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Spectral Continuity: (p, r) - $\mathcal{A}\ \mathcal{P}$ And (p, k) - $\mathcal{Q}$

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Abstract: *An operator $T \in B(H)$ is said to be absolute - (p, r) - paranormal operator if $\left\| |T|^p |T^*|^r x \right\|^r \|x\| \geq \left\| |T^*|^r x \right\|^{p+r}$ for all $x \in H$ and for positive real number p > 0 and r > 0, where T = U |T| is the polar decomposition of T. In this paper, we prove that continuity of the set theoretic functions spectrum, Weyl spectrum, Browder spectrum and essential surjectivity spectrum on the classes consisting of (p, k) - quasihyponormal operators and absolute - (p, r) - paranormal operators.*

Keywords: *absolute - (p, r) - paranormal operator, Weyl's theorem, Single valued extension property, Continuity of spectrum, Fredholm, B – Fredholm*

I. Introduction and Preliminaries

Let H be an infinite dimensional complex Hilbert space and B(H) denote the algebra of all bounded linear operators acting on H. Every operator T can be decomposed into T = U |T| with a partial isometry U, where |T| = $\sqrt{T^*T}$. In this paper, T = U |T| denotes the polar decomposition satisfying the kernel condition N(U) = N(|T|). Yamazaki and Yanagida [23] introduced absolute - (p, r) - paranormal operator. It is a further generalization of the classes of both absolute - k - paranormal operators and p - paranormal operators as a parallel concept of class A(p, r). An operator $T \in B(H)$ is said to be absolute - (p, r) - paranormal operator, denoted by (p, r) - $\mathcal{A}\ \mathcal{P}$, if $\left\| |T|^p |T^*|^r x \right\|^r \|x\| \geq \left\| |T^*|^r x \right\|^{p+r}$ for every unit vector x or equivalently $\left\| |T|^p |T^*|^r x \right\|^r \geq \left\| |T^*|^r x \right\|^{p+r}$ for all $x \in H$ and for positive real numbers p > 0 and r > 0. It is also proved that T = U |T| is absolute - (p, r) - paranormal operator for p > 0 and r > 0 if and only if $r\,|T|^r U^* |T|^{2p} U |T|^r - (p + r)\lambda^p |T|^{2r} + p\,\lambda^{p+r} I \geq 0$ for all real λ. Evidently,

a (k, 1) - $\mathcal{A}\ \mathcal{P}$ operator is absolute - k - paranormal;
a (p, p) - $\mathcal{A}\ \mathcal{P}$ operator is p - paranormal;
a (1, 1) - $\mathcal{A}\ \mathcal{P}$ operator is paranormal [23].

An operator $T \in B(H)$ is said to be (p, k) - quasihyponormal operator, denoted by (p, k) - $\mathcal{Q}$, for some 0 < p ≤ 1 and integer k ≥ 1 if $T^{*k}(|T|^{2p} - |T^*|^{2p})T^k \geq 0$. Evidently,

a (1, k) - $\mathcal{Q}$ operator is k - quasihyponormal;
a (1, 1) - $\mathcal{Q}$ operator is quasihyponormal;
a (p, 1) - $\mathcal{Q}$ operator is k - quasihyponormal or quasi - p - hyponormal ([8], [10]),
a (p, 0) - $\mathcal{Q}$ operator is p - hyponormal if 0 < p < 1 and hyponormal if p = 1.

If $T \in B(H)$, we write N(T) and R(T) for null space and range of T, respectively. Let $\alpha(T)$ = dim N(T) = dim ($T^{-1}(0)$), $\beta(T)$ = dim N(T^*) = dim (H / T(H)), $\sigma(T)$ denote the spectrum and $\sigma_a(T)$ denote the approximate point spectrum. Then $\sigma(T)$ is a compact subset of the set $\mathbb{C}$ of complex numbers. The function σ viewed as a function from B(H) into the set of all compact subsets of $\mathbb{C}$, with its hausdroff metric, is know to be an upper semi - continuous function [14, Problem 103], but it fails to be continuous [14, Problem 102].

Also we know that σ is continuous on the set of normal operators in B(H) extended to hyponormal operators [14, Problem 105]. The continuity of σ on the set of quasihyponormal operators (in B(H)) has been proved by Erevenko and Djordjevic [10], the continuity of σ on the set of p - hyponormal has been proved by Duggal and Djordjevic [9], and the continuity of σ on the set of G_1 - operators has been proved by Luecke [17].

An operator $T \in B(H)$ is called Fredholm if it has closed range, finite dimensional null space and its range has finite co - dimension. The index of a Fredholm operator is given by i(T) = $\alpha(T)$ - $\beta(T)$. The ascent of T, asc (T), is the least non - negative integer n such that $T^{-n}(0) = T^{-(n+1)}(0)$ and the descent of T, dsc (T), is the least non - negative integer n such that $T^n(H) = T^{(n+1)}(H)$. We say that T is of finite ascent (resp., finite descent) if asc (T - λ I) < ∞ (resp., dsc (T - λ I) < ∞) for all complex numbers λ. An operator T is said to be left semi - Fredholm (resp., right semi - Fredholm), $T \in \Phi_+(H)$ (resp., $T \in \Phi_-(H)$) if TH is closed and the deficiency index $\alpha(T)$ = dim ($T^{-1}(0)$) is finite (resp., the deficiency index $\beta(T)$ = dim (H \ TH) is finite); T is semi - Fredholm if it is either left semi - Fredholm or right semi - Fredholm, and T is Fredholm if it is both left and right semi - Fredholm. The semi - Fredholm index of T, ind (T), is the number ind (T) = $\alpha(T)$ - $\beta(T)$. An operator T is called Weyl if it is Fredholm of index zero and Browder if it is Fredholm of finite ascent and descent. Let $\mathbb{C}$ denote the set of complex numbers. The Weyl spectrum $\sigma_w(T)$ and the Browder spectrum $\sigma_b(T)$ of T are the sets $\sigma_w(T)$ = { $\lambda \in \mathbb{C}$: T - λ is not Weyl} and $\sigma_b(T)$ = { $\lambda \in \mathbb{C}$: T - λ is not Browder}.

Let $\pi_0(T)$ denote the set of Riesz points of T (i.e., the set of $\lambda \in \mathbb{C}$ such that T - λ is Fredholm of finite ascent and descent [7]) and let $\pi_{00}(T)$ and iso $\sigma(T)$ denotes the set of eigen values of T of finite geometric multiplicity and isolated points of the spectrum. The operator $T \in B(H)$ is said to satisfy Browder's theorem if $\sigma(T) \setminus \sigma_w(T) = \pi_0(T)$ and T is said to satisfy Weyl's theorem if $\sigma(T) \setminus \sigma_w(T) = \pi_{00}(T)$. In [15], Weyl's theorem for T implies Browder's theorem for T, and Browder's theorem for T is equivalent to Browder's theorem for T^*.

Berkani [5] has called an operatot $T \in B(X)$ as B - Fredholm if there exists a natural number n for which the induced operator $T_n : T^n(X) \to T^n(X)$ is Fredholm. We say that the B - Fredholm operator T has stable index if ind (T - λ) ind (T - μ) $\geq$ 0 for every λ, μ in the B - Fredholm region of T.

$\alpha(T)$ - $\beta(T)$

The essential spectrum $\sigma_e(T)$ of $T \in B(H)$ is the set $T \in B(H)$ = { $\lambda \in \mathbb{C}$: T - λ is not Fredholm}. Let acc $\sigma(T)$ denote the set of all accumulation points of $\sigma(T)$, then $\sigma_e(T) \subseteq \sigma_w(T) \subseteq \sigma_b(T) \subseteq \sigma_e(T) \bigcup$ acc $\sigma(T)$. Let $\pi_{a0}(T)$ be the set of $\lambda \in \mathbb{C}$ such that λ is an isolated point of $\sigma_a(T)$ and 0 < $\alpha(T-\lambda)$ < ∞, where $\sigma_a(T)$ denotes the approximate point spectrum of the operator T. Then $\pi_0(T) \subseteq \pi_{00}(T) \subseteq \pi_{a0}(T)$. We say that a - Weyl's theorem holds for T if $\sigma_{aw}(T) = \sigma_a(T) \setminus \pi_{a0}(T)$, where $\sigma_{aw}(T)$ denotes the essential approximate point spectrum of T (i.e., $\sigma_{aw}(T) = \bigcap\{\sigma_a(T+K) : K \in K(H)\}$ with $K(H)$ denoting the ideal of compact operators on H). Let $\Phi_+(H)$ = {$T \in B(H)$: $\alpha(T)$ < ∞ and T(H) is closed} and $\Phi_-(H)$ = {$T \in B(H)$: $\beta(T)$ < ∞} denote the semigroup of upper semi Fredholm and lower semi Fredholm operators in B(H) and let $\Phi_+^-(H)$ = {T$\in \Phi_+(H)$: ind (T) $\leq$ 0}. Then $\sigma_{aw}(T)$ is the complement in $\mathbb{C}$ of all those λ for which (T - λ) $\in \Phi_+^-(H)$ [19]. The concept of a - Weyl's theorem was introduced by Rakocvic [20]. The concept states that a - Weyl's theorem for T $\Rightarrow$ Weyl's theorem for T, but the converse is generally false. Let $\sigma_{ab}(T)$ denote the Browder essential approximate point spectrum of T.

$$\sigma_{ab}(T) = \bigcap\{\sigma_a(T+K) : TK = KT \text{ and } K \in K(H)\}$$
$$= \{ \lambda \in \mathbb{C} : T - \lambda \notin \Phi_+^-(H) \text{ or asc } (T - \lambda) = \infty \}$$

then $\sigma_{aw}(T) \subseteq \sigma_{ab}(T)$. We say that T satisfies a - Browder's theorem if $\sigma_{ab}(T) = \sigma_{aw}(T)$ [19].

An operator $T \in B(H)$ has the single valued extension property at $\lambda_0 \in \mathbb{C}$, if for every open disc Dλ_0 centered at λ_0 the only analytic function f : D$\lambda_0 \to$ H which satisfies

$$(T - \lambda) f(\lambda) = 0 \text{ for all } \lambda \in D\lambda_0.$$

is the function f $\equiv$ 0. Trivially, every operator T has SVEP at points of the resolvent $\rho(T) = \mathbb{C} / \sigma(T)$; also T has SVEP at $\lambda \in$ iso $\sigma(T)$. We say that T has SVEP if it has SVEP at every $\lambda \in \mathbb{C}$. In this paper, we prove that if $\{T_n\}$ is a sequence of operators in the class (p, k) - Q or (p, r) - $\mathcal{AP}$ which converges in the operator norm topology to an operator T in the same class, then the functions spectrum, Weyl spectrum, Browder spectrum and essential surjectivity spectrum are continuous at T. Note that if an operator T has finite ascent, then it has SVEP and $\alpha(T-\lambda) \le \beta(T-\lambda)$ for all λ [1, Theorem 3.8 and 3.4]. For a subset S of the set of complex numbers, let $\bar{S} = \{\bar{\lambda} : \lambda \in S\}$ where λ denotes the complex number and $\bar{\lambda}$ denotes the conjugate.

II. Main Results

Lemma 2.1 (i) If T $\in$ (p, k) - Q, then asc (T - λ) $\le$ k for all λ.
(ii) If T $\in$ (p, r) - $\mathcal{AP}$, then T has SVEP.

Proof:
(i) Refer [13, Page 146] or [22]
(ii) Refer [21, Theorem 2.8].

Lemma 2.2 If T $\in$ (p, k) - Q $\bigcup$ (p, r) - $\mathcal{AP}$ and $\lambda \in$ iso $\sigma(T)$, then λ is a pole of the resolvent of T.
Proof: Refer [22, Theorem 6] and [21, Proposition 2.1].

Lemma 2.3 If T $\in$ (p, k) - Q $\bigcup$ (p, r) - $\mathcal{AP}$, then T^* satisfies a - Weyl's theorem.
Proof: If T $\in$ (p, k) - Q, the T has SVEP, which implies that $\sigma(T^*) = \sigma_a(T^*)$ by [1, Corollary 2.45]. Then T satisfies Weyl's theorem i.e., $\sigma(T) \setminus \sigma_w(T) = \pi_0(T) = \pi_{00}(T)$ by [13, Corollary 3.7]. Since $\pi_{00}(T) = \overline{\pi_{00}(T^*)} = \overline{\pi_{a0}(T^*)}$, $\sigma(T) = \overline{\sigma(T^*)} = \overline{\sigma_a(T^*)}$ and $\sigma_w(T) = \overline{\sigma_w(T^*)} = \overline{\sigma_{ea}(T^*)}$ by [3, Theorem 3.6(ii)], $\sigma_a(T^*) \setminus \sigma_{ea}(T^*) = \pi_{a0}(T^*)$. Hence if T $\in$ (p, k) - Q, then T^* satisfies a - Weyl's theorem.

If T $\in$ (p, r) - $\mathcal{AP}$, then by [21, Theorem 2.18], T^* satisfies a - Weyl's theorem.

Corollary 2.4 If T $\in$ (p, k) - Q $\bigcup$ (p, r) - $\mathcal{AP}$ and $\lambda \in \sigma_a(T^*) \setminus \sigma_{ea}(T^*) \Rightarrow \lambda \in$ iso$\sigma_a(T^*)$.
Lemma 2.5 If T $\in$ (p, k) - Q $\bigcup$ (p, r) - $\mathcal{AP}$, then asc (T - λ) < ∞ for all λ.
Proof: Since T - λ is lower semi - Fredholm, it has SVEP. We know that from [1, Theorem 3.16] that SVEP implies finite ascent. Hence the proof.
Lemma 2.6 [6, Proposition 3.1] If σ is continuous at a $T^* \in$ B(H), then σ is continuous at T.
Lemma 2.7 [12, Theorem 2.2] If an operator $T \in B(H)$ has SVEP at points $\lambda \notin \sigma_w(T)$, then

$$\sigma \text{ is continuous at T} \Leftrightarrow \sigma_w \text{ is continuous at T} \Leftrightarrow \sigma_b \text{ is continuous at T.}$$

Lemma 2.7 If $\{T_n\}$ is a sequence in (p, k) - Q or (p, r) - $\mathcal{AP}$ which converges in norm to T, then T^* is a point of continuity of σ_{ea}.
Proof: We have to prove that the function σ_{ea} is both upper semi - continuous and lower semi - continuous at T^*. But by [11, Theorem 2.1], we have that the function σ_{ea} is upper semi - continuous at T^*. So we have to

prove that σ_{ea} is lower semi - continuous at T^* i.e., $\sigma_{ea}(T^*) \subset \liminf \sigma_{ea}(T_n^*)$. Assume the contradiction that σ_{ea} is not lower semi - continuous at T^*. Then there exists an $\varepsilon > 0$, an integer n_0, a $\lambda \in \sigma_{ea}(T^*)$ and an ε - neighbourhood $(\lambda)_\varepsilon$ of λ such that $\sigma_{ea}(T_n^*) \bigcap (\lambda)_\varepsilon = \phi$ for all $n \geq n_0$. Since $\lambda \notin \sigma_{ea}(T_n^*)$ for all $n \geq n_0$ implies $T_n^* - \lambda \in \Phi_+^-(H)$ for all $n \geq n_0$, the following implications holds:

$$\text{ind}(T_n^* - \lambda) \leq 0,\ \alpha(T_n^* - \lambda) < \infty \text{ and } (T_n^* - \lambda)H \text{ is closed}$$
$$\Rightarrow \text{ind}(T_n - \overline{\lambda}) \geq 0,\ \beta(T_n - \overline{\lambda}) < \infty$$
$$\Rightarrow \text{ind}(T_n - \overline{\lambda}) = 0,\ \alpha(T_n - \overline{\lambda}) < \beta(T_n - \overline{\lambda}) < \infty$$

(Since $T_n \in$ (p, k) - $\mathcal{Q} \bigcup$ (p, r) - $\mathcal{A}\,\mathcal{P} \Rightarrow \text{ind}(T_n - \overline{\lambda}) \leq 0$ by Lemma 2.1 and Lemma 2.5).

for all $n \geq n_0$. The continuity of the index implies that ind $(T - \overline{\lambda}) = \lim_{n\to\infty}$ ind $(T_n - \overline{\lambda}) = 0$, and hence that$(T - \overline{\lambda})$ is Fredholm with ind $(T - \overline{\lambda}) = 0$. But then $T^* - \lambda$ is Fredholm with ind $(T^* - \lambda) = 0 \Rightarrow T^* - \lambda \in \Phi_+^-(H)$, which is a contradiction. Therefore σ_{ea} is lower semi - continuous at T^*. Hence the proof.

Theorem 2.9 If $\{T_n\}$ is a sequence in (p, k) - $\mathcal{Q}$ or (p, r) - $\mathcal{A}\,\mathcal{P}$ which converges in norm to T, then σ is continuous at T.

Proof: Since T has SVEP by Lemma 2.1, $\sigma(T^*) = \sigma_a(T^*)$. Evidently, it is enough if we prove that $\sigma_a(T^*) \subset \liminf \sigma_a(T_n^*)$ for every sequence $\{T_n\}$ of operators in (p, k) - $\mathcal{Q}$ or (p, r) - $\mathcal{A}\,\mathcal{P}$ such that T_n converges in norm to T. Let $\lambda \in \sigma_a(T^*)$. Then either $\lambda \in \sigma_{ea}(T^*)$ or $\lambda \in \sigma_a(T^*) \setminus \sigma_{ea}(T^*)$.

If $\lambda \in \sigma_{ea}(T^*)$, then proof follows, since

$$\sigma_{ea}(T^*) \subset \liminf \sigma_{ea}(T_n^*) \subset \liminf \sigma_a(T_n^*)$$

If $\lambda \in \sigma_a(T^*) \setminus \sigma_{ea}(T^*)$, then $\lambda \in$ iso $\sigma_a(T^*)$ by Corollary 2.4. Consequently, $\lambda \in \liminf \sigma_a(T_n^*)$ i.e., $\lambda \in \liminf \sigma(T_n^*)$ for all n by [16, Theorem IV. 3.16], and there exists a sequence $\{\lambda_n\}$, $\lambda_n \in \sigma_a(T_n^*)$, such that λ_n converges to λ. Evidently $\lambda \in \liminf \sigma_a(T_n^*)$. Hence $\lambda \in \liminf \sigma_a(T_n^*)$. Now by applying Lemma 2.6, we obtain the result.

Corollary 2.10 If $\{T_n\}$ is a sequence in (p, k) - $\mathcal{Q}$ or (p, r) - $\mathcal{A}\,\mathcal{P}$ which converges in norm to T, then σ, σ_w and σ_b are continuous at T.

Proof: Combining Theorem 2.9 with Lemma 2.7 and Lemma 2.8, we obtain the results.

Let $\sigma_s(T) = \{\lambda : T - \lambda$ is not surjective$\}$ denote the surjectivity spectrum of T and let $\Phi_+^-(H) = \{\lambda : T - \lambda \in \Phi_-(H)$, ind $(T - \lambda) \geq 0\}$. Then the essential surjectivity spectrum of T is the set $\sigma_{es}(T) = \{\lambda : T - \lambda \notin \Phi_+^-(H)\}$.

Corollary 2.11 If $\{T_n\}$ is a sequence in (p, k) - $\mathcal{Q}$ or (p, r) - $\mathcal{A}\,\mathcal{P}$ which converges in norm to T, then $\sigma_{es}(T)$ is continuous at T.

Proof: Since T has SVEP by Lemma 2.1, $\sigma_{es}(T) = \sigma_{ea}(T^*)$ by [1, Theorem 3.65 (ii)]. Then by applying Lemma 2.8 , we obtain the result.

Let $K \subset B(H)$ denote the ideal of compact operators, B(H) / K the Calkin algebra and let $\pi : B(H) \to B(H) / K$ denote the quotient map. Then B(H) / K being a C^*- algebra, there exists a Hilbert space H_1 and an isometric * - isomorphism υ : B(H) / K $\rightarrow$ B(H_1) such that the essential spectrum $\sigma_e(T)$ = $\sigma(\pi(T))$ of T $\in$ B(H) is the spectrum of $\upsilon \circ \pi(T)$ ($\in$ B(H_1)). In general, $\sigma_e(T)$ is not a continuous function of T.

Corollary 2.12 If $\{\pi(T_n)\}$ is a sequence in (p, k) - $\mathcal{Q}$ or (p, r) - $\mathcal{A}\ \mathcal{P}$ which converges in norm to $\pi(T)$, then $\sigma_e(T)$ is continuous at T.

Proof: If $T_n \in$ B(H) is essentially (p, k) - $\mathcal{Q}$ or (p, r) - $\mathcal{A}\ \mathcal{P}$, i.e., if $\pi(T_n)$ $\in$ (p, k) - $\mathcal{Q}$ or (p, r) - $\mathcal{A}\ \mathcal{P}$, and the sequence {T_n} converges in norm to T, then $\upsilon \circ \pi(T)$ ($\in$ B(H_1)) is a point of continuity of σ by Theorem 2.9. Hence σ_e is continuous at T, since $\sigma_e(T)$ = $\sigma(\upsilon \circ \pi(T))$.

Let H(σ(T)) denote the set of functions f that are non - constant and analytic on a neighbourhood of σ(T).

Lemma 2.13 Let $T \in B(X)$ be an invertible (p, r) - $\mathcal{A}\ \mathcal{P}$ and let f $\in$ H(σ(T)). Then $\sigma_{bw}(f(T)) \subset f(\sigma_{bw}(T))$, and if the B - Fredholm operator T has stable index, then $\sigma_{bw}(f(T)) = f(\sigma_{bw}(T))$.

Proof: Let $T \in B(X)$ be an invertible (p, r) - $\mathcal{A}\ \mathcal{P}$, let f $\in$ H(σ(T)), and let g(T) be an invertible function such that $f(\mu) - \lambda = (\mu - \alpha_1)......(\mu - \alpha_n)g(\mu)$. If $\lambda \notin f(\sigma_{bw}(T))$, then $f(T) - \lambda = (T - \alpha_1)......(T - \alpha_n)g(T)$ and $\alpha_i \notin \sigma_{bw}(T)$, i = 1, 2,...., n. Consequently, T - α_i is a B - Fredholm operator of zero index for all i = 1, 2,...., n, which, by [5, Theorem 3.2], implies that $f(T) - \lambda$ is a B - Fredholm operator of zero index. Hence, $\lambda \notin$ $\sigma_{bw}(f(T))$.

Suppose now that T has stable index, and that $\lambda \notin$ $\sigma_{bw}(f(T))$. Then, $f(T) - \lambda = (T - \alpha_1)......(T - \alpha_n)g(T)$ is a B - Fredholm operator of zero index. Hence, by [4, Corollary 3.3], the operator g(T) and T - α_i, i = 1, 2,...., n, are B - Fredholm and

$$0 = \text{ind}\ (f(T) - \lambda) = \text{ind}\ (T - \alpha_1) + ...+ \text{ind}\ (T - \alpha_n) + \text{ind}\ g(T).$$

Since g(T) is an invertible operator, ind (g(T)) = 0; also ind $(T - \alpha_i)$ has the same sign for all i = 1, 2,...., n. Thus ind $(T - \alpha_i)$ = 0, which implies that $\alpha_i \notin \sigma_{bw}(T)$ for all i = 1, 2,...., n, and hence $\lambda \notin f(\sigma_{bw}(T))$.

Lemma 2.14 Let $T \in B(X)$ be an invertible (p, r) - $\mathcal{A}\ \mathcal{P}$ has SVEP, then ind (T - λ) $\leq$ 0 for every $\lambda \in \mathbb{C}$ such that T - λ is B - Fredholm.

Proof: Since T has SVEP by [21, Theorem 2.8]. Then $T|_M$ has SVEP for every invariant subspaces M $\subset$ X of T. From [4, Theorem 2.7], we know that if T - λ is a B - Fredholm operator, then there exist T - λ invariant closed subspaces M and N of X such that X = M $\oplus$ N, (T - λ)$|_M$ is a Fredholm operator with SVEP and (T- λ)$|_N$ is a Nilpotent operator. Since ind (T - λ)$|_M$} $\leq$ 0 by [18, Proposition 2.2], it follows that ind (T - λ) $\leq$ 0.

References

[1]. Aiena. P, Fredholm and Local Spectral Theory with Applications to Multipliers, Kluwer Acad. Pub., 2004.
[2]. Aiena. P and Monsalve. O, Operators which do not have the single valued extension property, J. Math. Anal. Appl., 250 (2000), 435 -- 438.
[3]. Apostol. C, Fialkow. L. A, Herrero. D. A, Voiculescu. D, Approximation of Hilbert space operators, Vol. II, Research Notes in Mathematics., 102, Pitman, Boston (1984).
[4]. Berkani. M, On a class of quasi - Fredholm operators, Inter. Equat. operator Theory., 34 (1999), 244 -- 249.
[5]. Berkani. M, Index of B - Fredholm operators and generalization of the Weyl theorem, Proc. Amer. Math. Soc., 130 (2002), 1717 -- 1723.
[6]. Burlando. L, Noncontinuity of the adjoint of an operator, Proc. Amer. Math. Soc., 128 (2000), 479 -- 486.

[7]. Caradus. S. R, Pfaffenberger. W. E, Bertram. Y, Calkin algebras and algebras of operators on Banach spaces, Marcel Dekker, New York, 1974.

[8]. Campbell. S. I, Gupta. B. C, On k - quasihyponormal operators, Math. Japonic., 23 (1978), 185 -- 189.

[9]. Djordjevic. S. V, On the continuity of the essential approximate point spectrum, Facta Math. Nis., 10 (1995), 69 -- 73.

[10]. Djordjevic. S. V, Continuity of the essential spectrum in the class of quasihyponormal operators, Vesnik Math., 50 (1998), 71 -- 74.

[11]. Djordjevic. S. V, Duggal. B. P, Weyl's theorem and continuity of spectra in the class of p - hyponormal operators, Studia Math., 143 (2000), 23 -- 32.

[12]. Djordjevic. S. V, Han. Y. M, Browder's theorem and spectral continuity, Glasgow Math. J., 42 (2000) 479 -- 486.

[13]. Duggal. B. P, Riesz projections for a class of Hilbert space operators, Lin. Alg. Appl., 407 (2005), 140 -- 148.

[14]. Halmos. P. R, A Hilbert space problem book, Graduate Texts in Mathematics, Springer - Verlag, New York, 1982.

[15]. Harte. R. E, Lee. W. Y, Another note on Weyl's theorem, Trans. Amer. Math. Soc. 349 (1997), 2115 -- 2124.

[16]. Kato. T, Perturbation theory for Linear operators, Springer - verlag, Berlin, 1966.

[17]. Luecke. G. R, A note on spectral continuity and spectral properties of essentially G_1 operators, Pac. J. Math., 69 (1977), 141 -- 149.

[18]. Oudghiri, Weyl's theorem and Browder's theorem for operators satisfying the SVEP, Studia Mathematica., 163 (2004), 85 -- 101.

[19]. Rakocevic. V, On the essential approximate point spectrum II, Mat. Vesnik, 36(1) (1984), 89 -- 97.

[20]. Rakocevic. V, Operators obeying a - Weyl's theorem, Rev. Roumaine Math. Pures Appl., 34 (1989), 915 -- 919.

[21]. Senthilkumar. D, Maheswari Naik. P, Absolute - (p, r) - paranormal operators, International J. of Math. Sci. & Engg. Appls. (IJMSEA), Vol. 5, No. III (May, 2011), 311 -- 322.

[22]. Tanahashi. K, Uchiyama. A, Cho. M, Isolated points of spectrum of (p, k) - quasihyponormal operators, Lin. Alg. Appl., 382 (2004), 221 -- 229

[23]. T. Yamazaki and M. Yanagida, A further generalization of paranormal operators, Scientiae Mathematicae, Vol. 3, No: 1 (2000), 23 -- 31.