = 0$; $\mu_B(y) = 1$, $\nu_B(y) = 0$ and $A \cap B = 0_\sim \Longrightarrow \mu_A(x) > 0$,$\nu_A(x) = 0$; $\mu_B(y) = 1$, $\nu_B(y) = 0$and$A \cap B = (0^\sim, \gamma^\sim)$ where $\gamma \in (0, 1]$.Which is $IF-T_2(iii)$ space. Hence $IF-T_2(i) \Longrightarrow IF-T_2(iii)$.

Furthermore, it can easily verify that $IF-T_2(i) \Longrightarrow IF-T_2(iv)$, $IF-T_2(ii) \Longrightarrow IF-T_2(iv)$ and$IF-T_2(iii) \Longrightarrow IF-T_2(iv)$.None of the reverse implications is true in general as can be seen from the following examples.

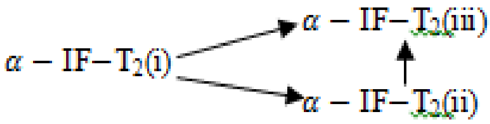
Example 2.3.1 Let$X = \{x, y\}$and$t$be the intuitionistic fuzzy topology on$X$generated by $\{A, B\}$ where$A = \{x, 1, 0\}$and$B = \{y, 0.5, 0\}$.We see that the IFTS $(X, t)$ is IF- $T_2$(ii) but not IF- $T_2$(i).

Example 2.3.2Let$X = \{x, y\}$ and$t$ be the intuitionistic fuzzy topology on$X$ generated by $\{A, B\}$ where$A = \{x, o.3, 0\}$ and$B = \{y, 1, 0\}$. We see that the IFTS $(X, t)$ is IF- $T_2$(iii) but not IF- $T_2$(i).

Example 2.3.3Let$X = \{x, y\}$and$t$be the intuitionisticfuzzy topology on$X$generated by$\{A, B\}$where$A = \{x, 1, 0\}$ and $B = \{y, 0.7, 0\}$.We see that the IFTS $(X, t)$ is IF- $T_2$(ii) but not IF- $T_2$(iii).

Example 2.3.4Let$X = \{x, y\}$ and$t$be the intuitionistic fuzzy topology on$X$ generated by $\{A, B\}$ where $A = \{x, o.6, 0\}$ and$B = \{y, 1, 0\}$. We see that the IFTS $(X, t)$ is IF- $T_2$(iii) but not IF- $T_2$(ii).

**Theorem 2.4** Let $(X, t)$ be an intuitionistic fuzzy topological space. Then we have the following implication:




**Proof:**Suppose $(X, t)$ is $\alpha - IF - T_2$ (i) space. We shall prove that $(X, t)$ is $\alpha - IF - T_2$ (ii).Since $(X, t)$ is $\alpha - IF - T_2$(i), then for all $x, y \in X$, $x \neq y$there exists$A = (\mu_A, \nu_A)$, $B = (\mu_B, \nu_B) \in t$ such that$\mu_A(x) = 1$, $\nu_A(x) = 0$; $\mu_B(y) \geq \alpha$, $\nu_B(y) = 0$ and $A \cap B = 0_\sim \Longrightarrow \mu_A(x) \geq \alpha$, $\nu_A(x) = 0$; $\mu_B(y) \geq \alpha$, $\nu_B(y) = 0$ for any $\alpha \in (0, 1)$ and $A \cap B = (0^\sim, \gamma^\sim)$ where $\gamma \in (0, 1]$. Which is $\alpha - IF - T_2$(ii)space. Hence $\alpha - IF - T_2$(i)$\Longrightarrow$ $\alpha - IF - T_2$(ii).

Again,suppose $(X, t)$ is $\alpha - IF - T_2$ (ii) space. We shall prove that $(X, t)$ is $\alpha - IF - T_2$ (iii).Since $(X, t)$ is $\alpha - IF - T_2$ (ii), then for all $x, y \in X$, $x \neq y$ there exists $A = (\mu_A, \nu_A)$, $B = (\mu_B, \nu_B) \in t$ such that$\mu_A(x) \geq \alpha$, $\nu_A(x) = 0$; $\mu_B(y) \geq \alpha$, $\nu_B(y) = 0$and $A \cap B = (0^\sim, \gamma^\sim)$ where $\gamma \in (0, 1]$ $\Longrightarrow \mu_A(x) > 0$, $\nu_A(x) = 0$; $\mu_B(y) \geq \alpha$, $\nu_B(y) = 0$for any $\alpha \in (0, 1)$ and $A \cap B = (0^\sim, \gamma^\sim)$ where $\gamma \in (0, 1]$. Which is $\alpha - IF - T_2$(iii) space. Hence $\alpha - IF - T_2$(ii) $\Longrightarrow$ $\alpha - IF - T_2$(iii).

Furthermore, one can easily verify that $\alpha - IF - T_2$(i) $\Longrightarrow$ $\alpha - IF - T_2$(iii).

None of the reverse implications is true in general as can be seen from the following examples.

Example 2.4.1 Let $X = \{x, y\}$and$t$be the intuitionistic fuzzy topology on X generated by $\{A, B\}$ where$A = \{x, 0.3, 0\}$ and $B = \{y, 0.4, 0\}$. For $\alpha = 0.3$, we see that the IFTS $(X, t)$ is $\alpha - IF - T_2$(ii) but not $\alpha - IF - T_2$(i).

Examples 2.4.2Let $X = \{x, y\}$ and$t$be the intuitionistic fuzzy topology on$X$generated by$\{A, B\}$ where$A = \{x, 0.2, 0\}$ and $B = \{y, 0.6, 0\}$. For $\alpha = 0.4$, we see that the IFTS $(X, t)$ is $\alpha - IF - T_2$(iii) but not $\alpha - IF - T_2$(ii).

**Theorem2.5**Let $(X, t)$ be an intuitionistic fuzzy topological space and $0 < \alpha \leq \beta < 1$, then
(a) $\beta - IF - T_2$(i) $\Longrightarrow$ $\alpha - IF - T_2$(i).
(b) $\beta - IF - T_2$(ii) $\Longrightarrow$ $\alpha - IF - T_2$(ii).
(c) $\beta - IF - T_2$(iii) $\Longrightarrow$ $\alpha - IF - T_2$(iii).

**Proof (a):**Suppose the intuitionistic fuzzy topological space $(X, t)$ is $\beta - IF - T_2$(i). We shall prove that $(X, t)$ is $\alpha - IF - T_2$(i). Since $(X, t)$ is $\beta - IF - T_2$(i), then for all$x, y \in X$, $x \neq y$with $\beta \in (0, 1)$ there exist $A = (\mu_A, \nu_A)$, $B = (\mu_B, \nu_B) \in t$ such that$\mu_A(x) = 1$, $\nu_A(x) = 0$; $\mu_B(y) \geq \beta$, $\nu_B(y) = 0$ and $A \cap B = 0_\sim \Longrightarrow \mu_A(x) = 1$, $\nu_A(x) = 0$; $\mu_B(y) \geq \alpha$, $\nu_B(y) = 0$ and $A \cap B = 0_\sim$as $0 < \alpha \leq \beta < 1$.Which is $\alpha - IF - T_2$(i). Hence $\beta - IF - T_2$(i) $\Longrightarrow$ $\alpha - IF - T_2$(i).The proofs that$\beta - IF - T_2$(ii) $\Longrightarrow$ $\alpha - IF - T_2$(ii) and$\beta - IF - T_2$(iii) $\Longrightarrow$ $\alpha - IF - T_2$(iii) are similar.
None of the reverse implications is true in general as can be seen from the following examples.

Example 2.5.1 Let$X = \{x, y\}$ and let$t$be the intuitionistic fuzzy topology on$X$generated by $\{A, B\}$ where$A = \{x, 1, 0\}$ and $B = \{y, 0.6, 0\}$. For $\alpha = 0.5$ and $\beta = 0.7$, it is clear that the IFTS $(X, t)$ is$\alpha - IF - T_2$(i) but not $\beta - IF - T_2$(i).

Example 2.5.2 Let $X = \{x, y\}$and let$t$be the intuitionistic fuzzy topology on$X$generated by $\{A, B\}$ where$A = \{x, 0.5, 0\}$ and $B = \{y, 0.4, 0\}$. For $\alpha = 0.4$ and $\beta = 0.8$, it is clear that the IFTS $(X, t)$ is $\alpha - IF - T_2$(ii) but not $\beta - IF - T_2$(ii).

Example 2.5.3 Let $X = \{x, y\}$ and let $t$ be the intuitionistic fuzzy topology on $X$ generated by $\{A, B\}$ where $A = \{x, 0.4, 0\}$ and $B= \{y, 0.5, 0\}$. For $\alpha = 0.5$ and $\beta=0.6$, it is clear that the IFTS $(X, t)$ is $\alpha - IF - T_2(iii)$ but not $\beta - IF - T_2(iii)$.

**Theorem2.6** Let $(X, t)$ be an intuitionistic fuzzy topological space, $U \subseteq X$ and $t_U=\{ A|U : A \in t\}$ and $\alpha \in (0, 1)$, then

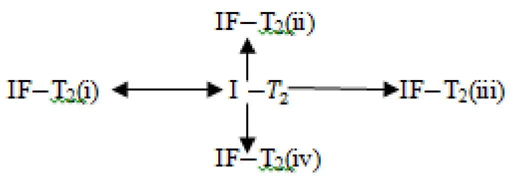
(a) $(X, t)$ is $IF - T_2(i) \Longrightarrow (U, t_U)$ is $IF - T_2(i)$.
(b) $(X, t)$ is $IF - T_2(ii) \Longrightarrow (U, t_U)$ is $IF - T_2(ii)$.
(c) $(X, t)$ is $IF - T_2(iii) \Longrightarrow (U, t_U)$ is $IF - T_2(iii)$.
(d) $(X, t)$ is $IF - T_2(iv) \Longrightarrow (U, t_U)$ is $IF - T_2(iv)$.
(e) $(X, t)$ is $\alpha - IF - T_2(i) \Longrightarrow (U, t_U)$ is $\alpha - IF - T_2(i)$.
(f) $(X, t)$ is $\alpha - IF - T_2(ii) \Longrightarrow (U, t_U)$ is $\alpha - IF - T_2(ii)$.
(g) $(X, t)$ is $\alpha - IF - T_2(iii) \Longrightarrow (U, t_U)$ is $\alpha - IF - T_2(iii)$.

The proofs (a), (b), (c), (d), (e), (f), (g) are similar. As an example we proved (e).

**Proof(e):** Suppose $(X, t)$ is the intuitionistic fuzzy topological space and is also $\alpha - IF - T_2(i)$. We shall prove that $(U, t_U)$ is $\alpha - IF - T_2(i)$. Let $x, y \in U$ with $x \neq y$, then $x, y \in X$ with $x \neq y$ as $U \subseteq X$. Since $(X, t)$ is $\alpha - IF - T_2(i)$, then there exists $A=(\mu_A, \nu_A)$, $B=(\mu_B, \nu_B) \in t$ such that $\mu_A(x) = 1, \nu_A(x) = 0$; $\mu_B(y) \geq \alpha$, $\nu_B(y) = 0$ and $A \cap B = (0^{\sim}, \gamma^{\sim})$ where $\gamma \in (0, 1] \Longrightarrow \mu_A|U(x) = 1, \nu_A|U(x) = 0; \mu_B|U(y) \geq \alpha, \nu_B|U(y) = 0$ and $A \cap B = (0^{\sim}, \gamma^{\sim})$ where $\gamma \in (0, 1]$. Since $\{(\mu_A|U, \nu_A|U), (\mu_B|U, \nu_B|U)\} \in t_U \Longrightarrow \{B|U, C|U\} \in t_U$. Hence, it is clear that the intuitionistic fuzzy topological space $(U, t_U)$ is $\alpha - IF - T_2(i)$.

**Definition2.7** An intuitionistic topological space ( ITS, in short) $(X, \tau)$ is called intuitionistic $T_2$ –space ($I - T_2$ space ) if for all $x, y \in X$, $x \neq y$ there exists $C = (C_1, C_2)$, $D = (D_1, D_2) \in \tau$ such that $x \in C_1$, $y \in D_1$ and $C \cap D = \phi_{\sim}$.

**Theorem2.8** Let $(X, \tau)$ be an intuitionistic topological space and let $(X, t)$ be an intuitionistic fuzzy topological space. Then we have the following implications:




**Proof:** Suppose $(X, \tau)$ is $I - T_2$ space. We shall prove that $(X, t)$ is $IF - T_2(i)$. Since $(X, \tau)$ is $I - T_2$, then for all $x, y \in X, x \neq y$ there exists $C = (C_1, C_2)$, $D = (D_1, D_2) \in \tau$ such that $x \in C_1, y \in D_1$ and $C \cap D = \phi_{\sim} \Longrightarrow 1_{C_1}(x) = 1$, $1_{D_1}(y) = 1$ and $C \cap D = \phi_{\sim} \Longrightarrow 1_{C_1}(x) = 1$, $1_{C_2}(x) = 0$; $1_{D_1}(y) = 1$, $1_{D_2}(y) = 0$ and $C \cap D = \phi_{\sim}$. Let $1_{C_1} = \mu_A$, $1_{C_2} = \upsilon_A$, $1_{D_1} = \mu_B$, $1_{D_2} = \upsilon_B$ then $\mu_A(x) = 1, \upsilon_A(x) = 0; \mu_B(y) = 1, \upsilon_B(y) = 0$ and $A \cap B = 0_{\sim}$. Since $\{(\mu_A, \upsilon_A), (\mu_B, \upsilon_B)\} \in t \Longrightarrow (X, t)$ is $IF - T_2(i)$. Hence $I - T_2 \Longrightarrow IF - T_2(i)$.

Conversely, suppose $(X, t)$ is $IF - T_2(i)$. We shall show that $(X, \tau)$ is $I - T_2$. Since $(X, t)$ is $IF - T_2(i)$, then for all $x, y \in X, x \neq y$ there exists $A = (\mu_A, \upsilon_A)$, $B = (\mu_B, \upsilon_B) \in t$ such that $\mu_A(x) = 1, \upsilon_A(x) = 0; \mu_B(y) = 1, \upsilon_B(y) = 0$ and $A \cap B = 0_{\sim}$. Let $C_1 = \mu_A^{-1}\{1\}$, $D_1 = \mu_B^{-1}\{1\} \Longrightarrow x \in C_1, y \in D_1$ and $C \cap D = \phi_{\sim}$. Since $\{(C_1, C_2), (D_1, D_2)\} \in \tau \Longrightarrow (X, \tau)$ is $I - T_2$. Hence $IF - T_2(i) \Longrightarrow I - T_2$. Therefore $I - T_2 \Longleftrightarrow IF - T_2(i)$

Furthermore, it can be shown that $I - T_2 \Longrightarrow IF - T_2(ii)$, $I - T_2 \Longrightarrow IF - T_2(iii)$ and $I - T_2 \Longrightarrow IF - T_2(iv)$.

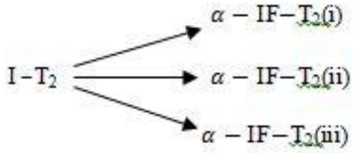
None of the reverse implications is true in general as can be seen from the following examples.

Examples2.8.1 Let $X = \{x, y\}$ and $t$ be the intuitionistic fuzzy topology on $X$ generated by $\{A, B\}$ where $A= \{x, 1, 0\}$ and $B= \{y, 0.3, 0\}$, it is clear that the IFTS $(X, t)$ is $IF - T_2(ii)$ but not corresponding $I - T_2$.

Examples2.8.2 Let $X = \{x, y\}$ and $t$ be the intuitionistic fuzzy topology on $X$ generated by $\{A, B\}$ where $A= \{x, o.5, 0\}$ and $B= \{y, 1, 0\}$, it is clear that the IFTS $(X, t)$ is $IF - T_2(iii)$ but not corresponding $I - T_2$ .

Examples2.8.3Let $X = \{x,\ y\}$ andtbe the intuitionistic fuzzy topology onXgenerated by$\{A,\ B\}$ where A= {x, o.2, 0} and B = {y, 0.6, 0}, it is clear that the IFTS $(X, t)$ is IF$-T_2$(iv) but not corresponding I$-$ $T_2$.

**Theorem2.9** Let $( X, \tau )$ be an intuitionistic topological space and let $( X, t )$ be the intuitionistic fuzzy topological space. Then we have the following implications:




**Proof:**Suppose $( X, \tau )$ is $I - T_2$ space. We shall prove that $( X, t )$ is $\alpha - IF - T_2$(i). Since $( X, \tau )$ is $I - T_2$, then for allx, $y \in X$, $x \neq y$ there exists $C = ( C_1, C_2 )$, $D = ( D_1, D_2 ) \in \tau$ such thatx $\in C_1$,y $\in D_1$ and $C \cap D = \phi_{\sim} \Rightarrow 1_{C_1}(x) = 1, 1_{D_1}(y) = 1$ and$C \cap D = \phi_{\sim} \Rightarrow 1_{C_1}(x) = 1, 1_{D_1}(y) \geq \alpha$for any $\alpha \in (0, 1)$ and $C \cap D = \phi_{\sim} \Rightarrow 1_{C_1}(x) = 1, 1_{C_2}(x) = 0$, $1_{D_1}(y) \geq \alpha$, $1_{D_2}(y) = 0$ for any $\alpha \in (0, 1)$ and $C \cap D = \phi_{\sim}$.Let$1_{C_1} = \mu_A$, $1_{C_2} = \upsilon_A$, $1_{D_1} = \mu_B$, $1_{D_2} = \upsilon_B$ then$\mu_A(x) = 1$, $\upsilon_A(x) = 0$;$\mu_B(y) \geq \alpha$, $\upsilon_B(y) = 0$ for any $\alpha \in (0, 1)$ and $A \cap B = 0_{\sim}$.Since $\{(\mu_A, \upsilon_A), (\mu_B, \upsilon_B)\} \in t \Rightarrow ( X, t )$ is $\alpha - IF - T_2$(i). Hence $I - T_2 \Rightarrow \alpha - IF - T_2$(i).

Furthermore, it can be easily shown that $I - T_2 \Rightarrow \alpha - IF - T_2$(ii) and $I - T_2 \Rightarrow \alpha - IF - T_2$(iii).

None of the reverse implications is true in general as can be seen from the following examples.

Example 2.9.1 Let $X = \{x,\ y\}$ and lettbe the intuitionistic fuzzy topology onX generated by $\{A,\ B\}$ whereA= {x, 1, 0} and B= {y, 0.8, 0}. For $\alpha$ =0.7, it is clear that the IFTS $(X, t)$ is $\alpha - IF - T_2$(i) but not corresponding I$-$ $T_2$.

Example 2.9.2 Let $X = \{x,\ y\}$ and lettbe the intuitionistic fuzzy topology onXgenerated by $\{A,\ B\}$ where A = {x, 0.5, 0} and B = {y, 0.6, 0}. For $\alpha$ =0.4, it is clear that the IFTS $(X, t)$ is $\alpha - IF - T_2$(ii) but not corresponding I$-$ $T_2$.

Example 2.9.3 Let $X = \{x,\ y\}$ and lett be the intuitionistic fuzzy topology onX generated by $\{A,\ B\}$ whereA ={x, 0.3, 0} and B ={y, 0.4, 0}. For $\alpha$ =0.4, it is clear that the IFTS $(X, t)$ is $\alpha - IF - T_2$(iii) but not corresponding I$-$ $T_2$.

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