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τ** - gs - Continuous Maps in Topological Spaces

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Abstract: *In this paper, we introduce a new class of maps called τ^{**}- generalized semi continuous maps in topological spaces and study some of its properties and relationship with some existing mappings.*
Key Words: *scl*, τ^{**} -topology, τ^{**}-gs-open set, τ^{**}-gs-closed set, τ^{**}- gs-continuous maps.*

I. Introduction

In 1970, Levine[7] introduced the concept of generalized closed sets as a generalization of closed sets in topological spaces. Using generalized closed sets, Dunham [3] introduced the concept of the closure operator cl* and a topology τ* and studied some of its properties. Pushpalatha, Easwaran and Rajarubi [10] introduced and studied τ*-generalized closed sets, and τ*-generalized open sets. Using τ*generalized closed sets, Easwaran and Pushpalatha [4] introduced and studied τ*-generalized continuous maps.

The purpose of this paper is to introduce and study the concept of a new class of maps, namely τ** -gs-continuous maps. Throughout this paper X and Y are topological spaces on which no separation axioms are assumed unless otherwise explicitly stated. For a subset A of a topological space X, cl(A), scl(A), scl*(A) and A^C denote the closure, semi-closure, generalized semi closure and complement of A respectively.

II. Preliminaries

Definition: 2.1
For the subset A of a topological space X, the generalized semi closure of A (i.e., scl*(A)) is defined as the intersection of all gs-closed sets containing A.

Definition: 2.2
For the subset A of a topological space X, the topology [6]

$$\tau^{**} = \{G : scl^*(G^C) = G^C\}.$$

Definition: 2.3
A subset A of a topological space X is called τ**-generalized semi closed set [6] (briefly τ**-gs-closed) if scl*(A) ⊆ G whenever A ⊆ G and G is τ**-open.

The complement of τ**-generalized semi closed set is called the τ**- generalized semi open set (briefly τ**- open).

Definition: 2.4
The τ**- generalized semi closure operator $cl_{\tau^{**}}$ for a subset A of a topological space (X,τ**) is defined by the intersection of all τ**- generalized semi closed sets containing A.
(i.e.,) $cl_{\tau^{**}}(A) = \cap\{G: A \subseteq G \text{ and } G \text{ is } \tau^{**}\text{-gs-closed}\}$

Definition: 2.5
A map f: X → Y from a topological space X into a topological space Y is called:
1) continuous if the inverse image of every closed set (or open set) in Y is closed(or open) in X.
(2) strongly gs-continuous if the inverse image of each gs-open set of Y is open in X.
(3) semi continuous [12] if the inverse image of each closed set of Y is semi-closed in X.
(4) sg-continuous [12] if the inverse image of each closed set of Y is sg-closed in X.
(5) gs-continuous [13] if the inverse image of each closed set of Y is gs-closed in X.
(6) gsp-continuous [2] if the inverse image of each closed set of Y is gsp-closed in x.
(7) αg-continuous [5] if the inverse image of each closed set of Y is αg –closed in X.
(8) pre-continuous [9] if the inverse image of each open set of Y is pre-open in X.
(9) α-continuous [10] if the inverse image of each open set of Y is α-open in X.

(10) sp-continuous [1] if the inverse image of each open set of Y is semi-preopen in X.

Remark: 2.6

1) In [6] it has been proved that every closed set is τ** - gs closed.
2) In [6] it has been proved that every gs-closed set in X is τ** -gs closed.

III. τ** - gs- Continuous Maps In Topological Spaces

In this section, we introduce a new class of map namely τ**-generalized semi continuous map in topological spaces and study some of its properties and relationship with some existing mappings.

Definition: 3.1

A map f:X →Y from a topological space X into a topological space Y is called τ** -generalized continuous map(briefly τ** - gs-continuous) if the inverse image of every closed set in Y is τ** -gs-closed in X.

Theorem: 3.2

Let f : X →Y be a map from a topological space (X, τ**) into a topological space (Y,σ**).
The following statements are equivalent:

a) f is τ** - gs-continuous.
b) the inverse image of each open set in Y is τ** - gs-open in X.

(ii) If f : X →Y is τ** - gs-continuous, then $f(cl_{\tau^{**}}(A)) \subseteq cl(f(A))$ for every subset A of X.

Proof:

Assume that f : X→ Y is τ** - gs-continuous. Let G be open in Y. Then G^C is closed in Y. Since f is τ** - gs-continuous, $f^{-1}(G^C)$ is τ** - gs-closed in X.
But $f^{-1}(G^C) = X – f^{-1}(G)$. Thus $X – f^{-1}(G)$ is τ** - gs-closed in X.
Therefore (a)⟹(b).

Conversely, assume that the inverse image of each open set in Y is τ** - gs-open in X.
Let F be any closed set in Y. Then F^C is open in Y. By assumption, $f^{-1}(F^C)$ is τ** - gs-open in X. But $f^{-1}(F^C) = X - f^{-1}(F)$. Therefore, $X - f^{-1}(F)$ is τ** - gs-open in X and so $f^{-1}(F)$ is τ** - gs-closed in X. Therefore, f is τ** - gs-continuous.
Hence (b) ⟹(a). Thus (a) and (b) are equivalent.

(ii) Assume that f is τ** - gs-continuous. Let A be any subset of X, f(A) is a subset of Y. Then cl(f(A))) is a closed subset of Y.
Since f is τ** - gs-continuous, $f^{-1}(cl(f(A)))$ is τ** - gs-closed in X and it containing A. But
$cl_{\tau^{**}}(A)$ is the intersection of all τ** - gs-closed sets containing A.
$cl_{\tau^{**}}(A) \subseteq f^{-1}(cl(f(A)))$
$\Longrightarrow \quad f(cl_{\tau^{**}}(A)) \subseteq cl(f(A))$.

Theorem: 3.3

If a map f : X →Y from a topological space X into a topological space Y is τ** - gs-continuous then f is not gs-continuous.

Proof:

Let V be a closed set in Y. Then $f^{-1}(V)$ is τ** - gs-closed in X, since f is τ** - gs-continuous. But every τ** - gs-closed set is not gs-closed. Therefore, $f^{-1}(V)$ is not gs-closed in X. Hence f is not gs-continuous.

Theorem: 3.4

If a map f : X →Y from a topological space X into a topological space Y is continuous then it is τ** - gs-continuous but not conversely.

Proof:

Let f : X →Y be continuous. Let V be a closed set in Y. Since f is continuous, $f^{-1}(V)$ is closed in X. By Remark: 2.6(2), $f^{-1}(V)$ is τ** - gs-closed. Thus, f is τ** - gs-continuous.
The converse of the theorem need not be true as seen from the following example.

Example:

Let X =Y = {a,b,c}, τ = {X,Φ,{c}} and **σ** = {Y,Φ, {b},{c},{b,c}}.

Let f :X →Y be an identity map. Then f is τ** - gs-continuous. But f is not continuous. Since for the closed set V = {a} in Y, $f^{-1}(V)$ = {a} is not closed in X.

Theorem: 3.5
If a map f : X →Y from a topological space X into a topological space Y is gs-continuous then it is τ** - gs-continuous but not conversely.

Proof:
Let f : X →Y be gs-continuous. Let V be a closed set in Y. Since f is gs-continuous, $f^{-1}(V)$ is gs-closed in X. Also, by Remark: 2.6(2), $f^{-1}(V)$ is τ** - gs-closed. Then, f is τ** - gs-continuous.
The converse of the theorem need not be true as seen from the following example.

Example:
Let X = Y = {a,b,c}, τ = {X,Φ,{a},{a,b}} and **σ** = {Y,Φ, {a},{c},{a,c}}.
Let f : X →Y be an identity map. Then f is τ** - gs-continuous. But it is not gs-continuous. Since for the closed set V = {a,b} in Y, $f^{-1}(V)$ = {a,b} is not gs-closed in X and V = {b} in Y, $f^{-1}(V)$ = {b} is not gs-closed in X.

Theorem: 3.7
If a map f : X→Y from a topological space X into a topological space Y is strongly gs-continuous then it is τ** - gs-continuous but not conversely.

Proof:
Let f : X →Y be strongly gs-continuous. Let V be a closed set in Y, then G is gs-closed. Hence G^C is gs-open in Y. Since f is strongly gs-continuous $f^{-1}(G^C)$ is open in X.
But $f^{-1}(G^c)$ = X – $f^{-1}(G)$. Therefore $f^{-1}(V)$ is closed in X. By Remark: 2.6(1), $f^{-1}(G)$ is τ** - gs-closed in X. Therefore f is τ** -gs-continuous.

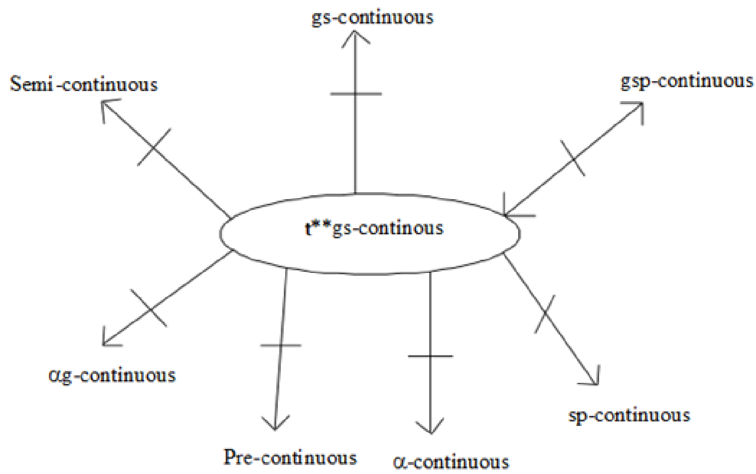
Note:
If f : X → Y and g : Y → Z are both τ** -gs-continuous then the composition gof ; x → z is not τ** -gs-continuous mapping.

Example:
Let X = Y = Z = {a,b,c}, τ = {X,Φ,{a},{b},{a,b}}and**σ** = {Y,Φ, {a}}.
Define f : X → Y by f(a) = c, f(b) = a and f(c) = b and define g : Y → Z by g(a) = a, g(b) = c, g(c) = b. Then f and g are τ** -gs-continuous mappings. The set {b} is closed in Z.
$(gof)^{-1}$ ({b}) = $f^{-1}(g^{-1}(\{b\}))$ = $f^{-1}(\{c\})$ = {a} which is not τ** -gs-closed in X. Hence gof is not τ** -gs-continuous.

Remark: 3.8
From the above discussion, we obtain the following implications.
Let X = Y = {a,b,c}. Let f : X →Y be an identity map.

1) Let τ = {X,Φ,{a},{a,c}} and **σ** = {Y,Φ,{a},{a,b},{a,c}}. Then f is both τ** - gs-continuous and semi-continuous.
2) Let τ = {X,Φ,{c}} and **σ** = {Y,Φ,{b},{c},{b,c}}. Then f is τ** - gs-continuous. But it is not semi-continuous. Since for the closed set V = {a,c} in Y, $f^{-1}(V)$ = {a,c} is not semi-closed in X.
3) Let τ = {X,Φ,{a},{b},{a,b},{a,c}} and **σ** = {Y,Φ, {a},{a,c}}. Then f is both τ** - gs-continuous and sg-continuous.
4) Let τ = {X,Φ,{a}} and **σ** = {Y,Φ,{b},{c},{b,c},{a,c}}. Then f is τ** - gs-continuous. But it is not gs-continuous. Since for the closed set V = {a} in Y, $f^{-1}(V)$ = {a} is not gs- closed in X.
5) Let τ = {X,Φ,{c}} and **σ** = {Y,Φ,{a},{b}{b,c},{a,b}}. Then f is τ** - gs-continuous. But it is not gsp-continuous. Since for the closed set V = {c} in Y, $f^{-1}(V)$ = {c} is not gsp-closed in X.
6) Let τ = {X,Φ,{a,c}} and **σ** = {Y,Φ,{a},{c},{a,b},{a,c}}. Then f is gsp-continuous. But it is not τ** - gs-continuous. Since for closed set V = {c} in Y, $f^{-1}(V)$ = {c} is not τ** - gs-closed in X.
7) Let τ = {X,Φ,{b}} and **σ** = {Y,Φ,{c},{a,c}}. Then f is τ** - gs-continuous. But it is not αg-continuous. Since for the closed set V = {b} in Y, $f^{-1}(V)$ = {b} is not αg-closed in X.
8) Let τ = {X,Φ,{a}} and **σ** = {Y,Φ,{c},{a,c},{b,c}}. Then f is τ** - gs-continuous. But it is not pre-continuous. Since for the closed set V = {b,c} in Y, $f^{-1}(V)$ = {b,c} is not pre-open in X.

9) Let τ= {X,Φ,{a},{b,c}} and **σ** = {Y,Φ,{a},{c},{a,c},{b,c}}. Then f is τ** - gs- continuous. But it is not α-continuous. Since for the closed set V = {a,c} in Y,$f^{-1}(V)$ = {a,c} is not α-open in X.
10) Let τ = {X,Φ,{a},{a,c} and **σ** = {Y,Φ,{a},{a,b},{a,c}}. Then f is both sp-continuous and τ** -gs-continuous.
11) Let τ = {X,Φ,{c}} and **σ** = {Y,Φ,{b},{c},{a,c},{b,c}}. Then f is τ** -gs-continuous. But it is not sp-continuous. Since for the open set V= {b} in Y,$f^{-1}(V)$ = {b} is not sp-open in X.



IV. Conclusion

The class of τ** -gs-continuous maps defined using τ** -gs-closed sets. The τ** -gs-closed sets can be used to derive a new homeomorphism, connectedness, compactness and new separation axioms. This concept can be extended to bitopological and fuzzy topological spaces.

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