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# On Generalized Complex Space Forms

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***Abstract:*** *The object of the present paper is to characterize generalized complex space forms satisfying certain curvature conditions on conhormonic curvature tensor and concrcular curvature tensor. In this paper we study conhormonic semisymetric curvature, conhormonic flat, concircular flat. Also we studied copmlex space form satisfying* $N.S = 0$, $\tilde{C}.S = 0$.

***Key Words:*** *Generalized complex space forms, Conhormonic semisymetric, Conhormonic flat, Concircular flat.*
***Ams Subject Classification (2010)**: 53C15, 53C20, 53C21, 53C25, 53D10, 53C55*;

## I. Introduction

In 1989 the author Olszak. Z. [7] has worked on existence of generalized complex space form. The authors U.C. De and A. Sarkar studied nature of a generalized Sasakian space form under some conditions on projetctive curvature tensor [3]. They also studied generalized Sasakian space forms with vanishing quasi-conformal curvature tensor and investigated quasi-conformal flat generalized Saskian space form. The authors Venkatesha and B.Sumangala [10], Mehmet Atceken [6] studied generalized space form satisfying certain conditions on M-projective curvature tensor and concircular curvature tensor. Motivated by these ideas, in this paper, we made an attempt to study conhormonic and concircular curvature tensors in generalized complex space forms.

## II. Preliminaries

A Kaehler manifold is an even-dimensional manifold $M^n$, where $n = 2k$ with a complex structure J and a positive-definite metric g which satisfies the following conditions [9]

$J^2 = -I$, $g(JX, JY) = g(X, Y)$ and $\nabla J = 0$.

Where $\nabla$ means covariant derivation according to the Levi-civita connection. Let $(M, J, g)$ be a Kaehlermanifold with constant holomorphic sectional curvature c. It is said to be a complex space form if the curvature tensor is of the form

$$R(X,Y)Z = \frac{c}{4}[g(Y,Z)X - g(X,Z)Y + g(Z,JY)JX - g(Z,JX)JY + g(X,JY)JZ]. \quad (2.1)$$

An almost Hermitian manifold M is called a generalized complex space form $M(f_1, f_2)$ if its Riemannian curvature tensor R satisfies

$$R(X,Y)Z = f_1\{g(Y,Z)X - g(X,Z)Y\} + f_2\{g(X,JZ)JY - g(Y,JZ)JX + 2g(X,JY)JZ\} \quad (2.2)$$

for all $X, Y, Z \in TM$ where $f_1$ and $f_2$ are smooth functions on M.

For generalized complex space form $M(f_1, f_2)$ we have

$$S(X,Y) = \{(n_1 - 1)f_1 + 3f_2\}g(X,Y), \quad \backslash \quad (2.3)$$

$$QX = [(n_1 - 1)f_1 + 3f_2]X, \quad (2.4)$$

$$r = n_1\{(n_1 - 1)f_1 + 3f_2\}, \quad \text{Where } n_1 = 2n \quad (2.5)$$

S is the Ricci tensor, Q is the Ricci operator and r is scalar curvature of the space form $M(f_1, f_2)$.

Given an $n_1$ dimensional where $n_1 = 2k$ a Kaehler manifold $(M, g)$ the concircular curvature tensor $\tilde{C}$ and the conhormonic curvature tensor N are given by

$$\tilde{C}(X,Y)Z = R(X,Y)Z - \frac{r}{n_1(n_1-1)}[g(Y,Z)X - g(X,Z)Y]. \quad (2.6)$$

$$N(X,Y)Z = R(X,Y)Z - \frac{1}{n_1-2}[S(Y,Z)X - S(X,Z)Y + g(Y,Z)QX - g(X,Z)QY]. \quad (2.7)$$

Two important curvature properties are flatness and symmetry. As a generalization of locally symmetric spaces, the notion of semisymmetric space is defined by $R(X,Y).R = 0$, where $R(X,Y)$ acts on as a derivation. A $n_1-$dimensional generalized complex space form is called conhormonically semisymmertic if it satisfies $R.N = 0$ where R is the Riemannian curvature and N is conhormonic curvature tensor of the space form.

**Theorem 2.1.** If $n_1$ −dimensional $(n_1 \geq 2)$ generalized complex space form $M(f_1, f_2)$ satisfies $R.N = 0$ then it is Einstein.

**Proof -** Consider

$$R.N = 0,$$
$$(R(X,Y).N)(U,V,W) = 0,$$
$$R(X,Y)N(U,V)W - N(R(X,Y)U,V)W - N(U,R(X,Y)V)W - N(U,V)R(X,Y)W = 0.$$

Taking inner product with Z we have

$$g(R(X,Y)N(U,V)W,Z) - g(N(R(X,Y)U,V)W,Z) - g(N(U,R(X,Y)V)W,Z) - g(N(U,V)R(X,Y)W,Z) = 0.$$

Using equations (2.2), (2.3), (2.4) and (2.7) and putting $X = V = Y = Z = e_i$ where $e_i$ is an orthonormal basis of the tangent space at each point of the manifold and taking summation over $i(1 \le i \le n_1)$ we get after simplification that

$$S(U,W) = \frac{-[(2n_1^2 - 8n_1 + 2)f_1 + (3n_1 - 12)f_2]}{[(4n_1^2 - n_1 + 3)f_1 + 6n_1 f_2 + 2]} r.g(U,W).$$

This implies $M(f_1, f_2)$ is an Einstein manifold.

**Theorem 2.2.** If $n_1$ −dimensional $(n_1 > 2)$ generalized complex space form $M(f_1,f_2)$ is conhormonically flat then $M(f_1,f_2)$ is an Einstein manifold.

**Proof:-** If $M(f_1,f_2)$ is conhormonically flat
i.e.,$N(X,Y)Z = 0$ equation (2.7) implies

$$R(X,Y)Z = \frac{1}{n_1 - 2}[S(Y,Z)X - S(X,Z)Y + g(Y,Z)QX - g(X,Z)QY].$$

Using equation (2.3) and (2.4)

$$R(X,Y)Z = \frac{2}{n_1-2}[(n_1 - 1)f_1 + 3f_2][g(Y,Z)g(X,W) - g(X,Z)g(Y,W)],$$

putting $Y = Z = e_i$ where $e_i$ is an orthonormal basis of the tangent space at each point of the manifold and taking summation over $i(1 \le i \le n_1)$ we get after simplification that

$$S(X,W) = \frac{r}{n_1(n_1-1)(n_1-2)} g(X,W).$$

Implies $M(f_1,f_2)$ is an Einstein manifold

**Theorem 2.3**. If generalized complex space form $M(f_1,f_2)$ satisfies $N.S = 0$ is either Ricci flat or $(n_1 - 1f1+3f2=0$

**Proof:-** Consider

$$N.S = 0,$$
$$S(N(X,Y)U,V) + S(U,N(X,Y)V) = 0.$$

Using equations (2.3), and (2.7) and putting $Y = U = e_i$ where $e_i$ is an orthonormal basis of the tangent space at each point of the manifold and taking summation over $i(1 \le i \le n_1)$ we get

$$\frac{2n_1-4}{n_1-2}\{(n_1 - 1)f_1 + 3f_2\}S(X,V) = 0, \text{ where } (n_1 > 2).$$

Implies either $S(X,V) = 0$ or $(2n_1 - 1)f_1 + 3f_2 = 0$

**Theorem 2.4**. If the generalized complex space form $M(f_1,f_2)$ is concircularly flat then $M(f_1,f_2)$ is Einstein manifold.

**Proof:-** If $M(f_1,f_2)$ is concircularally flat
i.e., $\tilde{C}(X,Y)Z = 0$ Using equation (2.6) we have

$$R(X,Y)Z = \frac{r}{n_1(n_1-1)}[g(Y,Z)X - g(X,Z)Y]$$
$$R(X,Y,Z,W) = \frac{r}{n_1(n_1-1)}[g(Y,Z)g(X,W) - g(X,Z)g(Y,W)]$$

putting $Y = Z = e_i$ where $e_i$ is an orthonormal basis of the tangent space at each point of the manifold and taking summation over $i(1 \le i \le n_1)$ we get after simplification that

$$S(X,W) = \frac{r}{n_1}.g(X,W).$$

Implies .$M(f_1,f_2)$ is an Einstein manifold.

**Theorem 2.5. .** If $n_1$ −dimensional $(n_1 > 2)$ generalized complex space form $M(f_1,f_2)$ stisfying $\tilde{C}.S = 0$, is either Ricci flat or $(n_1 - 1)f_1 + 3f_2 = 0$

**Proof:** Consider

$$\tilde{C}.S = 0,$$
$$S(\tilde{C},(X,Y)U,V) + S(U,\tilde{C}(X,Y)V) = 0.$$

Using equations (2.3), and (2.6) and putting $Y = U = e_i$ where $e_i$ is an orthonormal basis of the tangent space at each point of the manifold and taking summation over $i(1 \leq i \leq n_1)$ we get

$$\{(n_1 - 1)f_1 + 3f_2\}S(X,V) = 0.$$

Implies either Ricci flat or $\{(n_1 - 1)f_1 + 3f_2\} = 0.$

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