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Continuous Sampling Plan for Bulk Production and Bulk Sale

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Abstract: *This paper studies a continuous sampling plan to prevent entries of defective products for sale using theory of queues for bulk production and bulk sale systems. A machine produces random number of products at each production. Any product in a production lot may be good or defective. The sampling plan considered here to prevent defective products to go for sale has three inspection modes. Each lot of production is inspected in mode I and in modes II and III each lot is inspected with some probability $c \geq 0$ and $d > 0$ respectively in order to avoid high cost of inspecting all the lots of products produced. Matrix methods are used to derive the stock level probabilities, the rate of entry of defectives, the expected defective products in the stock, the standard deviation and the coefficient of variation. Two models are treated. In one model, the maximum production size is greater than the maximum sale size and in the second model the maximum sale size is greater than the maximum production size. Numerical cases are treated to illustrate the significance of the continuous sampling plan in reducing the entry rates of defectives. The expected defective products in the stock for sale and all the risk measures are listed and discussed.*

Keywords*: Block Partitioned Methods, Continuous Sampling Plan, Expected Defective Products, Infinitesimal Generator, Rate of Entry of Defective Products.*

I. Introduction

It has been very common that when products are manufactured by a machine in lots some products of the lots are found to be good and some are defective. Because of the cost involved in inspecting all the lots manufactured may be very high, various inspection policies are adopted to reduce this cost. Continuous Sampling Plan (CSP) is introduced to reduce the inspection cost to a manageable level. This may filter the entry of defective lots to some extent but may not be in full. It is of interest to policy makers to know in the long run, the expected number of products and defective products in the stock when a CSP is adopted and the risk values for the same. The CSP to be followed must have relation with number of good products or defectives noticed. The CSP studied in this paper has three inspection modes and the modes change depending on good and defective items found. It is a combination and extension of the CSP mentioned in Dodge [1], Mytalas and Zazanis [2] and Bowker [3] introduced for single production system where CSP was considered to have two inspection modes only with non-zero probability in mode II which is a restriction. Single production system with bulk sale has been studied by Sundar [4]. The risk involved in the CSP studied in this paper are presented and measured by finding the entry of defective rates, the expected defective products in the stock, the standard deviation and the coefficient of variation. Ken block [5] and James c. Cox and Vjollca Sadiraj [6] have discussed the coefficient of variation in detail. There are many measures of risk; one may refer Rockefeller [7]. The products produced form a queue for sale. The present paper examines the performance of the CSP in the case of a continuous time Markov chain model and presents results identifying Neuts [8] matrix structures. Numerical results for theory of queues one may refer to Bini, Latouche and Mein [9]. Matrix analytic methods have been treated by Latouche and Ramaswami [10] and M/M/1 bulk queues with varying rates have been studied by Rama Ganesan, Ramshankar and Ramanarayanan [11]. Fatigue models have been treated by Sundar [12] using matrix methods. Rama Ganesan and Ramanarayanan [13] have studied software issues and fixing the cause and Ramshankar and Ramanarayanan [14] have treated catrascopic models using matrix partition methods.

This paper considers bulk production by a machine which produces good and defective products. The bulk productions of products are inspected for sale. The CSP considered here has three inspection modes. In inspection mode I, the CSP inspects every product in a bulk production until k consecutive bulk productions contain only good products without any defective in any of them. At this point the CSP changes its inspection mode to II of inspecting the next r bulk productions where every bulk is inspected with probability $c \geq 0$ until the CSP finds a bulk with a defective product. When the CSP finds a bulk with a defective product in mode II, it changes its inspection mode to I. If the CSP finds no defective product in those r bulk productions in mode II, it changes its inspection mode to III where the CSP inspects a bulk with probability $d > 0$. If in inspection mode

III, the CSP finds a defective product, it changes its inspection mode to I and if no defective product is noticed by the CSP, it changes its inspection mode to II. The products found to be defective in the inspection modes I, II and III are rejected. The probabilistic inspection procedures considered by the CSP reduce the number of inspections considerably with a view to reduce the inspection cost. This paper treats a bulk production and bulk sale model with the above CSP. So far bulk production models adopting a CSP for preventing defectives have not been treated at any depth. The study here is organized in the following manner. In Section (2) the stock level probabilities are derived using matrix geometric approach when the CSP is adopted for the model in which the maximum bulk production size is greater than the maximum sale size. The performance measures and various stock level probabilities are presented in the stationary case. The expected stock level for sale and the various risk measures such as the expected defective stocks, the standard deviation, the rate of entry of defectives, the variance and the co-efficient of variation are obtained. In Section 3, the model in which the maximum bulk production size is less the maximum bulk sale size is studied. In section (4) numerical cases are treated for illustration.

II. Model (A). Continuous Sampling Plan With Three Modes Where Maximum Production Size Greater Than Maximum Sale Size

2.1Assumptions

(i) At each production a machine produces χ products for sale. The time between two consecutive productions has exponential distribution with parameter λ. The number χ of products at each production has the probability given by P (χ= j) = α_j for $1 \leq j \leq M$ and $\sum_{j=1}^{M} \alpha_j$=1. Each product produced is good with probability p > 0 and is defective with probability q > 0 where p + q =1.

(ii) The productions are inspected for sale by a continuous sampling plan (CSP). The CSP considered here has three inspection modes. In mode I the CSP inspects every single product in a bulk production χ until k consecutive bulk productions with only good products are found. At this point, the CSP changes its inspection mode to II. Here it inspects a bulk with probability c ≥ 0 for the next r bulk productions until it finds a defective product. When the CSP finds a defective product, it changes its mode to I. When the CSP finds no defective product in the r bulk productions then it changes its mode to III where the CSP inspects a bulk production with probability d > 0. In mode III if the CSP finds a bulk with a defective product, it changes its mode to I and if the CSP finds no defective it changes its mode to II. The products found to be defective by the CSP in the bulk productions in the inspection modes I, II and III are rejected and the products which are not rejected are stocks for sale.

(iii) The products are sold at sale epochs with the inter occurrence time between two consecutive sale epochs has exponential distribution with parameter μ. In a sale ψ products are sold at a time with probability P (ψ=i) =β_i , for $1 \leq i \leq N$ where $\sum_1^N \beta_i$ =1. When n products n < N are available, then i products are sold with P (ψ=i) =β_i , for $1 \leq i \leq$ n-1 and n products are sold with probability$\sum_n^N \beta_i$, as sales are only for available number n products.

(iv) The maximum size of production M is greater than the maximum size of sale N.

2.2Analysis

For studying the above model, the state of the system of the continuous time Markov chain X (t) may be defined as follows X (t)={(n, m, j): for $0 \leq n < \infty$; $0 \leq m \leq M-1$; $1 \leq j \leq k+r+1$} (1)

The first two co-ordinates are used to indicate the stock level of products for sale. If the stock level is r ≥ 0, then r is identified with (n, m) for r = n M + m where n and m are non negative integers with n ≥ 0 and 0 ≤ m ≤ M-1. The system X (t) is in the state (n, m, j) for 1 ≤ j ≤ k, n ≥ 0 and 0 ≤ m ≤ M-1, when n M + m products are available for sale, the CSP inspection mode is I and it has found no defective in (j-1) consecutive bulk productions. The system is in the state (n, m, k + i) for 1 ≤ i ≤ r, n ≥ 0, 0 ≤ j ≤ M-1, when n M + m products are available for sale, the CSP inspection mode is II and the CSP has found no defective in (i-1) consecutive bulk productions in mode II. The system is in the state (n, m, k + r +1) when n M + m for n ≥ 0 and 0 ≤ m ≤ M-1 products are available for sale and the CSP inspection mode is III. The joint probability of a bulk production has size j and it has r good products =P (The bulk size χ = j, the number of good products in the bulk is r) = $p_j^r = \alpha_j \binom{j}{r} p^r q^{j-r}$ for $0 \leq r \leq j$ and $1 \leq j \leq M$. Then the probability of no defective product in a bulk of size χ is $p_\chi = \sum_{j=1}^{M} p_j^j = \sum_{j=1}^{M} \alpha_j p^j$ and the probability of at least one defective in a bulk of size χ is $q_\chi = 1 - p_\chi = \sum_{j=1}^{M} \alpha_j \sum_{r=0}^{j-1} \binom{j}{r} p^r q^{j-r}$. (2)

The transition probability matrix governing transitions of the Markov chain describing the CSP is

$$P'=\begin{bmatrix} q_\chi & p_\chi & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 \\ q_\chi & 0 & p_\chi & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ q_\chi & 0 & 0 & \cdots & 0 & p_\chi & 0 & \cdots & 0 & 0 \\ cq_\chi & 0 & 0 & \cdots & 0 & 0 & \gamma & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ cq_\chi & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \gamma \\ dq_\chi & 0 & 0 & \cdots & 0 & \delta & 0 & \cdots & 0 & 0 \end{bmatrix} \tag{3}$$

In (3), γ = 1- c q_χ and δ = 1 – d q_χ and in the above matrix $(P')_{k+r+1,k+1}$ = δ ; $(P')_{i,1}$ =q_χ, for $1 \le i \le k$; $(P')_{i,1}$ = cq_χ, for k +1 $\le$ i $\le$ k + r, and $(Q')_{k+r+1,1} = dq_\chi$. The components of the probability vector w = $(w_1, w_2, w_3,\ldots w_{k+r+1})$ satisfying the equations wP'=1 and we=1 is given by the following w_{k+r+1} = $[\gamma^r p_\chi^k(1\text{-}\gamma)q_\chi]$ / $[(1\text{-}\gamma)(1\text{-}p_\chi^k)$ $(1\text{-}\delta\gamma^r)+q_\chi\, p_\chi^k(1\text{-}\gamma^{r+1})]$; w_i= $(1/p_\chi)^{k-i-1}[\left(\frac{1}{\gamma^r}\right)-\delta]$ w_{k+r+1}, for $1 \le i \le k$; w_i= $[1/\gamma^{k+r-i+1}]$ w_{k+r+1}, for $k+1 \le i \le k + r$. (4)

The fraction of the bulk productions that are inspected by the CSP out of all bulk productions is then given by f=1-$\sum_{i=1}^{r+1} w_{k+i}$+c$\sum_{i=1}^{r} w_{k+i}$+dw_{k+r+1}. (5)

The fraction of bulk productions that are not inspected by the CSP out of all bulk productions = 1-f. When the CSP inspects a bulk, the defective products in the bulk are removed and the number of defective in it is zero. When there is no CSP the expected number of defectives in a bulk is q E (χ). So the expected number of defectives in a production when the CSP is adopted = q E (χ) (1-f). On division by E (χ), this gives the probability of a product is defective when CSP is adopted = q (1-f). The rate of entry of defective products when the CSP is adopted = λ q E (χ) (1-f). (6)

The continuous time Markov chain describing the model has infinitesimal generator Q_A of infinite order which can be presented in block partitioned form with each block of order M (k+r+1). It is given below.

$$Q_A=\begin{bmatrix} B_1 & A_0 & 0 & 0 & . & . & . & \cdots \\ A_2 & A_1 & A_0 & 0 & . & . & . & \cdots \\ 0 & A_2 & A_1 & A_0 & 0 & . & . & \cdots \\ 0 & 0 & A_2 & A_1 & A_0 & 0 & . & \cdots \\ 0 & 0 & 0 & A_2 & A_1 & A_0 & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots & \ddots & \ddots & \ddots \end{bmatrix} \tag{7}$$

In (7) the states of the matrix are listed lexicographically as $\underline{0}, \underline{1}, \underline{2}, \underline{3}, \ldots \underline{n}, \ldots$ Here the state vectors are given as follows. $\underline{n}$ = ((n,0,1),(n,0,2)…(n,0,k+r+1),(n,1,1),(n,1,2)…(n,1,k+r+1),(n,2,1),(n,2,2)…(n,2,k+r+1),…, (n,M-1,1), (n,M-1,2)…(n,M-1, k+r+1) for $0 \le n < \infty$. The matrices B_1 and A_1 have negative diagonal elements, they are of order M (k+r+1) and their off diagonal elements are non- negative. The matrices A_0, and A_2 have nonnegative elements and are of order M (k+r+1) and they are given below. Let the survivor probability of bulk size production be P (χ > j) =P_j =1-$\sum_{i=1}^{j} \alpha_i$, and $P_0 = 1$ for $1 \le j \le M$ -1. (8)

Let the survivor probability of bulk size sale be P (ψ > j) = Q_j =1-$\sum_{i=1}^{j} \beta_i$, and $Q_0 = 1$ for $1 \le j \le N$ -1. (9)

The blocks of component block matrices of Q_A are listed below.

$$A_0 = \begin{bmatrix} \Lambda_M & 0 & \cdots & 0 & 0 & 0 \\ \Lambda_{M-1} & \Lambda_M & \cdots & 0 & 0 & 0 \\ \Lambda_{M-2} & \Lambda_{M-1} & \cdots & 0 & 0 & 0 \\ \Lambda_{M-3} & \Lambda_{M-2} & \ddots & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots & \vdots \\ \Lambda_3 & \Lambda_4 & \cdots & \Lambda_M & 0 & 0 \\ \Lambda_2 & \Lambda_3 & \cdots & \Lambda_{M-1} & \Lambda_M & 0 \\ \Lambda_1 & \Lambda_2 & \cdots & \Lambda_{M-2} & \Lambda_{M-1} & \Lambda_M \end{bmatrix} \tag{10}$$

$$\Lambda_j = \begin{bmatrix} \lambda\Lambda_{j,1} & \lambda p_j^j & 0 & \cdots & 0 & 0 & \cdots & 0 \\ \lambda\Lambda_{j,1} & 0 & \lambda p_j^j & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ \lambda\Lambda_{j,1} & 0 & 0 & \cdots & \lambda p_j^j & 0 & \cdots & 0 \\ c\lambda\Lambda_{j,1} & 0 & 0 & \cdots & 0 & \lambda\gamma_j & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ c\lambda\Lambda_{j,1} & 0 & 0 & \cdots & 0 & 0 & \cdots & \lambda\gamma_j \\ d\lambda\Lambda_{j,1} & 0 & 0 & \cdots & \lambda\delta_j & 0 & \cdots & 0 \end{bmatrix} \tag{12}$$

$$\Lambda_M = \begin{bmatrix} 0 & \lambda p_M^M & 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & 0 & \lambda p_M^M & \cdots & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \lambda p_M^M & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 & \lambda\gamma_M & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & \lambda\gamma_M \\ 0 & 0 & 0 & \cdots & \lambda\delta_M & 0 & \cdots & 0 \end{bmatrix} \tag{11}$$

$$A_2 = \begin{bmatrix} 0 & \cdots & 0 & U_N & U_{N-1} & \cdots & U_2 & U_1 \\ 0 & \cdots & 0 & 0 & U_N & \cdots & U_3 & U_2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & 0 & 0 & 0 & \cdots & U_N & U_{N-1} \\ 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & U_N \\ 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix} \tag{13}$$

The component matrices of (10) are given by (11) and (12) which are matrices of order k+r+1. The symbols of (11), (12) and (13) are as follows. In (11), γ_M = $(1-c)\sum_{r=0}^{M-1} p_M^r + p_M^M$= (1-c)$\alpha_M$ +c$\alpha_M p^M$ and $\delta_M = (1-d)\sum_{r=0}^{M-1} p_M^r$ + p_M^M= (1-d)α_M+d$\alpha_M p^M$. In (12), $\gamma_j = (1-c)\sum_{r=0}^{j-1} p_j^r + p_j^j$= (1-c)$\alpha_j$ +c$\alpha_j p^j$;

$\delta_j = (1-d)\sum_{r=0}^{j-1} p_j^r + p_j^j$ = (1-d)α_j+d$\alpha_j p^j$; $p_j^j = \alpha_j p^j$ and $\Lambda_{j,1} = \sum_{i=j}^{M-1} p_{i+1}^j = \sum_{i=j+1}^{M} \alpha_i \binom{i}{j} p^j q^{i-j}$ for $1 \le j \le$ M-1. In (13), $U_j = \mu\beta_j I$ is a matrix of order k+r+1 for $1 \le j \le N$.

$$A_1 = \begin{bmatrix} \mathcal{Q}_1' & \Lambda_1 & \Lambda_2 & \cdots & \Lambda_{M-N-2} & \Lambda_{M-N-1} & \Lambda_{M-N} & \cdots & \Lambda_{M-2} & \Lambda_{M-1} \\ U_1 & \mathcal{Q}_1' & \Lambda_1 & \cdots & \Lambda_{M-N-3} & \Lambda_{M-N-2} & \Lambda_{M-N-1} & \cdots & \Lambda_{M-3} & \Lambda_{M-2} \\ U_2 & U_1 & \mathcal{Q}_1' & \cdots & \Lambda_{M-N-4} & \Lambda_{M-N-3} & \Lambda_{M-N-2} & \cdots & \Lambda_{M-4} & \Lambda_{M-3} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ U_N & U_{N-1} & U_{N-2} & \cdots & \mathcal{Q}_1' & \Lambda_1 & \Lambda_2 & \cdots & \Lambda_{M-N-2} & \Lambda_{M-N-1} \\ 0 & U_N & U_{N-1} & \cdots & U_1 & \mathcal{Q}_1' & \Lambda_1 & \cdots & \Lambda_{M-N-3} & \Lambda_{M-N-2} \\ 0 & 0 & U_N & \cdots & U_2 & U_1 & \mathcal{Q}_1' & \cdots & \Lambda_{M-N-4} & \Lambda_{M-N-3} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & U_N & U_{N-1} & U_{N-2} & \cdots & \mathcal{Q}_1' & \Lambda_1 \\ 0 & 0 & 0 & \cdots & 0 & U_N & U_{N-1} & \cdots & U_1 & \mathcal{Q}_1' \end{bmatrix} \tag{14}$$

The matrix $\mathcal{Q}_1'$ is given below where $\Lambda_{0,1} = \sum_{i=0}^{M-1} p_{i+1}^0 = \sum_{r=1}^{M} \alpha_r q^r$.

$$\mathcal{Q}_1' = \begin{bmatrix} \lambda\Lambda_{0,1} - (\lambda+\mu) & 0 & \cdots & 0 & \cdots & 0 \\ \lambda\Lambda_{0,1} & -(\lambda+\mu) & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \lambda\Lambda_{0,1} & 0 & \cdots & 0 & \cdots & 0 \\ c\lambda\Lambda_{0,1} & 0 & \cdots & -(\lambda+\mu) & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ c\lambda\Lambda_{0,1} & 0 & \cdots & 0 & \cdots & 0 \\ d\lambda\Lambda_{0,1} & 0 & \cdots & 0 & \cdots & -(\lambda+\mu) \end{bmatrix} \tag{15}$$

$$B_1 = \begin{bmatrix} \mathcal{Q}_1' + U & \Lambda_1 & \Lambda_2 & \cdots & \Lambda_{M-N-2} & \Lambda_{M-N-1} & \Lambda_{M-N} & \cdots & \Lambda_{M-2} & \Lambda_{M-1} \\ U & \mathcal{Q}_1' & \Lambda_1 & \cdots & \Lambda_{M-N-3} & \Lambda_{M-N-2} & \Lambda_{M-N-1} & \cdots & \Lambda_{M-3} & \Lambda_{M-2} \\ V_1 & U_1 & \mathcal{Q}_1' & \cdots & \Lambda_{M-N-4} & \Lambda_{M-N-3} & \Lambda_{M-N-2} & \cdots & \Lambda_{M-4} & \Lambda_{M-3} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ V_{N-1} & U_{N-1} & U_{N-2} & \cdots & \mathcal{Q}_1' & \Lambda_1 & \Lambda_2 & \cdots & \Lambda_{M-N-2} & \Lambda_{M-N-1} \\ 0 & U_N & U_{N-1} & \cdots & U_1 & \mathcal{Q}_1' & \Lambda_1 & \cdots & \Lambda_{M-N-3} & \Lambda_{M-N-2} \\ 0 & 0 & U_N & \cdots & U_2 & U_1 & \mathcal{Q}_1' & \cdots & \Lambda_{M-N-4} & \Lambda_{M-N-3} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & U_N & U_{N-1} & U_{N-2} & \cdots & \mathcal{Q}_1' & \Lambda_1 \\ 0 & 0 & 0 & \cdots & 0 & U_N & U_{N-1} & \cdots & U_1 & \mathcal{Q}_1' \end{bmatrix} \tag{16}$$

Here $V_j = \mu Q_j I$ for $1 \le j \le$ N-1 and U= μ I. They are matrices of order k+r+1. The basic generator $\mathcal{Q}_A''$ which is concerned with only the bulk production, bulk sale and the CSP, is a matrix of order N(k+r+1) given below in (19) where $\mathcal{Q}_A'' = A_0 + A_1 + A_2$ (17)

Its probability vector w' gives, $w'\mathcal{Q}_A''$ =0 and w 'e = 1 (18)

$$\mathcal{Q}_A'' = \begin{bmatrix} \mathcal{Q}_1' + \Lambda_M & \Lambda_1 & \cdots & \Lambda_{M-N-2} & \Lambda_{M-N-1} & \Lambda_{M-N} + U_N & \cdots & \Lambda_{M-2} + U_2 & \Lambda_{M-1} + U_1 \\ \Lambda_{M-1} + U_1 & \mathcal{Q}_1' + \Lambda_M & \cdots & \Lambda_{M-N-3} & \Lambda_{M-N-2} & \Lambda_{M-N-1} & \cdots & \Lambda_{M-3} + U_3 & \Lambda_{M-2} + U_2 \\ \Lambda_{M-2} + U_2 & \Lambda_{M-1} + U_1 & \cdots & \Lambda_{M-N-4} & \Lambda_{M-N-3} & \Lambda_{M-N-2} & \cdots & \Lambda_{M-4} + U_3 & \Lambda_{M-3} + U_3 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \Lambda_{M-N+2} + U_{N-2} & . & \cdots & . & . & . & \cdots & \Lambda_{M-N} + U_N & \Lambda_{M-N+1} + U_{N-1} \\ \Lambda_{M-N+1} + U_{N-1} & . & \cdots & . & . & . & . & \Lambda_{M-N-1} & \Lambda_{M-N} + U_N \\ \Lambda_{M-N} + U_N & . & \cdots & \mathcal{Q}_1' + \Lambda_M & \Lambda_1 & \Lambda_2 & \cdots & \Lambda_{M-N-2} & \Lambda_{M-N-1} \\ \Lambda_{M-N-1} & \Lambda_{M-N} + U_N & \cdots & \Lambda_{M-1} + U_1 & \mathcal{Q}_1' + \Lambda_M & \Lambda_1 & \cdots & \Lambda_{M-N-3} & \Lambda_{M-N-2} \\ \Lambda_{M-N-2} & \Lambda_{M-N-1} & \cdots & \Lambda_{M-2} + U_2 & \Lambda_{M-1} + U_1 & \mathcal{Q}_1' + \Lambda_M & \cdots & \Lambda_{M-N-4} & \Lambda_{M-N-3} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \Lambda_2 & \Lambda_3 & \cdots & \Lambda_{M-N} + U_N & \Lambda_{M-N+1} + U_{N-1} & \Lambda_{M-N+2} + U_{N-2} & \cdots & \mathcal{Q}_1' + \Lambda_M & \Lambda_1 \\ \Lambda_1 & \Lambda_2 & \cdots & \Lambda_{M-N-1} & \Lambda_{M-N} + U_N & \Lambda_{M-N+1} + U_{N-1} & \cdots & \Lambda_{M-1} + U_1 & \mathcal{Q}_1' + \Lambda_M \end{bmatrix} \tag{19}$$

It is well known that a square matrix in which each row (after the first) has the elements of the previous row shifted cyclically one place right, is called a circulant matrix. It is very interesting to note that the matrix $\mathcal{Q}_A'' = A_0 + A_1 + A_2$ is a block circulant matrix where each block matrix is rotated one block to the right relative to the preceding block partition. The first block row is $W = (\mathcal{Q}_1' + \Lambda_M, \Lambda_1, \Lambda_2, \ldots \Lambda_{M-N-2}, \Lambda_{M-N-1}, \Lambda_{M-N} + U_N, \ldots \Lambda_{M-2} + U_2, \Lambda_{M-1} + U_1)$. This gives as the sum of the blocks $(\mathcal{Q}_1' + \Lambda_M) + \Lambda_1 + \Lambda_2 + \ldots + \Lambda_{M-N-2} + \Lambda_{M-N-1} + \Lambda_{M-N} + U_N + \ldots \ldots + \Lambda_{M-2} + U_2 + \Lambda_{M-1} + U_1 = \mathcal{Q}'_1 = \lambda\,(P' - I)$ where P' is given by (3). So the stationary probability vector of $\mathcal{Q}'_1$ is w. This gives w $\mathcal{Q}'_1$ =0 and $w(\mathcal{Q}_1' + \Lambda_M) + w\sum_{i=1}^{M-N-1}\Lambda_i + w\sum_{i=1}^{N}(\Lambda_{M-i} + U_i)$ = 0 which implies (w, w… w, w). W = 0 = (w, w… w, w) W' where W' is the transpose of block-row vector W. Since all blocks, in any block-row are seen somewhere in each and every column block due to block circulant structure, the above equation shows the left eigen vector of the matrix $\mathcal{Q}_A''$ is (w, w…w). Using (18) $w' = \left(\frac{w}{M}, \frac{w}{M}, \frac{w}{M}, \ldots, \frac{w}{M}\right)$. (20)

The stability condition for the CSP adopted production and sale system to have a stationary distribution as per Neuts [8] is the inequality $w' A_0 e < w' A_2$ e.

This means w' A_0 e = $\frac{1}{M} w' (\sum_{n=1}^{M} n\Lambda_n)e < w' A_2$ e = $\frac{1}{M}$ $w'(\sum_{n=1}^{N} nU_n)e = \frac{1}{M}$ $w'.\left(\mu E(\psi), \mu E(\psi), \ldots, \mu E(\psi)\right)$

The inequality reduces to w' ($\sum_{n=1}^{M} n\Lambda_n)e < \mu E(\psi)$. (21)

The left side of inequality (21) may be simplified as follows. Let $\Lambda_n = \lambda\, \Lambda'_n$. It may be noted that $\Lambda'_n e$ for $1 \le n \le M$, is the probability that the CSP allows n products of a bulk production of size χ to join the stocks for sale in various testing states 1 to k of mode I, in various testing states k+1 to k + r of mode II and the testing state k+r+1 of mode III. It is a column vector of type (k + r +1) x 1 of probabilities of n of products joining the stock for sale by adopting the CSP in a bulk production. The column vector $\sum_{n=1}^{M} n\Lambda'_n\, e$ is the expected number of products cleared by the CSP for sale in the modes I, II and III with regard to the interim states 1 to k, k+1 to k + r and k + r + 1 respectively. It is the column vector of type (k + r+1) x 1 and it equals $E(\chi)(p, p,\ldots p, p + q(1\text{-}c), p + q(1\text{-}c),\ldots,p + q(1\text{-}c), p + q(1\text{-}d))'$ where p appears in the first k rows as all products are tested, p + q(1-c) appears in the next r rows since with probability 1-c defectives also enter the stocks for sale along with good products and in the last row the probability c changes to d. So the stability condition for the CSP to have a stationary distribution as proved by Neuts [8] using (21) is

$\lambda\, E(\chi)\, [p + q\,(1\text{-}c)\sum_{i=1}^{r} w_{k+i} + q\,(1\text{-}d)w_{k+r+1}] < \mu\, E(\psi).$ (22)

Let π (n, m, j), for $0 \le n < \infty$, $0 \le m \le M\text{-}1$, for $1 \le j \le k+r+1$ be the stationary probability of the states in (1) and π_n be the vector of type 1xM(k+r+1) be the stationary probability vector as defined for the states of the matrix (7) for $n \ge 0$ when the inequality (22) is satisfied. Then stationary probability vector $\pi = (\pi_0, \pi_1, \pi_3, \ldots\ldots)$ satisfies the equations $\pi Q_A = 0$ and $\pi e = 1$ (23)

From (23), it can be seen that $\pi_0 B_1 + \pi_1 A_2 = 0$ and $\pi_{n-1}A_0 + \pi_n A_1 + \pi_{n+1}A_2 = 0$, for $n \ge 1$ (24)

Introducing the rate matrix R as the minimal non-negative solution of $A_0 + RA_1 + R^2A_2 = 0$ (25)

it can be proved using Neuts [8] that the stationary probability vector π_n satisfies $\pi_n = \pi_0 R^n$ for $n \ge 1$ (26)

Using (24) and (26), π_0 satisfies $\pi_0\, [B_1 + RA_2] = 0$ (27)

The vector π_0 can be calculated up to multiplicative constant by (27). From (23) and (26) $\pi_0\, (I - R)^{-1} e = 1.$ (28)

Replacing the first column of the matrix multiplier of π_0 in equation (27) by the column vector multiplier of π_0 in (28), a matrix which is invertible may be obtained. The first row of the inverse of that same matrix is π_0 and this gives along with (26) all the stationary probabilities of the system. The matrix R given in (25) is computed by substitutions in the recurrence relation starting with $R(0) = 0$;

$R(n + 1) = -A_0A_1^{-1} - R^2(n)A_2A_1^{-1}$, $n \ge 0$ (29)

The iteration may be terminated to get a solution of R at an approximate level where $\|R(n + 1) - R(n)\| < \varepsilon$

2.3. Performance Measures

(1) The probability of the stock level (S = s), P(S = s), can be seen as follows. Let $n \ge 0$ and m for $0 \le m \le M\text{-}1$ be non-negative integers such that s = n M + m. Then using (21) (22) and (23) it is noted that P (S=s) $=\sum_{i=1}^{k+r+1} \pi\,(n, m, i,)$, where s = n M + m and P (Stock level is 0) = P (S=0) = $\sum_{i=1}^{k+r+1} \pi(0, 0, i)$.

(2) The expected stock level E(S) can be calculated as follows. $E(S) = \sum_{n=0}^{\infty}\sum_{m=0}^{M-1}\sum_{j=1}^{k+r+1} \pi(n, m, j)\ (Mn + m)$ $=\sum_{n=0}^{\infty}\pi_n$. (Mn… Mn, Mn+1… Mn+1, Mn+2…Mn+2… Mn+M-1… Mn+M-1) where in the multiplier vector Mn appears k+r+1 times, Mn+1 appears k+r+1 times and so on and finally Mn+M-1appears k+r+1 times. So $E(S) = M\sum_{n=0}^{\infty} n\pi_n\, e + \pi_0 (I - R)^{-1}\xi$. Here M(k+r+1)x1 column vector $\xi = (0, \ldots 0, 1, \ldots, 1, 2, \ldots, 2, \ldots, M - 1, \ldots, M - 1)'$. This gives $E(S) = \pi_0 (I - R)^{-1}\xi + M\pi_0 (I - R)^{-2} Re$ (30)

The expected number of defectives in the stock when CSP is adopted using (6) is E(Def) = q(1-f)E(S)

(3) Variance of stock level can be seen using VAR (S) = E (S^2) – $E(S)^2$. Let η be column vector $\eta = [0, \ldots 0, 1^2, \ldots 1^2\ 2^2, \ldots 2^2, \ldots\ (M-1)^2, \ldots\ (M-1)^2]'$ of type M(k+r+1) x 1. Then it can be seen that the second moment, $E(S^2) = \sum_{n=0}^{\infty}\sum_{m=0}^{M-1}\sum_{j=1}^{k+r+1} \pi(n, m, j)\, [M\, n + m]^2 = M^2[\sum_{n=1}^{\infty} n(n-1)\pi_n\, e + \sum_{n=0}^{\infty} n\pi_n\, e] + \sum_{n=0}^{\infty}\pi_n\eta\ + 2M\sum_{n=0}^{\infty} n\, \pi_n\, \xi$.

This gives $E(S^2) = M^2[\pi_0 (I - R)^{-3} 2R^2\, e + \pi_0 (I - R)^{-2} Re] + \pi_0 (I - R)^{-1}\eta + 2M\, \pi_0 (I - R)^{-2} R\xi$ (31)

(4)The rate of entry of defective products when the CSP is adopted is $\lambda\, q\, E(\chi)\,(1\text{-}f)$ as seen in (6) where f is given by (5). The rate of entry of defective products at various states $\underline{n}$ is given by $\lambda\, q\, E(\chi)\, \pi_n\tau$ where τ is a column vector of type M(k+r+1) x 1 and $\tau = (\zeta, \zeta,\ldots\zeta)'$ with $\zeta = (0,\ldots 0,(1\text{-}c), (1\text{-}c),\ldots(1\text{-}c), (1\text{-}d))$ is vector of type 1 x (k+r+1) in which 0 appears k times,1-c appears r times and 1-d appears once. The rate of entry when the stock level is s =M n + m is $\pi(n, m)\zeta$ where $\pi(n, m) = (\pi(n,m,1), \pi(n,m,2)\ldots\pi(n,m,k+r+1))$.

III. Model (B). Continuous Sampling Plan with Three Modes where Maximum Production Size Less than Maximum Sale Size

In this Model (B) the dual of Model (A), namely the case, M < N is treated. (When M =N both models are applicable and one can use any one of them). In Model (B) the assumption (iv) of Model (A) is alone replaced.

3.1Assumption.

(iv)The maximum size of production M is less than the maximum size of sale N.

3.2Analysis

Since this model is dual, the analysis is similar to that of Model (A). The differences are noted below. The state space of the chain X (t) = {(n, m, j): for $0 \le m \le N-1$ for $1 \le j \le k+r+1$ and $0 \le n < \infty$}. (32)
The system X (t) is in the state (n, m, j) for $1 \le j \le k$, $n \ge 0$ and $0 \le m \le N-1$, when n N + m products are available for sale, the CSP inspection mode is I and it has found no defective in (j-1) consecutive bulk productions. The system is in the state (n, m, k + i) for $1 \le i \le r$, $n \ge 0$, $0 \le j \le N-1$, when n N + m products are available for sale, the CSP inspection mode is II and the CSP has found no defective in (i-1) consecutive bulk productions in mode II. The system is in the state (n, m, k + r +1) when n N + m products are available for sale for $n \ge 0$ and $0 \le m \le N-1$ and the CSP inspection mode is III. The infinitesimal generator Q_B of the model has the same block partitioned structure given in (7) for Model (A) but the inner matrices are of different orders and elements.

$$Q_B=\begin{bmatrix} B'_1 & A'_0 & 0 & 0 & . & . & . & \cdots \\ A'_2 & A'_1 & A'_0 & 0 & . & . & . & \cdots \\ 0 & A'_2 & A'_1 & A'_0 & 0 & . & . & \cdots \\ 0 & 0 & A'_2 & A'_1 & A'_0 & 0 & . & \cdots \\ 0 & 0 & 0 & A'_2 & A'_1 & A'_0 & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots & \ddots & \ddots & \ddots \end{bmatrix} \tag{33}$$

In (33) the states of the matrices are listed lexicographically as $\underline{0}, \underline{1}, \underline{2}, \underline{3}, \ldots . \underline{n}$, ...Here the state vectors are given as follows. $\underline{n}$ = ((n, 0, 1),…(n, 0, k+r+1),(n, 1, 1),…(n, 1, k+r+1),(n, 2, 1),…(n, 2, k+r+1),…(n, N-1, 1),…(n, N-1, k+r+1)), for $0 \le n < \infty$. The matrices, B'_1, A'_0, A'_1 and A'_2 are all of order N (k+r+1). The matrices B'_1 and A'_1 have negative diagonal elements and their off diagonal elements are non- negative. The matrices A'_0 and A'_2 have nonnegative elements. They are all given below. Letting Λ_j, for $1 \le j \le M$ and U_j, V_j for $1 \le j \le N$, as in (12), (13), (16), U as in (16) and letting $\mathcal{Q}'_1$ as in (15) the partitioning matrices are defined as follows

$$A'_0=\begin{bmatrix} 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ \Lambda_M & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ \Lambda_{M-1} & \Lambda_M & \cdots & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \Lambda_2 & \Lambda_3 & \cdots & \Lambda_M & 0 & 0 & \cdots & 0 \\ \Lambda_1 & \Lambda_2 & \cdots & \Lambda_{M-1} & \Lambda_M & 0 & \cdots & 0 \end{bmatrix} \tag{34}$$

$$A'_2=\begin{bmatrix} U_N & U_{N-1} & U_{N-2} & \cdots & U_3 & U_2 & U_1 \\ 0 & U_N & U_{N-1} & \cdots & U_4 & U_3 & U_2 \\ 0 & 0 & U_N & \cdots & U_5 & U_4 & U_3 \\ 0 & 0 & 0 & \ddots & U_6 & U_5 & U_4 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & U_N & U_{N-1} & U_{N-2} \\ 0 & 0 & 0 & \cdots & 0 & U_N & U_{N-1} \\ 0 & 0 & 0 & \cdots & 0 & 0 & U_N \end{bmatrix} \tag{35}$$

$$A'_1=\begin{bmatrix} \mathcal{Q}'_1 & \Lambda_1 & \Lambda_2 & \cdots & \Lambda_M & 0 & 0 & \cdots & 0 & 0 \\ U_1 & \mathcal{Q}'_1 & \Lambda_1 & \cdots & \Lambda_{M-1} & \Lambda_M & 0 & \cdots & 0 & 0 \\ U_2 & U_1 & \mathcal{Q}'_1 & \cdots & \Lambda_{M-2} & \Lambda_{M-1} & \Lambda_M & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ U_{N-M-1} & U_{N-M-2} & U_{N-M-3} & \cdots & \mathcal{Q}'_1 & \Lambda_1 & \Lambda_2 & \cdots & \Lambda_{M-1} & \Lambda_M \\ U_{N-M} & U_{N-M-1} & U_{N-M-2} & \cdots & U_1 & \mathcal{Q}'_1 & \Lambda_1 & \cdots & \Lambda_{M-2} & \Lambda_{M-1} \\ U_{N-M+1} & U_{N-M} & U_{N-M-1} & \cdots & U_2 & U_1 & \mathcal{Q}'_1 & \cdots & \Lambda_{M-3} & \Lambda_{M-2} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ U_{N-2} & U_{N-3} & U_{N-4} & \cdots & U_{N-M-2} & U_{N-M-3} & U_{N-M-2} & \cdots & \mathcal{Q}'_1 & \Lambda_1 \\ U_{N-1} & U_{N-2} & U_{N-3} & \cdots & U_{N-M-1} & U_{N-M-2} & U_{N-M-1} & \cdots & U_1 & \mathcal{Q}'_1 \end{bmatrix} \tag{36}$$

$$B'_1=\begin{bmatrix} \mathcal{Q}'_1+U & \Lambda_1 & \Lambda_2 & \cdots & \Lambda_M & 0 & 0 & \cdots & 0 & 0 \\ U & \mathcal{Q}'_1 & \Lambda_1 & \cdots & \Lambda_{M-1} & \Lambda_M & 0 & \cdots & 0 & 0 \\ V_1 & U_1 & \mathcal{Q}'_1 & \cdots & \Lambda_{M-2} & \Lambda_{M-1} & \Lambda_M & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ V_{N-M-2} & U_{N-M-2} & U_{N-M-3} & \cdots & \mathcal{Q}'_1 & \Lambda_1 & \Lambda_2 & \cdots & \Lambda_{M-1} & \Lambda_M \\ V_{N-M-1} & U_{N-M-1} & U_{N-M-2} & \cdots & U_1 & \mathcal{Q}'_1 & \Lambda_1 & \cdots & \Lambda_{M-2} & \Lambda_{M-1} \\ V_{N-M} & U_{N-M} & U_{N-M-1} & \cdots & U_2 & U_1 & \mathcal{Q}'_1 & \cdots & \Lambda_{M-3} & \Lambda_{M-2} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ V_{N-3} & U_{N-3} & U_{N-4} & \cdots & U_{N-M-2} & U_{N-M-3} & U_{N-M-2} & \cdots & \mathcal{Q}'_1 & \Lambda_1 \\ V_{N-2} & U_{N-2} & U_{N-3} & \cdots & U_{N-M-1} & U_{N-M-2} & U_{N-M-1} & \cdots & U_1 & \mathcal{Q}'_1 \end{bmatrix} \tag{37}$$

$$\mathcal{Q}''_B=\begin{bmatrix} \mathcal{Q}'_1+U_N & \Lambda_1+U_{N-1} & \cdots & \Lambda_{M-1}+U_{N-M+1} & \Lambda_M+U_{N-M} & U_{N-M-1} & \cdots & U_2 & U_1 \\ U_1 & \mathcal{Q}'_1+U_N & \cdots & \Lambda_{M-2}+U_{N-M+2} & \Lambda_{M-1}+U_{N-M+1} & \Lambda_M+U_{N-M} & \cdots & U_3 & U_2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ U_{N-M-2} & U_{N-M-3} & \cdots & \mathcal{Q}'_1+U_N & \Lambda_1+U_{N-1} & \Lambda_2+U_{N-2} & \cdots & \Lambda_M+U_{N-M} & U_{N-M-1} \\ U_{N-M-1} & U_{N-M-2} & \cdots & U_1 & \mathcal{Q}'_1+U_N & \Lambda_1+U_{N-1} & \cdots & \Lambda_{M-1}+U_{N-M+1} & \Lambda_M+U_{N-M} \\ \Lambda_M+U_{N-M} & U_{N-M-1} & \cdots & U_2 & U_1 & \mathcal{Q}'_1+U_N & \cdots & \Lambda_{M-2}+U_{N-M+2} & \Lambda_{M-1}+U_{N-M+1} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \Lambda_2+U_{N-2} & \Lambda_3+U_{N-3} & \cdots & U_{N-M-1} & U_{N-M-2} & U_{N-M-3} & \cdots & \mathcal{Q}'_1+U_N & \Lambda_1+U_{N-1} \\ \Lambda_1+U_{N-1} & \Lambda_2+U_{N-2} & \cdots & \Lambda_M+U_{N-M} & U_{N-M-1} & U_{N-M-2} & \cdots & U_1 & \mathcal{Q}'_1+U_N \end{bmatrix} \tag{38}$$

The basic generator which is concerned with only production, sale and the CSP is $Q''_B = A'_0 + A'_1 + A'_2$. This is also block circulant. Using similar arguments given for Model (A) it can be seen that its probability vector is $\left(\frac{w}{N}, \frac{w}{N}, \frac{w}{N}, \dots, \frac{w}{N}\right)$ and the stability condition remains the same.

Following the arguments given for Model (A), one can find the stationary probability vector for Model (B) also in matrix geometric form. All performance measures given in section 2.3 including the expectation and the variance for Model (B) have the same form as given in Model (A) except M is replaced by N.

IV. Numerical Cases

Six numerical cases two each for three models namely (i) M=4, N=4 (ii) M=4, N-3 and (iii) M=3, N=4 are treated using the analysis given for models (A) and (B) where M and N are the maximum production and sale sizes respectively. The bulk production requirement of k number of consecutive productions without defective for mode I is k= 3 and in mode II r=4 bulk productions are tested with probability c =.5 and in mode III the probability of testing a bulk is d =.8. The probability p of a product is good .8 and the defective probability q=.2.

For the three cases (i), (ii) and (iii) above, two different values for the production and sale parameters (λ, μ) are fixed as (λ=2, μ=3) and (λ=2.5, μ=3).For the cases (i) and (ii) the bulk production size probabilities are α_1= .5, α_2= .2, α_3=.2 α_4=.1 and for case (iii) they are α_1= .5, α_2= .3, α_3=.2 α_4=0. For the cases (i) and (iii) the bulk sale size probabilities are β_1= .4, β_2= .2, β_3=.2, β_4=.2 and for case (ii) they are β_1= .4, β_2= .4, β_3=.2, β_4 =0. As same CSP is adopted for the six cases the fraction of the bulk productions that are inspected out of all bulk productions is seen as f= 0.795440162.

The pairs of expected size of production and expected size of sale (E (χ), E (ψ)) for the three cases are respectively obtained as (1.9, 2.2), (1.9, 1.8) and (1.7, 2.2). Table1 presents the results obtained for various measures for Model (A) and Model (B) for the six numerical cases. Fifteen iterations are performed for finding the rate matrix R and the norm values are presented in row S.No.15. Various individual probability levels when stock levels are 0,1,2,3 and above 3 are presented in S.Nos. 1,2,3,4 and 5. The block level probabilities for blocks 0,1,2,3 and above 3 are listed in S.Nos. 6,7,8,9 and 10. The rate of entry of defective products (REDP) is λ q E (χ) if CSP is not adopted for the six cases are presented as S.No. 23. The REDP when the CSP is adopted is presented in S.No. 22 and its value calculated using iterated R matrix is presented in S.No. 21which shows substantial reduction in the rate of entry of defectives for sale. The expected stock level and expected defective products in the stocks are given in S.No 11 and S.No.24.

The ratios E (Def) / E(S) are .04 approximately and less than the defective probability q= .2. This ratio along with [S.nos.12, 14, 21, 22,24], standard deviations , Coefficient of variation, estimated Rate of Entry of Defective Products by rate matrix R, the REDP of the CSP and the E(Defectives) indicate the risk involved in adopting the CSP is very much less. Figure (1) presents various probabilities for stock levels and block levels. Figure (2) gives the rates of Entries of defectives for different situations and E (Defectives). The reduction in the rate of entry of defective products when all the products are allowed for sale without CSP is substantial as seen inFigure2.

Table.1 Results Obtained for the Six Numerical Cases

S.No	(λ,μ) (M,N)	(2,3) (4, 4)	(2.5, 3) (4, 4)	(2, 3) (4, 3)	(2.5,3) (4,3)	(2,3) (3,4)	(2.5,3) (3,4)
1	P(S=0)	0.40595741	0.29680114	0.34062481	0.21109948	0.4431943	0.34172495
2	P(S=1)	0.13394278	0.11597578	0.12634731	0.0937049	0.14805564	0.13508528
3	P(S=2)	0.10499984	0.09796881	0.10361331	0.08373954	0.12076333	0.11827768
4	P(S=3)	0.09005805	0.08932819	0.09109751	0.07922874	0.09088518	0.09660198
5	P(S>3)	0.26504192	0.39992608	0.33831706	0.53222735	0.19710154	0.3083101
6	π0e	0.73495808	0.60007392	0.66168294	0.46777265	0.80289846	0.6916899
7	π1e	0.180997	0.21913781	0.20607769	0.22459328	0.15017406	0.19979037
8	π2e	0.05718609	0.09871757	0.08028139	0.12941185	0.03577678	0.07034883
9	π3e	0.01827105	0.04480441	0.03153684	0.07495951	0.00850012	0.02474195
10	P(Block>3)	0.00858778	0.03726629	0.02042115	0.10326271	0.00265058	0.01342896
11	E(S)	2.47107994	4.00419884	3.23953339	6.25960994	1.90790959	2.92867633
12	σ(S)	3.2192414	4.63622292	3.92559153	6.71303546	2.58199365	3.53345006
13	VAR(S)	10.3635152	21.494563	15.4102688	45.064845	6.66669121	12.4852694
14	CV	1.302767	1.1578403	1.121177684	1.0724367	1.35331027	1.20650071
15	NORM	4.1086E-07	1.24840E-05	6.62810E-06	1.34820E-04	3.69450E-08	1.8304E-06
16	REDP block 0	0.10910589	0.10958904	0.09723651	0.08397256	0.11523945	0.12192548
17	REDP block 1	0.03128588	0.04581699	0.03504945	0.04588092	0.02622781	0.04201566
18	REDP block 2	0.01023695	0.02120566	0.01404483	0.02699571	0.00651131	0.01527112
19	REDP block 3	0.00328854	0.00965989	0.00553833	0.01567411	0.00155052	0.00538023
20	Sum REDP>3	0.00154778	0.0080412	0.00358919	0.02160175	0.00048392	0.00292165
21	Total REDP	0.15546503	0.19431277	0.1554583	0.19412506	0.15001301	0.18751413
22	REDP for CSP	0.15546548	0.19433185	0.15546548	0.19433185	0.15001304	0.1875163

23	Non CSP REDP	0.76	0.95	0.76	0.95	0.68	0.85
24	E(Def)	0.10109674	0.16381965	0.13253568	0.25609296	0.0841798	0.12921754

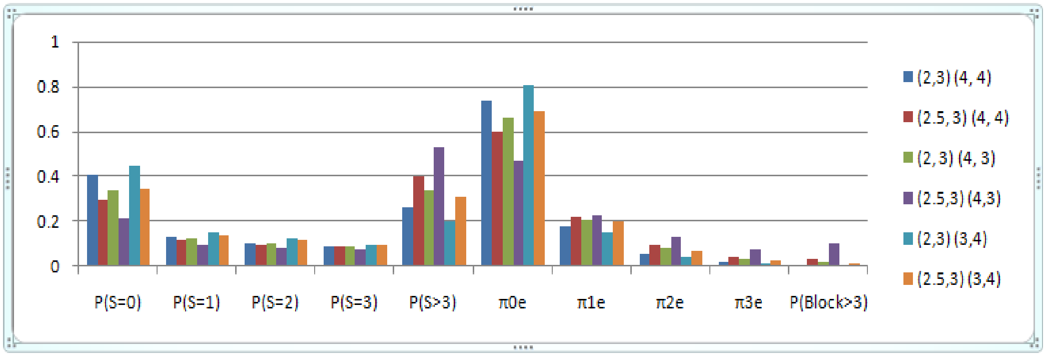

Figure.1.Various Probabilities for Six Numerical Cases for Individuals and Bulks

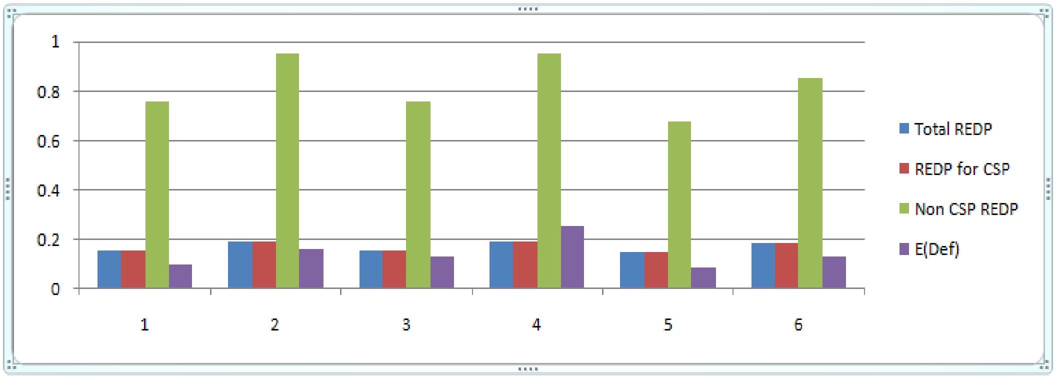

Figure.2.Rate of Entries of defectives for different situations and E (Defectives)

V. Conclusion

In this paper a continuous sampling plan (CSP) has been studied in preventing the entry of defective products into stocks for sale in bulk production and bulk sale system. The CSP considered here has three modes. In mode I every product in a bulk production is inspected until k consecutive good bulk productions are found. At this point the CSP changes its inspection mode to II of inspecting the next r bulk productions where every bulk is inspected with probability c until a defective product appears. When a defective one appears, the CSP changes its mode to I and if no defective product is noticed in those r bulk productions, the CSP changes to mode III where the bulk production is inspected with probability d. In mode III, if a defective is found, the CSP changes to I and if no defective is found then it changes to mode II. In the above continuous sampling plan, the probabilistic inspection procedures reduce the number of inspections considerably with a view to reduce the inspection cost. Giving different values for production rates, sale rates, bulk production and bulk sale sizes six numerical cases are treated. The rate of entry of defective products is reduced by the continuous sampling plan. All the risks measures involved in the CSP are obtained. For future studies models with catastrophic sale may produce further interesting results.

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